Adverse Selection in the Annuity Market and the Role for Social Security

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Social Security

- The largest government program in the U.S.

- Many debates over reform/privatization

- Central question to this debate
  - What useful aspects are lost (that market can’t replicate)?

- This paper talks about one
  - Mandatory annuity insurance
Mandatory annuity insurance

- Is a **key** feature in almost all social security systems

- Can be **desirable** when there is adverse selection
Why is it desirable?

- If there is private information about mortality
  - High mortality types
    - Annuitize smaller portion of their wealth

- Insurers recognize this self-selection
  - Offer high prices that reflect mortality rate of long-lived
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• A mandatory annuity insurance
  **Forces everyone** (including high mortality) to join
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- A mandatory annuity insurance
  Forces everyone (including high mortality) to join

- Thereby
  Provides insurance at higher (implicit) rate of return
Question

• We know
  ◦ Social security has mandatory annuitization
  ◦ It can be a desirable feature
  ◦ Private markets cannot replicate it

• Question
  How important is it quantitatively?
This paper

- Develops model of annuity market with adverse selection
  - Heterogeneous mortality
  - Private information
  - Market structure: linear contracts

Calibrates the model to match US facts

Compares welfare between three benchmarks
This paper

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- Annuities: financial contracts, difficult to observe/monitor
- Lack of observability ⇒ Contracts are non-exclusive
- Little evidence on screening in the market
This paper

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- Calibrates the model to match US facts

- Compares welfare between three benchmarks
Three benchmarks

- ‘Private annuity markets’
  - No social security
  - Annuity is available only through private markets

- ‘Current U.S. system’
  - ‘Stylized’ features of U.S. social security
  - Private markets

- ‘Ex ante efficient allocations’
  - Solution to utilitarian planner’s problem
Overall ex ante gains

- If welfare is evaluated ex ante
  i.e., before mortality type is realized, then ... 

- Welfare gains between
  - ‘Private annuity markets’ and ‘current US system’
  - ‘Current US system’ and ‘ex ante efficient’
Overall ex ante gains

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- Welfare gains between
  - ‘Private annuity markets’ and ‘current US system’  0.27%
  - ‘Current US system’ and ‘ex ante efficient’
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- Welfare gains between
  - ‘Private annuity markets’ and ‘current US system’ 0.27%
  - ‘Current US system’ and ‘ex ante efficient’ 0.64%
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  0.91%
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- Welfare gains between
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  - ‘Current US system’ and ‘ex ante efficient’ 0.64%
  ——— 0.91%

- Who loses and who gains ex post,
  i.e., after mortality type is realized?
Social Security has two effects

1. Transfers from high mortality types to low mortality types
   - About 9% suffer losses: high mortality - low survival
   - About 91% percent gain: low mortality - high survival
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   - These are good risk types
   - Market pool is populated by bad risk types $\Rightarrow$ high prices
   - This price effect has negative welfare impact of 0.29 percent
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- Can we use alternative policy to minimize this effect? Yes!
Related literature

- **Theoretical models**: Abel (1986); Eichenbaum and Peled (1987); Eckstein, Eichenbaum and Peled (1985)
  - Welfare enhancing role for mandatory annuitization
  - Evidence for adverse selection in the annuity market
  - Measure the value of access to actuarially fair annuity
- **Estimate welfare cost of asymmetric information**: Einav, Finkelstein and Shrimpf (2010)
  - Preference heterogeneity as well as risk heterogeneity
- **Benefits of annuitization in social security**: Hubbard and Judd (1987)
Model
Environment: information

- Individuals have private type $\theta$ known at date zero
  - $\theta$ indexes their mortality
  - It determines their individual survival probabilities
  - Distribution at date zero: $G_0(\theta)$

- The only heterogeneity is in $\theta$

- The only risk is time of death
Environment: preferences

- Everyone lives between 0 and $T$ and has preferences

$$\sum_{t=0}^{T} \beta^t P_t(\theta)[u(c_t) + \beta(1 - x_{t+1}(\theta))\xi u(b_t)]$$

- Where
Environment: preferences

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- $P_t(\theta)$: probability that type $\theta$ is alive at age $t$
- $x_{t+1}(\theta)$: One period conditional survival $x_{t+1}(\theta) = \frac{P_{t+1}(\theta)}{P_t(\theta)}$
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  - $x_{t+1}(\theta)$: One period conditional survival $x_{t+1}(\theta) = \frac{P_{t+1}(\theta)}{P_t(\theta)}$
  - $\xi$: weight on bequest, $b_t$
Technology

