A Bird in the Hand is Worth Two in the Grave
Risk Aversion and Life-Cycle Savings

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Jon M. Huntsman School of Business, Utah State University
Logan, Utah, May 2016
Question: How does risk aversion impact life-cycle saving and portfolio choice?

First answer: Depends on the risks considered

- Labor income risk: ↑
- Financial return risk: depends on IES
- Mortality risk: ↓

With multiple risks: ambiguous

⇒ Need quantitative analysis

Focus on risk aversion + income, financial and mortality risks
Modelling approach

- Kreps-Porteus recursive preferences:
  - Epstein-Zin (1989)
  - Risk-sensitive: Hansen and Sargent (1995) in their work on robustness
  - Allow us to vary risk aversion without changing IES
- Quantitative life-cycle model with incomplete markets
- Partial equilibrium analysis
- Calibrated to U.S. data
- ... and in particular to value of a statistical life: Viscusi and Aldy (2003) for a review
Main results

- Higher risk aversion
  - Decreases life-cycle savings
  - Decreases participation in the stock market
  - Decreases the conditional share in stock
- With mortality risk, give up homotheticity of Epstein-Zin
  \[ \rightarrow \text{intuition: we cannot "scale" death.} \]
- Risk-sensitive and Epstein-Zin qualitatively similar and quantitatively close
### Literature

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>... increases savings</th>
<th>... decreases savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income risk</td>
<td>e.g., BCL</td>
<td></td>
</tr>
<tr>
<td>Investment risk</td>
<td>Kihlstrom and Mirman (1974) and BCL if IES &lt; 1</td>
<td>Kihlstrom and Mirman (1974) and BCL if IES &gt; 1</td>
</tr>
<tr>
<td>Mortality risk</td>
<td><strong>HPSA if IES &lt; 1</strong></td>
<td>Bommier (2006, 2013), BCL, Drouhin (2015), HPSA if IES &gt; 1</td>
</tr>
<tr>
<td>All three risks</td>
<td><strong>Gomes and Michaelides (2005, 2008)</strong></td>
<td>This paper</td>
</tr>
</tbody>
</table>

- BCL: Bommier, Chassagnon, and LeGrand (2012)
Relationship between risk aversion and savings (1/2)

Simple framework (see Bommier, Chassagnon, LeGrand, 2012)

- Consumption-saving problem with 2 periods: 0 and 1; 2 states in period 1: $G$ and $B$
- Saving $s_B$ (resp. $s_G$) if $B$ (resp. $G$) for sure
- Saving $s^*$ if uncertain future ($B$ or $G$)

Role of risk aversion:

- $s^* = \text{convex combination of } s_B \text{ and } s_G$
- Weight on $s_B$ increases with risk aversion
  $\Rightarrow$ the more risk averse, the more important bad state realizations
Relationship between risk aversion and savings (2/2)

- Income risk
  - Bad state = low income
  - $s_B > s_G$
  - Risk aversion *increases* savings.

- Mortality risk
  - Bad state = living for one period only
  - saving = bet on living 2 periods
  - $s_B < s_G$
  - Risk aversion *decreases* savings.

- Investment risk: depends on IES

⇒ All three risks, ambiguous relationship → quantitative exercise
Back of the envelope calculation (1/2)

Magnitudes of income vs. mortality risks?

- Income risk from a lifecycle perspective
  - Lifecycle labor income = per period labor incomes discounted to age 20 at the risk-free rate
  - With our calibration, average lifetime labor income of $1.1 million with a standard deviation of $0.8 million
  - Income risk $\approx$ $0.8$ million
Magnitudes of income vs. mortality risks?

- Mortality risk.
  - Life expectancy at age 20 = 58.5 years with a standard deviation of 14.5 years.
  - Mortality risk ≈ 14.5 years.
  - Using the value of a statistical life, one year alive ≈ $186k
    (VSL = $6.5m at 45).
  - Mortality risk ≈ $2.7 millions.

