Optimal Public Debt
with Life Cycle Motives

William Peterman
Federal Reserve Board

Erick Sager
Bureau of Labor Statistics

QSPS

May 20, 2016

**The views herein are the authors’ and not necessarily those of the BLS, US DOL, Board of Governors or their staffs.**
Motivation

Q: What level of debt should the Government hold?

Government Debt

- Welfare Costs:
  - Crowds out capital ⇒ lower output
  - Financed by distortionary taxes
- Welfare Benefits (financial liquidity):
  - ↑ return to savings ⇒ reduces cost of holding precautionary savings


- Incomplete markets, infinitely lived
- Optimal debt = \( \frac{2}{3} \) of output
- Ignores life cycle
  - Agents transition through different phases of life cycle
**This Paper**

**Question:** What is optimal level of gov’t debt in life cycle model?

**Effect of Life Cycle on Optimal Public Debt**

- Large effect on optimal public debt
  - Life cycle model: savings = 160% of output
  - Infinitely lived agent model: debt = 87% of output
- Welfare of adopting misspecified optimal tax policy: CEV = 3.5%
- Different policies due to different phases of life cycle
Mechanism: Example (I)

<table>
<thead>
<tr>
<th>Age</th>
<th>Accumulation</th>
<th>Stationary Phase</th>
<th>Decumulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consumption</td>
<td>Savings</td>
<td>Hours</td>
</tr>
</tbody>
</table>

- Life cycle all three phases; Infinitely lived only one phase
- Changing prices has different effects
Mechanism: Example (II)

Affect of Gov’t Debt on Factor Prices:
- Decreases Government Debt (increases Gov’t. savings)
- Crowds in Productive Capital
- Interest rate ↓
- Wage ↑

Infinitely Lived Agent Model
- Only stationary phase
- Lower interest rate decreases liquidity

Life Cycle Model
- Accumulation, Stationary, Decumulation Phases
- Higher wage more accommodative during accumulation phase
Effects of government debt with incomplete markets

1. Steady State
   - Aiyagari & McGrattan (1998) - optimal debt large
   - Floden (2001) - if transfers below optimal then ↑ gov’t debt
   - Dyrda & Pedroni (2015) - if taxes optimized then less debt optimal
   - Winter & Roehrs (2015) - skewed wealth leads to gov’t savings being optimal

2. Transition
   - Dydra & Pedroni (2015); Winter and Roehrs (2015); Desbonnet & Weitzenblum (2012): Considerable welfare costs in transition

Previous analysis of question done with infinitely lived agent model
Outline

1. Introduction
2. Life cycle Model with Public Debt
3. Calibration
4. Results
5. Conclusion
Life cycle Model with Public Debt
Overview of Model

- General Equilibrium incomplete markets model
- Overlapping generations of heterogeneous agents
- Idiosyncratic uninsurable shocks:
  - Agent's labor productivity
  - Unemployment spells
  - Mortality
- Labor is supplied elastically
- Agents choose when to retire
- Social Security and UI programs modeled similar to U.S.
Production

- Representative Firm:
  - Large number of firms
  - Sell consumption good
  - Perfectly competitive product market

- Technology:
  - Cobb-Douglas: \( Y = K^\varsigma L^{1-\varsigma} \)
  - No aggregate uncertainty

- Resource Constraint: \( C + (K' - (1 - \delta)K) + G = Y \)
Demographics

- $J$ overlapping generations
- $s_j$ probability of living to $j + 1$ given one is alive in $j$
- Remaining assets are accidental bequests ($T_{rt}$).
- If still alive agents die with certainty at age $J$
- Agents retire at endogenously determined age ($J_{ret}$), irreversible
  - $J_{ret} \in [\underline{J}_{ret}, \overline{J}_{ret}]$
- Population growth $= g_n$
Labor Earnings (I)

Earnings: \( y_{ij} = w e_{ij} h_{ij} (1 - \bar{h}_{ij}) \)

- Labor productivity, \( e_{ij} \)
- Choice of hours, \( h_{ij} \in [0, 1] \)
- Unemployment shocks, \( \bar{h}_{ij} \)

