Retirement Financing: An Optimal Reform Approach

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QSPS Summer Workshop 2016
May 19-21
Background and Motivation

• U.S. government has a big role in retirement financing

• Social security benefits are
  ○ 40 percent of all elderly income
  ○ main source of income for almost half of elderly
  ○ 30 percent of federal expenditures

• Social security taxes are 30 percent of federal tax receipts

• Demographic changes pose serious fiscal challenge
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- Social security taxes are 30 percent of federal tax receipts

- Demographic changes pose serious fiscal challenge

⇒ reform needed
Question

• Question: How do we reform retirement system?

• We propose optimal reform:
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• We propose *optimal reform*: Polices that
  
  ◦ minimize cost of tax and transfers to the government, while
  ◦ respect individual behavioral responses
  ◦ respect distribution of welfare in the economy
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• We propose *optimal reform*: Polices that
  - minimize cost of tax and transfers to the government, while
  - respect individual behavioral responses
  - respect distribution of welfare in the economy

• To do this, we need:
  - a model that is a good description of the US economy
  - an approach that puts no ad hoc restriction on policy instruments
What We Do

- OLG model with many periods and heterogeneous agent
  - heterogeneous in labor productivity and mortality
  - labor productivity and mortality are correlated
  - no annuity market
  - US tax and transfer, and social security

- Model is calibrated to US aggregates
  - Consistent with distributional aspects

- We use the model to compute
  - lifetime welfare for each individual, i.e. status-quo welfare
What We Do

- A Mirrlees optimal nonlinear tax exercise
  - taxes cannot be conditioned on individual characteristics
  - no other restrictions on tax instruments

- We look for policies that
  1. minimize the NPDV of transfers to each generation
  2. do not lower anyones lifetime welfare relative to status-quo
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What We Find

• *Progressive* asset *Subsidies* – especially post retirement
  
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  cannot improve upon status-quo using only tax and transfer reform

• Ignoring progressivity is costly
  
  linear asset subsidies achieve only a fraction of cost saving
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  rates are higher than status-quo (not by much)
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Related Literature

  study reforms in limited set of instruments, not necessarily optimal

  maximize social welfare ⇒ mix redistribution with improving efficiency

- **Pareto efficient taxation**: Werning (2007)
  theoretical framework, static model

- **Imperfect annuity market and the effect of social security**: Hubbard-Judd (1987), Hong and Rios-Rull (2007), Hosseini (2015), Caliendo et al. (2014),...
  social security does not provide large efficiency gains
Outline

• Model

• Optimal Reform: Theory
  qualitative properties of efficient allocation

• Calibration

• Optimal Reform: Numbers
  distortions: efficient allocation vs status-quo
  optimal policies
  aggregate effects

• Conclusion
Individuals

- Large number of finitely lived individuals born each period
  - Population grows at constant rate $n$
  - There is a maximum age $T$

- Individuals are indexed by their type $\theta$:  
  - Drawn from distribution $F(\theta)$
  - Fixed through their lifetime

- Individual of type $\theta$
  - Has – deterministic – earnings ability $\varphi_t(\theta)$ at age $t$
  - Has survival rate $p_{t+1}(\theta)$ at age $t$

- Assumption: $\varphi'_t(\theta) > 0$ and $p'_{t+1}(\theta) > 0$ for all $t, \theta$
Preferences and Technology

- Individual $\theta$ has preference over consumption and leisure

$$\sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t) - v(l_t)]$$

where $P_t(\theta) = \Pi_{s=0}^{t} p_s(\theta)$

- Everyone retires at age $R$:

$$\phi_t(\theta) = 0 \text{ for } t > R \text{ for all } \theta$$

- Aggregate production function

$$\bar{Y} = (\bar{r} + \delta)K + L$$

$\bar{r}$: pre-tax rate of return net of depreciation

$\delta$: depreciation rate
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• Everyone retires at age $R$: $\phi_t(\theta) = 0$ for $t > R$ for all $\theta$

• Aggregate production function

$$Y = (\tilde{r} + \delta)K + L$$

$\delta$: depreciation rate

$\tilde{r}$: pre-tax rate of return net of depreciation
Markets and Government

- There is no annuity and/or life insurance, only risk free assets
  - upon death, the risk-free assets convert to bequest
  - bequest is transferred equally to all individuals alive