⇒ Back of the envelope calculation: Mortality risk ≫ income risk
⇒ Impact of risk aversion should be dominated by mortality risk
1 Motivation and mechanisms

2 Model

3 Computation and calibration

4 Results

5 Conclusion and outlook
Endowments

- Working age $t = 1$, retirement age $t = T_R$, max age $t = T_M$
- Mortality risk: survival probabilities $(p_{t+1|t})_t$
- Labor income $(1 \leq t < T_R)$

$$y^L_t = y_0 \exp(\mu_t + \pi_t + \varepsilon^y_t)$$

$$\pi_t = \rho \pi_{t-1} + \varepsilon^\pi_t$$

$$\varepsilon^y_t \sim \mathcal{N}(0, \sigma^2_y), \quad \varepsilon^\pi_t \sim \mathcal{N}(0, \sigma^2_\pi)$$

- Social security pension income $(T_R \leq t \leq T_M)$, $y^R_t$
Asset markets

- Bond: risk-free gross return $R^f$

- Stock: risky gross return
  \[
  \ln R^s_t = \ln (R^f_t + \nu) + \varepsilon^R_t, \quad \varepsilon^R_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_R)
  \]
  \[
  \varepsilon^R_t \text{ correlated with both labor income shocks with } \kappa_{R,y} \text{ and } \kappa_{R,\pi}
  \]

- No short-selling

- Stock-market participation cost, $F \geq 0$, paid once in life
Choices and constraints

- **Choices** $\{c_t, s_t, b_t, \eta_t\}$

- **Constraints**

  \[
  c_t + b_t + s_t + F1_{\eta_t=1}1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R^s_t s_{t-1},
  \]

  \[
  y_t = \begin{cases} 
  y^L_t & \text{if } t < t_R, \\
  y^R_t & \text{else},
  \end{cases}
  \]

  \[s_t = 0 \text{ if } \eta_t = 0,\]

  \[c_t > 0, \quad b_t \geq 0, \quad s_t \geq 0.\]

  and bequests are $w_t = R^f b_{t-1} + R^s_t s_{t-1}$. 

Risk Aversion and Life-Cycle Savings
Preferences (1/2)

- Felicity (alive) from consumption: \( u(c) = \frac{c^{1-\sigma}-1}{1-\sigma} \)

- Felicity (dead) from bequests:

\[
v(w) = -v_0 + \frac{\theta}{1-\sigma} \left[ (\hat{w} + w)^{1-\sigma} - \hat{w}^{1-\sigma} \right]
\]

- Kreps-Porteus recursive preferences

\[
U_t^A = (1 - \beta)u(c_t) + \beta \Phi^{-1} \left( p_{t+1|t} \mathbb{E}_t \left[ \Phi \left( U_{t+1}^A \right) \right] + (1 - p_{t+1|t}) \mathbb{E}_t \left[ \Phi \left( U_{t+1}^D \right) \right] \right)
\]

\[
U_t^D = (1 - \beta) v(w_t) + \beta v(0)
\]
Preferences (2/2)

Why is $v_0$ important?

- difference between being alive consuming 1 unit and being dead without leaving bequest
- strongly connected to the value of life
- cannot be set to zero without a loss of generality (and a strong constraint on value of life)
- does not “go away” with non-additive preferences
- (does not affect choices in case of additive preferences)

$$U_t^A = (1 - \beta) u(c_t) + \beta p_{t+1|t} \mathbb{E}_t \left[ U_{t+1}^A \right] - \beta (1 - p_{t+1|t}) v_0$$

$$+ (1 - p_{t+1|t}) \beta \mathbb{E}_t [(1 - \beta) \frac{\theta}{1 - \sigma} \left( (\hat{w} + w)^{1-\sigma} - \hat{w}^{1-\sigma} \right)]$$
Epstein-Zin and risk-sensitive preferences (1/2)