Labor Productivity: \( \log(e_{ij}) = \theta_j + \alpha_i + \epsilon_{ij} + \nu_{ij} \)

- Age-profile: \( \{\theta_j\}_{j=1}^{\bar{J}_{ret}} \)
- Idiosyncratic type: \( \alpha_i \sim iid \mathcal{N}(0, \sigma_{\alpha}^2) \)
- Transitory shock: \( \epsilon_{ij} \sim iid \mathcal{N}(0, \sigma_{\epsilon}^2) \)
- Persistent shock: \( \nu_{ij+1} = \rho \nu_{ij} + \eta_{ij+1} \)

\[ \eta_{ij+1} \sim iid \mathcal{N}(0, \sigma_{\nu}^2) \]
\[ \nu_{i1} = 0 \]
Labor Earnings (II)

**Earnings:** \( y_{ij} = w_{e_{ij}} h_{ij} (1 - \bar{h}_{ij}) \)

- Labor productivity, \( e_{ij} \)
- Choice of hours, \( h_{ij} \in [0, 1] \)
- Unemployment shocks, \( \bar{h}_{ij} \)

**Unemployment Shock:** \( \bar{h}_{i,j} \)

- Fraction of period unemployed
  - Either 0 or \( d_j \)
  - Probability of non zero: \( p_j \)
  - Probability and duration are age specific
- Receive unemployment benefits
  - \( b_{ui}(w_{e_{ij}}) \)
Asset Markets

Incomplete Asset Markets:

- Incomplete w.r.t. idiosyncratic productivity risk, unemployment risk, mortality risk
- Agents save using non-contingent bond, $a \geq 0$
- Before tax rate of return, $r$

Market Clearing: $A = K + B$

- Supply = Aggregate Savings
- Demand = Productive Capital ($K$) + Gov’t Debt ($B$)
Government Policy

Budget Constraint:

\[ G + UI + rB = (B' - B) + \Upsilon_y \]

1. G: Consumes in an unproductive sector
2. UI: Pays insurance when unemployed
3. B: Borrows or saves at interest \( r \)
4. \( \Upsilon_y \): Finances with progressive income taxation

Self Financing Programs:

5. Runs Social Security Program
6. Distributes accidental bequests
Social Security

Overview:

- Finances SS with a flat tax on labor income $\tau^{ss}$
- Half payed by employer (up to cap)
- Pays benefit $b^{ss}_i$ based on
  - Past income AIME: $x_i$
  - Age of retirement: $J_{ret}$
Competitive Equilibrium

1. Agents optimize utility s.t. budget constraint
2. Prices set by marginal product of capital and labor
3. Social Security budget clears
4. General Government budget clears
5. Capital and labor market clear
6. Stationary distribution of individuals over state space
   - Accounting for GDP growth: $g$

Dynamic Programming
Calibration
Firm

Production: \( Y = K^\zeta N^{1-\zeta} \)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Share</td>
<td>( \zeta )</td>
<td>.36</td>
<td>CKK</td>
</tr>
<tr>
<td>Depreciation</td>
<td>( \delta )</td>
<td>.0833</td>
<td>( \frac{I}{Y} = 25.5% )</td>
</tr>
<tr>
<td>Growth</td>
<td>( g )</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>
Demographics

- Agents enter the model at age 20
- \( s_j \) - Bell and Miller (2002)
- Remaining agents die with certainty age 100(\( J \))
- Population growth: \( g_n = 1.1\% \)
Idiosyncratic Labor Productivity

Labor Productivity:  \( \log(e_{ij}) = \theta_j + \alpha_i + \nu_{ij} + \epsilon_{ij} \)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence Shock</td>
<td>( \sigma^2_{\nu} )</td>
<td>0.017</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Persistence</td>
<td>( \rho )</td>
<td>0.958</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Ability</td>
<td>( \sigma^2_{\alpha} )</td>
<td>0.065</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Transitory Shock</td>
<td>( \sigma^2_{\epsilon} )</td>
<td>0.081</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Age Profile</td>
<td>( { \theta_j }<em>{j=1}^{J</em>{ret}} )</td>
<td></td>
<td>Kaplan (2012)</td>
</tr>
</tbody>
</table>
Unemployment Rates and Duration by Age
(March CPS, 1990-2005)