- Inelastic labor supply up to age $J < T$

- $n$ units of labor produces $wn$ units of consumption good

- Saving technology $R = \frac{1}{\beta}$
Annuity contracts

- Can be purchased at age $J$ (last period before retirement)

- Makes survival contingent payment starting age $J + 1$

- Unit cost of annuity is $q$
Individual’s problem

\[
\max_{c_t, k_{t+1}, a \geq 0} \sum_{t=0}^{T} \beta^t P_t(\theta)[u(c_t) + \beta(1 - x_{t+1}(\theta))\xi u(Rk_{t+1})]
\]

subject to

\[
c_t + k_{t+1} = Rk_t + w(1 - \tau) \quad \text{for } t < J
\]

\[
c_t + k_{t+1} + qa = Rk_t + w(1 - \tau) \quad \text{for } t = J
\]

\[
c_t + k_{t+1} = Rk_t + a + z \quad \text{for } t > J
\]
Insurers

- Insurers do not observe individual demand for each type $\theta$
- However, they know the demand function $a(\theta, q)$
- They anticipate the fraction of total sales, purchased by $\theta$

$$dF(\theta) = \frac{a(\theta; q)dG_J(\theta)}{\int a(\theta; q)dG_J(\theta)}$$

- Insurers use $F(\theta)$ to evaluate their profit
Annuity insurers problem

\[
\max_{y \geq 0} \quad qy - y \int \left( \sum_{t=J+1}^{T} \frac{P_t(\theta)}{P_J(\theta) R_s^{s-t}} \right) dF(\theta)
\]

- \(F(\theta)\) is anticipated distribution of pay-outs
  - Determines fraction of \(y\) sold to type \(\theta\)
  - Taken as given by the insurer
Government Budget Constraint

\[ \int \tau w \left( \sum_{t=0}^{J} \frac{P_t(\theta)}{R^t} \right) dG_0(\theta) = \int z \left( \sum_{t=J+1}^{T} \frac{P_t(\theta)}{R^t} \right) dG_0(\theta) \]
Equilibrium

- Households and firms optimize + markets clear

- $F(\theta)$ is consistent with individual decisions

$$dF(\theta) = \frac{a(\theta)dG_J(\theta)}{\int a(\theta)dG_J(\theta)}$$

- Government budget constraint
Properties of Equilibrium: Two period case
Two lessons

Use two period example to illustrate two properties

1 In this environment there is adverse selection

   - Equilibrium price is higher than aggregate risk

2 Increasing social security tax and benefit

   - Crowds out annuity market
   - Increases equilibrium price of annuity
A two period example

\[
\max u(c_1) + Pu(c_2)
\]

subject to

\[
c_1 + qa \leq w(1 - \tau)
\]
\[
c_2 \leq a + z
\]

- \(P\) is probability of survival (with distribution \(G(P)\))
- Aggregate risk of survival is \(\int P dG(P)\)
- The goal is to show in equilibrium

\[
q > \int P dG(P)
\]
• Consider the zero profit condition

\[ q \int a(P; q) dG(P) = \int Pa(P; q) dG(P) \]

Total sale

Total expected payment
Adverse selection

- Consider the zero profit condition

\[ q = \frac{\int Pa(P; q)dG(P)}{\int a(P; q)dG(P)} \]
Adverse selection

- Consider the zero profit condition

\[ q = \int P \frac{a(P; q) dG(P)}{\int a(P; q) dG(P)} \]
Adverse selection

- Consider the zero profit condition

\[ q = \int P \frac{a(P; q)dG(P)}{\int a(P; q)dG(P)} dF(P) \]

- Insurers use \( F(P) \) to evaluate risk
Adverse selection

- Consider the zero profit condition

\[ q = \int P \frac{a(P; q)dG(P)}{\int a(P; q)dG(P)} \]

- Insurers use \( F(P) \) to evaluate risk

- \( a(P; q) \) is increasing in \( P \)

\[ \Rightarrow F(P) \text{ is more skewed relative to } G(P) \]
Adverse selection

- Consider the zero profit condition

\[ q = \int PdF(P) > \int PdG(P) \]