- Both Kreps-Porteus
- Epstein-Zin preferences (EZ)
  \[ \Phi(u) = \frac{1}{1 - \gamma} \left( 1 + (1 - \sigma)u \right)^{\frac{1 - \gamma}{1 - \sigma}} - \frac{1}{1 - \gamma}, \text{ if } \gamma, \sigma \neq 1 \]
- Risk-sensitive preferences (RS)
  \[ \Phi(u) = -\frac{1}{k} \left( \exp(-ku) - 1 \right), \text{ if } k \neq 0 \]
- Limit cases \((k = 0, \gamma = 1, \sigma = 1)\) by continuity
- Coincide if
  - \(\gamma = \sigma\) and \(k = 0\) \(\Rightarrow\) additively separable case
  - \(\sigma = 1\)
Epstein-Zin and risk-sensitive preferences (2/2)

- EZ: homothetic but not monotone (with respect to FSD)
- RS: non-homothetic but monotone.

⇒ Not monotone, what does that mean?

- RS: the only KP preferences that are monotone and disentangle risk aversion from IES

In our setting:

- Homotheticity has to be given up, because of value of life.
- Non-monotonicity little impact
Value of a statistical life

- Standard definition (see Johansson 2002): Marginal rate of substitution between survival rate and consumption

\[
VSL_t = \frac{\partial U_t^A}{\partial p_{t+1|t}} - \frac{\partial U_t^A}{\partial c_t}
\]

⇒ how much consumption to give up for increasing the likelihood to live one more year

- Viscusi and Aldy (2003) for empirical estimates
Computation

- Reformulate model
  - Cash-at-hand, $x_t = R^f b_{t-1} + R^s_t s_{t-1} + y_t$
  - Total savings, $a_t$, and share in stock $\alpha \in [0, 1]$
- Persistent productivity, $\pi_t$: continuous state variable
- State space $(x_t, \pi_t, \eta_t, t)$
- Not differentiable
- Standard VFI very long $\rightarrow$ calibration hardly feasible.
  $\Rightarrow$ Refinement of VFI
  $\Rightarrow$ Use 3D cubic B-spline to interpolate expected continuation value
- Calibration: consider 3 agents: $add$, $EZ$, $RS$
## Calibration of preferences

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<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Exog. endowment, $\hat{\bar{w}}$</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.96</td>
<td>Assets_{45}^{add} = US$ 100'000</td>
</tr>
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<td>Life-death gap, $v_0$</td>
<td>30.0</td>
<td>VSL_{45}^{add} = US$ 6.5m</td>
</tr>
<tr>
<td>Bequest motive, $\theta$</td>
<td>20.0</td>
<td>Bequests_{85}^{add}</td>
</tr>
<tr>
<td>Risk aversion, EZ, $\gamma$</td>
<td>3.0</td>
<td>Assets_{45}^{RS} = Assets_{45}^{EZ}</td>
</tr>
<tr>
<td>Risk aversion, RS, $k$</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>
### Parameterization of endowments and asset markets

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/ counterpart/ target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working age, retirement age, maximum age</td>
<td>21, 65, 100</td>
<td></td>
</tr>
<tr>
<td>Survival rates, ( p_{t+1</td>
<td>t} )</td>
<td>( {p_{t+1</td>
</tr>
<tr>
<td>Age productivity, ( \mu_t )</td>
<td>( {\mu_t}^T_1 )</td>
<td>Earnings profiles 2007, PSID</td>
</tr>
<tr>
<td>Average wage, ( y_0 )</td>
<td>21 756 USD</td>
<td>Net compensation 2007, SSA</td>
</tr>
<tr>
<td>Pensions, ( y_R )</td>
<td>0.3</td>
<td>Replacement rate, preliminary</td>
</tr>
<tr>
<td>Autocorrelation, ( \rho )</td>
<td>0.95</td>
<td>Storesletten, et al. (2004)</td>
</tr>
<tr>
<td>Var. persistent shocks, ( \sigma^2_\pi )</td>
<td>0.03</td>
<td>Storesletten, et al. (2004)</td>
</tr>
<tr>
<td>Correlation with stock, ( \kappa_{R,\pi} )</td>
<td>0.15</td>
<td>Gomes and Michaelides (2005)</td>
</tr>
<tr>
<td>Var. transitory shocks, ( \sigma^2_y )</td>
<td>0.00</td>
<td>Preliminary</td>
</tr>
<tr>
<td>Inheritance, ( w_0 )</td>
<td>0.0</td>
<td>Preliminary</td>
</tr>
<tr>
<td>Gross risk-free return, ( R^f )</td>
<td>1.01</td>
<td>Bond return, Shiller data</td>
</tr>
<tr>
<td>Equity premium, ( \nu )</td>
<td>0.02</td>
<td>Preliminary</td>
</tr>
<tr>
<td>Stock volatility, ( \sigma_R )</td>
<td>0.18</td>
<td>Shiller data</td>
</tr>
<tr>
<td>Participation cost, ( F )</td>
<td>0.2</td>
<td>Preliminary</td>
</tr>
</tbody>
</table>
Lifecycle profiles *without* mortality risk