Age

Unemployment Rate (right axis)

Average Unemployment Duration

Weeks

Pct
Unemployment Insurance

- Base Benefit: $b_{ui}(we) = rr(we)we \ h_{\text{average}} \overline{h}$
- Replacement rate: $rr(we) = \phi_{ui,0} \ln(we) \phi_{ui,1}$
- $b_{ui} \in [.13 \times \text{avg. earnings} \times \overline{h}, 1.1 \times \text{avg. earnings} \times \overline{h}]$
**Preferences**

Preferences: \( u(c) + v(h, \bar{h}) = \frac{c^{1-\gamma}}{1-\gamma} - \chi_1 \frac{((1-\bar{h})^\xi h)^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} - \chi_2 1(j < J_{ret}) \)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional Discount</td>
<td>( \beta )</td>
<td>1.0</td>
<td>( \frac{K}{Y} = 2.7 )</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \gamma )</td>
<td>2.2</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Frisch Elasticity</td>
<td>( \sigma )</td>
<td>0.41</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Utility during unemployment</td>
<td>( \xi )</td>
<td>0</td>
<td>Kaplan (2012)</td>
</tr>
<tr>
<td>Disutility to Labor</td>
<td>( \chi_1 )</td>
<td>70.0</td>
<td>( \text{Avg. } h_j = \frac{1}{3} )</td>
</tr>
<tr>
<td>Fixed Cost to Working</td>
<td>( \chi_2 )</td>
<td>1.105</td>
<td>(70% \text{ retire by } J_{nr} )</td>
</tr>
</tbody>
</table>
## Government

Income tax function: $T(\tilde{y}_t; \tau_0, \tau_1, \tau_2) = \tau_0(\tilde{y}_t - (\tilde{y}_t - \tau_1 + \tau_2)^{-\frac{1}{\tau_1}})$

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Tax</td>
<td>$\tau_0$</td>
<td>0.258</td>
<td>Gouveia &amp; Strauss (1994)</td>
</tr>
<tr>
<td>Progressiveness</td>
<td>$\tau_1$</td>
<td>0.768</td>
<td>Gouveia &amp; Strauss (1994)</td>
</tr>
<tr>
<td>Progressiveness</td>
<td>$\tau_2$</td>
<td>8.99</td>
<td>Balance budget</td>
</tr>
<tr>
<td>Gov’t Consumption</td>
<td>$\frac{G}{Y}$</td>
<td>15.5%</td>
<td>Data</td>
</tr>
<tr>
<td>Debt to GDP</td>
<td>$\frac{B}{Y}$</td>
<td>$\frac{2}{3}$</td>
<td>Aiyagari &amp; McGrattan (1998)</td>
</tr>
<tr>
<td>UI</td>
<td>$\phi_{ui,0}$</td>
<td>0.38</td>
<td>March CPS</td>
</tr>
<tr>
<td>UI</td>
<td>$\phi_{ui,1}$</td>
<td>-0.80</td>
<td>March CPS</td>
</tr>
</tbody>
</table>
Results

Outline:

1. Illustrative Example
2. Social Welfare Function
3. Optimal Policy
4. Welfare Effects
5. Decompose Mechanisms
6. Transfer Programs & Borrowing Constraints
7. Sensitivity to Social Welfare Function
Illustrative Example

- Infinitely lived: only stationary
- Life cycle: three phases
Accumulation Phase

- Accumulating assets
- Labor income more important
Stationary Phase

- May not exist (shorter) in life cycle model
- Only phase in infinitely lived
Comparative Static: Holding less debt

- Less crowd-out $\rightarrow$ more productive capital
  - Higher wage, $w = (1 - \alpha)(K/L)^\alpha$
  - Lower interest rate $r = \alpha(K/L)^{\alpha-1} - \delta$