- Insurers use \( F(P) \) to evaluate risk

- \( a(P; q) \) is increasing in \( P \)

\[ \Rightarrow F(P) \text{ is more skewed relative to } G(P) \]

Therefore, equilibrium price is higher than aggregate risk
Effect of increasing social security

- SS benefit is a substitute for annuity
  \[ \Rightarrow \text{increasing SS reduces demand for annuity} \]
Effect of increasing social security

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Effect of increasing social security

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  ⇒ increasing SS reduces demand for annuity

- \( a(P; q) \) is increasing in \( P \)
- As SS tax goes up
  
  \( a(P; q) \) shifts down
  
  And becomes steeper
Effect of increasing social security

- SS benefit is a substitute for annuity
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  ⇒ price goes up
Effect of increasing social security

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  \[ a(P; q) \] shifts down
  And becomes steeper

  ⇒ price goes up

What about welfare?

\[ a(P; q) \]
Calibration
Calibration

- Mortality parameters
  - Survival probabilities, $P_t(\theta)$, for each $t$ and $\theta$
  - Initial distribution of $\theta$: $G_0(\theta)$

- Preference/technology parameters
  - Curvature of utility function
  - Weight on bequest
  - Return on saving and time preference

- Policy parameters
  - Social security tax and benefits
Calibrating mortality parameters

- Observe data on
  - Average survival probabilities (from life tables)
  - Individuals’ own assessment about longevity (from HRS)

- Use these observations to back out
  - $P_t(\theta)$ for each $\theta$
  - The distribution $G_0(\theta)$

- Need to impose restriction on $P_t(\theta)$
  - Standard assumptions from demography
Assumptions on $P_t(\theta)$

- Let $H_t(\theta)$ be cumulative mortality hazard for type $\theta$, define
  \[ P_t(\theta) = \exp(-H_t(\theta)) \]

- **Assumption 1**: $\theta$ shifts mortality hazard
  \[ H_t(\theta) = \theta H_t \]

- **Assumption 2**: Initial distribution of $\theta$ is gamma
  \[ g_0(\theta) \sim Gamma\left(\frac{1}{k}, k\right) = k^k \theta^{k-1} \frac{\exp(-k\theta)}{\Gamma(k)} \]
Assumptions on $P_t(\theta)$

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  \[ g_0(\theta) \sim Gamma\left(\frac{1}{k}, k\right) = k^k \theta^{k-1} \exp(-k\theta) \frac{1}{\Gamma(k)} \]

- What are implications of these assumptions?
Implication of the assumption $H_t(\theta) = \theta H_t$

- Suppose type $\theta$ has 50% chance of surviving to age $t$

- Then, type $2\theta$ has 25% chance of surviving to the same age
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- Suppose type $\theta$ has 50% chance of surviving to age $t$

- Then, type $2\theta$ has 25% chance of surviving to the same age

- Once $P_t(\theta)$ (or $H_t(\theta)$) is known for one $\theta$

  It is known for all $\theta$
Identifying survival probabilities

- Unknowns are
  - $H_t$
  - Parameter of distribution $G_0$
Identifying survival probabilities

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  - $H_t$
  - Parameter of distribution $G_0$

$T + 1$ unknowns

1 unknown
Identifying survival probabilities

- Unknowns are
  - $H_t$
  - Parameter of distribution $G_0$

- Life table gives population survival probabilities
  \[
  \overline{P}_t = \int dG_0(\theta)
  \]

- Given $G_0(\theta)$ the above identity can be solved to find $H_t$

- How do we find $G_0(\theta)$?

$T + 1$ unknowns
1 unknown

$T + 2$ unknowns
Identifying survival probabilities

- Unknowns are
  - $H_t$ (1 unknown)
  - Parameter of distribution $G_0$ ($T + 1$ unknowns)
  - Life table gives population survival probabilities

If $G_0(\theta)$ is given, the above identity can be solved to find $H_t$.