- Only labor income and asset return risks

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**Total savings**

- **Additive**: $EZ, \gamma > \sigma$
- **EZ, \gamma > \sigma**: $RS, k > 0$

**Stock market participation**

- **Additive**: $EZ, \gamma > \sigma$
- **EZ, \gamma > \sigma**: $RS, k > 0$
Lifecycle profiles with mortality risk (1/3)

- Baseline with all risks

- Total savings
- Stock market participation

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Lifecycle profiles *with* mortality risk (2/3)

- Baseline with all risks

![Graph of total savings and conditional share in stock](image)

**Total savings**
- additive
- EZ, $\gamma > \sigma$
- RS, $k > 0$

**Conditional Share in Stock**
- additive
- EZ, $\gamma > \sigma$
- RS, $k > 0$
Lifecycle profiles with Mortality risk (3/3)

- Baseline with all risks

![Graphs of Consumption and Value of a Statistical Life](image)

Risk Aversion and Life-Cycle Savings
Typical Epstein-Zin specification

- Many different variants, e.g. [GM 2005]. See [Literature Overview].

\[ \Omega_t = \left( (1 - \beta) c_t^{1-\sigma} + \beta \left( \mathbb{E}_t \left[ p_{t+1|t} \Omega_{t+1}^{1-\gamma} + (1 - p_{t+1|t}) \theta w_{t+1}^{1-\gamma} \right] \right) \frac{1-\sigma}{1-\gamma} \right) \frac{1}{1-\sigma} \]

- Bequests explicit and homothetic,

- ...but VSL not necessarily > 0

- In our framework, set \( v_0 = -\theta \hat{\omega}^{1-\sigma} \) (and \( \hat{\omega} = 0.0 \))

- In addition, if no bequests: \( \theta = 0 \)

  If \( \gamma > 1 \): \( \frac{\partial \Omega_t}{\partial p_{t+1|t}} < 0 \) \( \Rightarrow \) VSL < 0. The term
  \[ +(1 - p_{t+1|t})(\infty)^{1-\gamma} \]
  can be added in the recursion, where
  \( \infty = \text{utility of death} \).
Typical Epstein-Zin specification, $\theta = 0$ (1/2)

- Like baseline with all risks

![Graph showing total savings and stock market participation over age]

**Total savings**
- $\text{additive}$
- $\text{EZ, } \gamma > \sigma$

**Stock market participation**
- $\text{additive}$
- $\text{EZ, } \gamma > \sigma$
Typical Epstein-Zin specification, $\theta = 0$ (2/2)

Consumption

Value of a Statistical Life
Conclusion

- Mortality = main risk in life
  - importance of value of life
  - saving = risk-taking behavior
  - Higher risk aversion decreases lifecycle savings

- EZ vs. RS
  - EZ can accommodate positive VSL, but lose homotheticity
  - Typical EZ implementation may yield negative VSL

- Observed low levels of saving may be rational and explained by higher risk-aversion. Alternative explanation to time-inconsistency (e.g., Caliendo and Findley, 2013)