- During *accumulation phase*:
  - Labor earnings is majority of income
  - Higher wage increases income
  - Life cycle only

- During *stationary phase*:
  - Lower interest rate decreases interest income
  - Accumulate fewer total assets (less liquid)
  - Less emphasis in life cycle model
Computational Experiment

Choose B to maximize social welfare function:

\[ S(v, \lambda) \equiv \max_B E_0 v_0(a, \epsilon, x; B) \] (1)

Utilitarian SWF: maximizing expected utility of newborn

- Adjust taxes to clear budgets
  - \( \tau_{ss} \) to satisfy Social Security budget
  - \( \tau_0 \) to clear government general budget (G held fixed)
Experiment 1: Optimal Policy

- Compute optimal policy in life cycle model
- Compute optimal policy in infinitely lived agent analogue
Experiment 1: Optimal Policy

Optimal Policy:
- Life cycle - savings = 160% of output
- Infinitely lived - debt = 87% of output
Experiment 2: Welfare Decomposition

- Consumption equivalence (CEV)
  - Optimal (160% savings) vs optimal from infinitely lived (87% debt)
- Decompose into:
  1. **Level effect:** difference in aggregate consumption
  2. **Insurance effect:** difference in volatility of consumption paths
  3. **Redistribution effect:** difference in cross-sectional spread
  4. **Labor effect:** difference in consumption-labor substitution
Welfare Decomposition, ex ante

<table>
<thead>
<tr>
<th>Effect</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEV (% Change)</td>
<td>3.47%</td>
</tr>
<tr>
<td>Levels Effect</td>
<td>5.62%</td>
</tr>
<tr>
<td>Insurance Effect</td>
<td>-0.46%</td>
</tr>
<tr>
<td>Redistribution Effect</td>
<td>0.14%</td>
</tr>
<tr>
<td>Labor Disutility Effect</td>
<td>-1.72%</td>
</tr>
</tbody>
</table>

- Optimal policy has strong positive Levels Effect
- Optimal policy somewhat mitigated by labor disutility
Level Effect:
- Higher wages → more consumption early
- Lower r → less consumption later, work longer
The Effect on Life Cycle Profiles

Optimal policy: More government savings, $\uparrow$ wage, $\downarrow$ r

---

### Hours Profile

- Misspecified
- Optimal

### Savings Profile

- Misspecified
- Optimal

### Consumption Profile

- Misspecified
- Optimal
Experiment 3

Decompose the Effect of Life Cycle Features:

- Sequentially remove life cycle features
  1. Age-varying aspects
  2. Demographics
  3. Endowment

- Recalibrate each model

- Calculate optimal policy
## Models

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Retirement</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Soc. Sec</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age H.C.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age Unemp</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Mort. Risk</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pop. Growth</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Life Length</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>400</td>
<td>Infinite</td>
</tr>
<tr>
<td>Save Endow.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Avg. IV</td>
</tr>
</tbody>
</table>

- Age-specific I
- Demographics II-IV
- Endowment V
## Optimal Policy (Age-specific)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal (% of GDP)</td>
<td>160%</td>
<td>173%</td>
<td>287%</td>
<td>307%</td>
<td>360%</td>
<td>100%</td>
</tr>
<tr>
<td>Retirement</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Soc. Sec</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age H.C.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age Unemp</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Mort. Risk</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pop. Growth</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Life Length</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Save Endow.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Avg. IV</td>
</tr>
</tbody>
</table>

↑ optimal savings because work throughout whole life
Competing effects on optimal policy

- Wage more important
- Less building time
## Optimal Policy (Demographics II)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>160%</td>
<td>173%</td>
<td>287%</td>
<td>307%</td>
<td>360%</td>
<td>-100%</td>
<td>-87%</td>
<td></td>
</tr>
<tr>
<td>Retirement</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Soc. Sec</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age H.C.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age Unemp</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Mort. Risk</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pop. Growth</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Life Length</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>400</td>
<td>400</td>
<td>Infinite</td>
</tr>
<tr>
<td>Save Endow.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Avg. IV</td>
<td>Dist.</td>
</tr>
</tbody>
</table>