How do we find $G_0(\theta)$?
Identifying survival probabilities

• Unknowns are
  ○ $H_t$  \hspace{2cm} $T + 1$ unknowns
  ○ Parameter of distribution $G_0$  \hspace{2cm} 1 unknown
  ○ Life table  \hspace{2cm} $-(T+1)$ unknowns
  \hspace{2cm} 1 unknown

• Life table gives population survival probabilities

$$\bar{P}_t = \int \exp(-\theta H_t) dG_0(\theta)$$

• Given $G_0(\theta)$ the above identity can be solved to find $H_t$
Identifying survival probabilities

- Unknowns are
  - $H_t$  \( T + 1 \) unknowns
  - Parameter of distribution $G_0$  1 unknown
  - Life table  \(-(T + 1)\) unknowns
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- Given $G_0(\theta)$ the above identity can be solved to find $H_t$

- How do we find $G_0(\theta)$?
Subjective survival prob. in HRS

- HRS asks individuals their subjective prob. of living to 75

- Hurd & McGarry (1995, 2002): responses are consistent with
  - Life tables
  - Ex post mortality experience
  - Individuals’ health data

- Use Gan-Hurd-McFadden (2003)’s method to estimate $G_0(\theta)$
Individual survival curves: \( P_t(\theta) \)
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Average Life Expectancy at 30: 44 yrs (74 years old)
Standard deviation: 4 yrs
Individual survival curves: $P_t(\theta)$

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Average Life Expectancy at 30: 44 yrs (74 years old)

Standard deviation : 4 yrs
Profile of Life Expectancy by age
Profile of Life Expectancy by age

Life expectancy at each age

One Standard Deviation
Profile of Life Expectancy by age
Calibration: preferences + social security

• CRRA utility function

\[ u(c) = \frac{c^{1-\gamma}}{1 - \gamma} \]

• Preference parameters are chosen to match
  - Fraction of pension wealth for 70 yrs old in HRS \( \xi = 0.8 \)
  - Fraction of SS wealth for 70 yrs old in HRS \( \gamma = 1.47 \)

• Social security tax: chosen to match %45 replacement ratio
## Calibration summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk aversion, $\gamma$</td>
<td>1.47</td>
</tr>
<tr>
<td>weight on bequest, $\xi$</td>
<td>0.8</td>
</tr>
<tr>
<td>discount factor, $\beta$</td>
<td>0.975</td>
</tr>
<tr>
<td>return on savings, $R$</td>
<td>1.035</td>
</tr>
<tr>
<td>SS tax, $\tau$</td>
<td>0.08</td>
</tr>
<tr>
<td>variance of $g_0(\theta)$, $\sigma_\theta^2 = \frac{1}{k}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Fraction of wealth annuitized, average

Annuitized Wealth/Total Wealth

SS Wealth/Total Wealth

Pension Wealth/Total Wealth

Age

%
Fraction of wealth annuitized, average

![Graph showing the fraction of wealth annuitized, with lines for Annuitized Wealth/Total Wealth, SS Wealth/Total Wealth, and Pension Wealth/Total Wealth. The x-axis represents age from 65 to 100, and the y-axis represents the percentage. The graph illustrates how the fraction of wealth annuitized decreases with age.](image-url)
Fraction of wealth annuitized, average

![Graph showing the fraction of wealth annuitized, with age on the x-axis and percentage on the y-axis. The graph compares annuitized wealth to total wealth, Social Security (SS) wealth to total wealth, and pension wealth to total wealth. The curves are labeled for annuitized wealth, SS wealth, and pension wealth. The points are chosen to match certain ages and percentages.](image)
Findings
Use the model to ask

- How does annuitization decision vary by mortality type?
- How do these decisions change by removing SS?
- Welfare comparison
Fraction of wealth annuitized at 70, by type

- Approximately 60% hold annuity
- Consistent with evidence in HRS

- Johnson-Burman-Kobes (2004) evidence from HRS

- 43% of all adults (52% of males) hold pensions

SS wealth / Total wealth

% of Wealth Annuitized

Low Mortality High Mortality

\( \theta \)

Low Mortality
High Mortality

SS wealth / Total wealth
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![Diagram showing annuitized wealth, Social Security (SS) wealth, and pension wealth as a percentage of total wealth across different mortality rates.](image_url)
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Graph:
- % of Wealth Annuitized
  - Low Mortality
  - High Mortality
  - Annuitized wealth / Total wealth
  - SS wealth / Total wealth
  - Pension wealth / Total wealth
Fraction of wealth annuitized at 70, by type

- Annuitized wealth / Total wealth
- SS wealth / Total wealth
- Pension wealth / Total wealth

60% hold annuity, consistent with evidence in HRS. Johnson-Burman-Kobes (2004) evidence from HRS. 43% of all adults (52% of males) hold pensions.
Fraction of wealth annuitized at 70, by type

60% hold annuity

Consistent with evidence in HRS
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Consistent with evidence in HRS

- Johnson-Burman-Kobes(2004) evidence from HRS

43% of all adults (52% of males) hold pensions
Only market vs Current U.S. 