- In paper, also explain the different results of Hugonnier, Pelgrin, and Saint-Amour (2012)
Thank you!
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6 Appendix
Literature

- Epstein-Zin preferences:

- Risk aversion and savings:

- Value of a statistical life:
  Kaplow (2005), Viscusi and Aldy (2003), Bommier and Villeneuve (2010), Cordoba and Ripoll (2013)
Investment risk

- Bad state = low rate of return
- If IES < 1
  - Income effect dominates
  - $s_B > s_G$
  - Risk aversion increases savings
- Else if IES > 1
  - Substitution effect dominates
  - $s_B < s_G$
  - Risk aversion decreases savings
General Kreps-Porteous Recursion

\[ U_t = (1 - \beta) u_t + \beta \Phi^{-1} \left( \mathbb{E}^{F \times G}_t [\Phi(U_{t+1})] \right), \]

with \[ u_t = \begin{cases} 
  u(c_t) & \text{if alive at } t \\
  v(w_t) & \text{if dead at } t 
\end{cases} \]
Numerical Example of Non-Monotonic Preferences

- Consider EZ utility: \( V(c_0, \tilde{c}_1) = c_0^\frac{1}{2} + (\mathbb{E}[\tilde{c}_1^\frac{1}{2}])^{-1} \).

- Lotteries \( i = \ell_1, \ell_2 \) paying off \((c_0^i, c_d^i)\) or \((c_0^i, c_u^i)\) (50%–50%):

<table>
<thead>
<tr>
<th>Lottery</th>
<th>( c_0^i )</th>
<th>( c_d^i )</th>
<th>( c_u^i )</th>
<th>( V(c_0^i, c_d^i) )</th>
<th>( V(c_0^i, c_u^i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = \ell_1 )</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>9.00</td>
<td>21.58</td>
</tr>
<tr>
<td>( i = \ell_2 )</td>
<td>2</td>
<td>2.5</td>
<td>9</td>
<td>8.97</td>
<td>19.49</td>
</tr>
</tbody>
</table>

\( \Rightarrow \) \( \ell_1 \) always pays off more than \( \ell_2 \).

- BUT, ex ante, \( V(c_0^{\ell_1}, \tilde{c}_1^{\ell_1}) = 11.91 < 12.15 = V(c_0^{\ell_2}, \tilde{c}_1^{\ell_2})! \)
Implications for consumption-saving problems

- Two states $B$, $G$, two periods, constant rate $R$
- $y_B < y_G$ and $s_B > s_G$
- With monotone preferences: $s_B > s^*_m > s_G$
- With EZ preferences, it may be the case that: $s^*_{EZ} > s_B > s_G$, while saving $s_B$ offers a greater lifetime utility in both states $B$ and $G$. 
Re-calibration Without Mortality

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<td>$0.08 \rightarrow 0.58$</td>
<td>$\text{Assets}<em>{45}^{RS} = \text{Assets}</em>{45}^{EZ}$</td>
</tr>
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</table>
\[ V_t = \left( (1 - \beta p_t) c_t^{1-\frac{1}{\varepsilon}} + \beta E_t \left( p_t V_{t+1}^{1-\rho} + (1 - p_t) b \frac{(X_{t+1}/b)^{1-\rho}}{1-\rho} \right)^{\frac{1-\frac{1}{\varepsilon}}{1-\rho}} \right)^{\frac{1}{1-\frac{1}{\varepsilon}}} \]

- Derivative ambiguous if \( \rho > 1 \) and \( \varepsilon < 1 \)
Re-calibration for ‘typical’ EZ Specification

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<td>30.0 $\rightarrow$ 0.0</td>
<td><em>not targeted</em></td>
</tr>
<tr>
<td>Bequest motive, $\theta$</td>
<td>20.0 $\rightarrow$ 0.0</td>
<td><em>exogenous</em></td>
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<td></td>
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<td>0.08 $\rightarrow$ 0.71</td>
<td>$Assets_{45}^{RS} = Assets_{45}^{EZ}$</td>
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