↑ optimal savings because agents live to older age
Removing mortality lengthens accumulation phase.
Optimal Policy (Demographics III)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal</strong> (% of GDP)</td>
<td>160%</td>
<td>173%</td>
<td>287%</td>
<td><strong>307%</strong></td>
<td>360%</td>
<td>-100%</td>
<td>-87%</td>
</tr>
<tr>
<td>Retirement</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Soc. Sec</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age H.C.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age Unemp</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Mort. Risk</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pop. Growth</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Life Length</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>400</td>
<td>400</td>
<td>Infinite</td>
</tr>
<tr>
<td>Save Endow.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Avg. IV</td>
</tr>
</tbody>
</table>

↑ optimal savings: more old agents affects aggregate dynamics
Increased Population of Old

Elasticity of Private Savings wrt Government Savings

<table>
<thead>
<tr>
<th>Model II</th>
<th>Model III</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.923</td>
<td>-0.900</td>
</tr>
</tbody>
</table>

- Young are more responsive to interest rates changes
- Model III compared to II:
  - Fewer young agents
  - Government savings crowds out less private savings
  - Public saving is more productive
  - Government saves more
Optimal Policy (Demographics IV)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>160%</td>
<td>173%</td>
<td>287%</td>
<td>307%</td>
<td>360%</td>
<td>-100%</td>
<td>-87%</td>
<td></td>
</tr>
</tbody>
</table>

- **Retirement**: Yes/No
- **Soc. Sec**: Yes/No
- **Age H.C.**: Yes/No
- **Age Unemp**: Yes/No
- **Mort. Risk**: Yes/No
- **Pop. Growth**: Yes/No
- **Life Length**: 81
- **Save Endow.**: 0

⇑ optimal savings: extend building period
→ Lengthens accumulation phase
Optimal Policy (Endowment)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal (%) of GDP</td>
<td>160%</td>
<td>173%</td>
<td>287%</td>
<td>307%</td>
<td>360%</td>
<td>-100%</td>
</tr>
<tr>
<td>Retirement</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Soc. Sec</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age H.C.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Age Unemp</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Mort. Risk</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pop. Growth</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Life Length</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Save Endow.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Avg. IV</td>
</tr>
</tbody>
</table>

- Eliminate building phase
- Optimal to hold debt
Why savings optimal in life cycle and debt in infinitely lived?

- In infinitely lived no accumulation phase
  - Link between stationary phase (endowment) and gov’t savings/debt
  - Less gov’t savings increases agents liquidity
- In life cycle agents experience an accumulation phase
  - More public savings increases wage
  - Particularly helpful during accumulation phase
  - Liquidity not affected until stationary phase
Experiments 4 & 5

(4) Interactions With Government Transfers
• Remove UI and solve for optimal
• Remove Social Security and solve for optimal
• Recalibrate each model
• Very small effect on optimal debt

(5) Interaction With Borrowing Constraint
• Allow for individual borrowing, ad hoc constraint
• Optimal public savings increases from 160% to 220%
• Precautionary savings less important when borrowing allowed
Experiment 6

Social Welfare Criteria

- We use ex ante Utilitarian social welfare function
  - Equivalent to welfare weight of 1 for newborn and 0 for others
- What if put different weight on cohorts?
Welfare weights

Allow for welfare weights on each generation $\{\alpha_j\}_{j=20}^J$:

$$
\sum_{j=20}^J \alpha_j E_0[v_j(a_j, \epsilon_j, x_j)] = \sum_{j=20}^J \left( \sum_{t=20}^j \alpha_t \beta^{j-t} \mu_j \right) E_j[U_j(c_j, h_j, J_j)]
$$

- We assumed $\alpha_{j=20} = 1$ and $\alpha = 0$ for other $j$
Illustrative example

What is relationship between cohorts’ weights and optimal policy?