![Graph showing the annuitized wealth to total wealth ratio for low and high mortality scenarios in the current U.S. system. The x-axis represents mortality levels (Low Mortality to High Mortality), and the y-axis represents the percentage of wealth annuitized.]
Only market vs Current U.S. 

![Graph showing annuitized wealth vs total wealth for low and high mortality scenarios. The graph includes two lines: one for the annuitized wealth/total wealth in the only private market scenario and another for the current U.S. system. The x-axis represents low and high mortality, while the y-axis shows the percentage of wealth annuitized.]
Ex post gain/loss

Welfare Gains/Losses from Introducing SS Across Mortality Types

91 percent of population gain on average 0.34%.

9 percent of population lose on average 0.39%.

Ex ante gain = 0.27%
Counter-factual: fix price at the equilibrium level without SS

Without price increase the ex ante gain is 0.56%
Can we do better?

- Social security forces individuals to pool their mortality risk
  
  But keeps this pool separate from market pool

- This derives good risk types out of the market.

- Alternative policy:
  - Return contributions to people at retirement
  - Force them to buy annuity in the market
Can we do better?

- Social security forces individuals to pool their mortality risk
  
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- Alternative policy:
  
  - Return contributions to people at retirement
  
  - Force them to buy annuity in the market

- Ex ante welfare gain increases to **0.36%**
Gains from implementing ex ante efficient

• What is the maximum ex ante welfare gain from policy?

• We need to find the solution to utilitarian planner’s problem
Planner’s problem

\[
\max \int \left[ \sum_{t=0}^{T} \beta^t P_t(\theta)[u(c_t(\theta)) + \beta(1 - x_{t+1}(\theta))\xi u(b_t(\theta))] \right] dG_0(\theta)
\]

subject to

\[
\int \sum_{t=0}^{T} \frac{P_t(\theta)}{R^t} \left[ c_t(\theta) + \frac{(1 - x_{t+1}(\theta))}{R} b_t(\theta) \right] dG_0(\theta) = \int w \sum_{t=0}^{J} \frac{P_t(\theta)}{R^t} dG_0(\theta)
\]
Planner’s problem

\[
\max \int \left[ \sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t(\theta)) + \beta (1 - x_{t+1}(\theta)) \xi u(b_t(\theta))] \right] dG_0(\theta)
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Planner chooses consumption and bequest
Planner’s problem

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Notice: No I.C constraints!
Planner’s problem

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subject to

\[
\int \sum_{t=0}^{T} \frac{P_t(\theta)}{R^t} \left[ c_t(\theta) + \frac{(1 - x_{t+1}(\theta))}{R} b_t(\theta) \right] dG_0(\theta) = \int w \sum_{t=0}^{J} \frac{P_t(\theta)}{R^t} dG_0(\theta)
\]

It turns out they don’t bind
Planner’s problem

$$\max \int \left[ \sum_{t=0}^{T} \beta^t P_t(\theta)[u(c_t(\theta)) + \beta(1 - x_{t+1}(\theta))\xi u(b_t(\theta))] \right] dG_0(\theta)$$

subject to

$$\int \sum_{t=0}^{T} \frac{P_t(\theta)}{R_t} \left[ c_t(\theta) + \frac{(1 - x_{t+1}(\theta))}{R} b_t(\theta) \right] dG_0(\theta) = \int w \sum_{t=0}^{J} \frac{P_t(\theta)}{R_t} dG_0(\theta)$$

Ex ante efficient allocations have very simple form
Ex ante efficient allocations

- Perfect insurance against risk type $\theta$
  \[ c_t(\theta) = c_t(\theta') = c_t \]
  \[ b_t(\theta) = b_t(\theta') = b_t \]

- Perfect insurance against time of death,
  \[ u'(c_t) = \beta Ru'(c_{t+1}) = \beta R\xi u'(b_t) \]
Ex ante efficient allocations

- Perfect insurance against risk type $\theta$
  \[ c_t(\theta) = c_t(\theta') = c_t \]
  \[ b_t(\theta) = b_t(\theta') = b_t \]