Assuming \( \hat{\beta}^j \mu_j \propto \sum_{t=20}^{j} \alpha_t \beta^{j-t} \mu_j \) can rewrite:

\[
S_{\hat{\beta}}(v, \lambda) = \max_B \sum_{j=20}^{J} \hat{\beta}^j \mu_j E_j \left[ U_j(c_j, h_j, J_j; v_j(\cdot ; B)) \right] \left| \lambda_j(\cdot ; B) \right]
\]

- Allows us to reweight each age’s stream
- Demonstrates effect of different weights
- Larger \( \hat{\beta} \) more weight on older generations
Effect of Cohort Weights

• ↑ weights on older less savings (more debt) optimal
• Putting more weight on ages after building phase
Alternative Criteria

- SWF = total expected future utility from population
- $\alpha_j = 1 \forall \ j$

$$\sum_{j=20}^{J} \alpha_j E_0[v_j(a_j, \epsilon_j, x_j)]$$
• Examine population average expected future utility

• Optimal debt is 100% of GDP
Conclusion

• Optimal debt policy is different in life cycle model
• Instead holding debt optimal for government to save
  • Facilitates accumulation phase
  • Stationary phase less important
• Large welfare consequences to ignoring life cycle model
  • Overall conclusion not sensitive to gov’t transfers or agents allowed some borrowing

For optimal debt assuming infinitely lived for tractability has large economic consequences
Thank you
Optimal Policy (With Endowment Shock)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>160%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>173%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>287%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>307%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>360%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>233%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>273%</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

**Endowment**

- Save Endow.: 0 0 0 0 0 Avg. IV Dist.

Removing age-specific: competing effects
- Exposed more periods to idiosyncratic shock
- No need to accumulate for retirement
Social Security

Benefit Formula: \( b^{ss} = \text{[Replacement Rate]} \times \text{[Past Earnings}(x)\text{]} \)

1. Past earnings: \( x \)

\[
x' = \begin{cases} 
\frac{y+(j-1)x}{j} & \text{if } j \leq 35, \\
\max\{x, \frac{y+(j-t)x}{j}\} & \text{if } 35 < j < J_{ret}, \\
x & \text{if } j \geq J_{ret},
\end{cases}
\]

2. Replacement rate (piecewise linear)

\[
\begin{cases} 
\tau_{r1} & \text{for } 0 \leq x_R < b_1 \\
\tau_{r2} & \text{for } b_1 \leq x_R < b_2 \\
\tau_{r3} & \text{for } b_2 \leq x_R < b_3 \\
0 & \text{for } b_3 \leq x_R,
\end{cases}
\]

3. Retirement Age Credits/Deductions (\( b^{ss} \) adjusted s.t.):

- 64-66: 6.7% reduction per year
- 62-63: 5% reduction per year
- 67-70: 8% increase per year
Dynamic Programming: Worker

\[ v_j(a, \epsilon, x) = \max_{c, a', h} \left[ u(c, h) \right] + \beta s_j \sum_{\epsilon'} \pi_j(\epsilon' | \epsilon) v_{j+1}(a', \epsilon', x') \]

\[ \begin{align*}
  c + a' & \leq \quad \text{we}(\epsilon) h (1 - \bar{h}) + (1 + r)(a + Tr) - T(h, a, \epsilon) + b_{ui}(\text{we}) \bar{h} \\
  a' & \geq \quad 0 \\
  \epsilon & \equiv (\theta_j, \alpha_i, \nu_{ij}, \epsilon_{ij}, \bar{h}_{ij})
\end{align*} \]
Dynamic Programming: Could Retire

Agents could retire \((j \in [J_{ret}, \overline{J}_{ret}])\) but have not:

\[
v_j(a, \epsilon, x) = \max_{c,a',h,\mathbb{1}(j=J_{ret})} \left[ u(c, h) \right] + \\
\beta s_j \sum_{\epsilon'} \pi_j(\epsilon'|\epsilon)(\mathbb{1}(j < J_{ret})v_{j+1}(a', \epsilon', x') + (1 - \mathbb{1}(j < J_{ret}))v_{j+1}^{ret}(a', x'))
\]

\[
c + a' \leq (1 + r)(a + Tr) - T(a) + b_{ss}(x) \quad \text{if } j \geq J_{ret}
\]

s.t.

\[
c + a' \leq we(\epsilon)h(1 - \bar{h}) + (1 + r)(a + Tr) - T(h, a, \epsilon) + b_{ui}(we)\bar{h} \quad \text{else}
\]

\[
a' \geq 0
\]
Dynamic Programming: Retired

\[ v_{j}^{ret}(a, x) = \max_{c, a'} u(c) + \beta s_j v_{j+1}^{ret}(a', x) \]

s.t. \[ c + a' \leq (1 + r)(a + Tr) - T(a) + b_{ss}(x) \]

\[ a' \geq 0 \]
# Social Security

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_{1a}$ Year 1 - 3</td>
<td>6.7%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\kappa_{1b}$ Year 4 &amp; 5</td>
<td>5%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>8%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.21 x Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.29 x Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$b_3$</td>
<td>2.42 x Avg Earnings</td>
<td>Huggett and Parra (2010)</td>
</tr>
<tr>
<td>$\tau_{r1}$</td>
<td>90%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{r2}$</td>
<td>32%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{r3}$</td>
<td>15%</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>10.3%</td>
<td>Mrkt Clearing</td>
</tr>
<tr>
<td>$j_{nr}$</td>
<td>66</td>
<td>Data</td>
</tr>
<tr>
<td>$J_{ret}$</td>
<td>62</td>
<td>U.S. SS Program</td>
</tr>
<tr>
<td>$\overline{J}_{ret}$</td>
<td>70</td>
<td>U.S. SS Program</td>
</tr>
</tbody>
</table>
Decomposition Details

Define Welfare:

\[ S = S_c + S_h \equiv \int E_0 \left[ \sum_{j=1}^{J} \beta^{j-1} s_j u(c_j) \right] d\lambda_1 + \int E_0 \left[ \sum_{j=1}^{J} \beta^{j-1} s_j \varphi(h_j) \right] d\lambda_1 \]

CEV Decomposition:

\[ (1 + \Delta_{CEV}) = (1 + \Delta_{level}) (1 + \Delta_{insure}) (1 + \Delta_{distr}) (1 + \Delta_{hours}) \]

\[ \left( \frac{S^{opt} - S_h}{S_c} \right)^{\frac{1}{1-\sigma}} = \frac{C^{opt}}{C} \frac{\tilde{C}^{opt}/\tilde{C}}{C^{opt}/\tilde{C}} \left( \frac{S^{opt}/S_c}{S^{opt}/S_c} \right)^{\frac{1}{1-\sigma}} \left( \frac{S^{opt} - S_h}{S_c^{opt}} \right)^{\frac{1}{1-\sigma}} \]

where:

- **Consumption Equivalent**: \( (1 + \Delta_{CEV})^{1-\sigma} S_c + S_h = S^{opt} \)
- **Labor Substitution Effect**: \( (1 + \Delta_{hours})^{1-\sigma} S^{opt} = S^{opt}_c + (S^{opt}_h - S_h) \)
- **Certainty Equivalent**: \( \tilde{C} = \sum_j \mu_j \int \tilde{c}(a, \varepsilon, x) d\lambda_1 \)
Welfare Decomposition

Welfare Decomposition, ex ante

<table>
<thead>
<tr>
<th>Effect</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEV (% Change)</td>
<td>2.33 %</td>
</tr>
<tr>
<td>Levels Effect</td>
<td>4.36 %</td>
</tr>
<tr>
<td>Insurance Effect</td>
<td>-0.47 %</td>
</tr>
<tr>
<td>Redistribution Effect</td>
<td>0.11 %</td>
</tr>
<tr>
<td>Labor Disutility Effect</td>
<td>-1.59 %</td>
</tr>
</tbody>
</table>

Similar to misspecified
Level Effect:
• Higher wages → more consumption early
• Lower r → less savings and consumption later