- Perfect insurance against time of death, assume $R_\beta = 1$
  \[ u'(c_t) = u'(c_{t+1}) = \xi u'(b_t) \]
Ex ante efficient allocations

- Perfect insurance against risk type $\theta$

\[
c_t(\theta) = c_t(\theta') = c_t
\]

\[
b_t(\theta) = b_t(\theta') = b_t
\]

- Perfect insurance against time of death, assume $R_\beta = 1$

\[
c_t = c, \ b_t = b \ \text{and} \ u'(c) = \xi u'(b)
\]
Ex ante efficient allocations 

- Perfect insurance against risk type $\theta$
  \[ c_t(\theta) = c_t(\theta') = c_t \]
  \[ b_t(\theta) = b_t(\theta') = b_t \]

- Perfect insurance against time of death, assume $R_\beta = 1$
  \[ c_t = c, b_t = b \text{ and } u'(c) = \xi u'(b) \]

- Can be implemented by
  - Type-independent social security tax and benefit
  - Type-independent survivors benefit
Ex ante efficient allocation can be implemented using:

- Type-independent taxes: 0.14 (compare this to 0.08)
- Replacement ratio: 0.71 (compare to 0.45)
- Survival benefit before retirement (small)
Comment

• There are two key assumptions

1. Only heterogeneity is in mortality

2. Individuals (and planner) are expected utility maximizers

⇒ Type-independent policy is optimal
Ex post gain/loss

Welfare Gains/Losses from Introducing SS Across Mortality Types

Ex ante gain = 0.91%
Conclusion

- Goal of the paper
  - Measure the gains from mandatory annuitization in S.S

- Welfare gain from mandatory annuitization
  - ‘current U.S. system’ over ‘private markets’: 0.27%

- Large impact on price with negative welfare implications

- Simple policy change can alleviate this negative price effect
Extensions

- Introducing other heterogeneities
  - Heterogeneity in preference for bequest
  - The link between measures of income and mortality
- Detailed model of altruism and intergenerational link
- Alternative equilibrium notions
Backup slides
Sensitivity: Risk aversion

Overall Welfare Gains for Various Levels of Risk Aversion, $\gamma$

- Ex ante gains in the benchmark model
- Ex ante gains under autarky
Overall Welfare Gains for Various Levels of Bequest Parameter, $\xi$

- Ex ante welfare gains in the benchmark model
- Ex ante welfare gains under autarky

Weight on Bequest, $\xi$
Consumption/Saving profiles (w/ SS) ______

**Consumption Profile Across Mortality Types, with SS**

- Lowest 5% of Mortality (●)
- Median Mortality (●)
- Highest 5% of Mortality (●)
- Mean (solid line)

**Profile of Liquid Asset Holdings Across Mortality Types, with SS**

- Lowest 5% of Mortality (●)
- Median Mortality (●)
- Highest 5% of Mortality (●)
- Mean (solid line)
Consumption/Saving profiles (w/o SS)
• Welfare gains going from
  - Private saving to current US system 2.85%
  - Current US system to ex ante efficient 0.84%
  \[ \text{Total: } 3.71\% \]

• When there is no annuity market, gains are large
Welfare Gains/Losses from Introducing SS Across Mortality Types

Gains/Losses when Annuity Market Does not Exist

Gains/Losses when Annuity Market Exists
Estimation procedure

• What is observed in HRS
  ◦ Response to the question on subjective survival prob.
  ◦ Ex post mortality/survival

• Problem: there are many 0’s and 1’s in responses

• Solution: assume error in reports
  ◦ Type $\theta$ at age $t$ makes report $r$ with prob. $f(r|\frac{P_{75}(\theta)}{P_t(\theta)})$
Estimation procedure (cont.)

• Observing report, $r$, we can estimate $\theta$ using Baye’s rule

  ◦ Prior on $\theta$ is given by $G_t(\theta)$
  ◦ Report, $r$ and $f(\cdot|\cdot)$ can be used to form a posterior
  ◦ Use posterior mean as estimate for $\theta$

• Use estimates to form likelihood functions for survival

• Estimate parameters of $G_t(\theta)$ and $f(\cdot|\cdot)$ using MLE
“the existence of asymmetric information may justify a social insurance program (a government annuity in this case) but does not necessarily do so. The case for a mandatory annuity program depends on calculations that could be done but that have not yet been done.”

Martin Feldstein, presidential address (2005)