- Government
  - Collects taxes on labor earnings, consumption and corporate profit
  - Makes transfers to individuals in pre- and post-retirement ages
  - Makes exogenously given purchases

\[ G + (r - n)D + \text{All Transfers} = \text{All Taxes} \]
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government consumption purchases – exogenous
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\[ G + (r - n)D + All 

\textit{All Transfers} = \textit{All Taxes} \]

steady state government debt – exogenous
Individual Optimization Problem

- Individual of type $\theta$ solves

$$U(\theta) = \max \sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t) - v(l_t)]$$

subject to

$$(1 + \tau_c) c_t + a_{t+1} = \varphi_t(\theta) l_t - T_y(\varphi_t(\theta) l_t) + Tr_t + S_t(E_t)$$

$$(1 + r) a_t - T_a((1 + r) a_t)$$
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$$a_{t+1} : \text{asset holding}$$
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$$(1 + r) a_t - T_a ((1 + r) a_t)$$

$\varphi_t(\theta) l_t$: labor earning
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$Tr_t$ : transfer to workers pre-retirement
Individual Optimization Problem

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$$(1 + r)a_t - Ta ((1 + r)a_t)$$

$E_t$: the average labor earning history
Individual Optimization Problem

- Individual of type \( \theta \) solves

\[
U(\theta) = \max \sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t) - v(l_t)]
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subject to

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(1 + \tau_c)c_t + a_{t+1} = \varphi_t(\theta)l_t - T_y(\varphi_t(\theta)l_t) + Tr_t + S_t(E_t)
\]

\[
(1 + r)a_t - T_a((1 + r)a_t)
\]

\( S_t \): social security benefit – paid only after retirement
Individual Optimization Problem

- Individual of type $\theta$ solves

$$U(\theta) = \max \sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t) - v(l_t)]$$

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$r$: after tax return on asset
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$$(1 + r) a_t - T_a ((1 + r) a_t)$$

- There is a corporate tax profit $\tau_K$

$$r = (1 - \tau_K) \tilde{r}$$
Equilibrium

- Equilibrium is set of allocations, factor prices and policies such that
  - Individuals optimize – taking policies as given
  - factors are paid marginal product
  - government budget holds
  - markets clear and allocations are feasible

- Once we know equilibrium allocations we can find status-quo welfare

\[ W_s(\theta) \equiv \sum_{t=0}^{T} \beta^t P_t(\theta) [u(c_t) - v(l_t)] \]
Optimal Policy Reform

• So far we have imposed no restriction on policies

• We can choose them to match the US system

• Or, we can choose them to be *optimal*

• Optimal means
  
  they deliver status-quo welfare at the lowest cost

• We characterize optimal policies next
A Cost Minimization Problem

\[
\min \{ \text{PDV of Net Transfers to a Generation} \}
\]

s.t.

1- given policies \( \{ T_y(\cdot), T_a(\cdot), \ldots \} \), individual optimize

2- resulting allocation delivers no less welfare than status-quo
A Cost Minimization Problem

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\min \left\{ \text{PDV of Net Transfers to a Generation} \right\}
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s.t.

1- given policies \( \{T_y(\cdot), T_a(\cdot), \ldots\} \), individual optimize

2- resulting allocation delivers no less welfare than status-quo

- This is a very complicated problem
  - choice variables are functions
  - constraint set is function of those functions!
A Cost Minimization Problem

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2- resulting allocation delivers no less welfare than status-quo

- This is a very complicated problem
  
  choice variables are functions
  
  constraint set is function of those functions!

- Instead, we use \textit{primal approach}
  
  write the problem only in terms of allocations
A Cost Minimization Problem
Planning Problem

\[
\min \int \sum_{t=0}^{T} \frac{P_t(\theta)}{(1 + r)^t} \left[ c_t(\theta) - \phi_t(\theta) l_t(\theta) \right] dF(\theta)
\]

s.t.
A Cost Minimization Problem
Planning Problem

\[
\begin{align*}
\min & \int \sum_{t=0}^{T} \frac{P_t(\theta)}{(1 + r)^t} \left[ c_t(\theta) - \varphi_t(\theta) l_t(\theta) \right] dF(\theta) \\
\text{s.t.} & \quad U(\theta) = \sum_{t=0}^{T} \beta^t P_t(\theta) \left[ u(c_t(\theta)) - v(l_t(\theta)) \right]
\end{align*}
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\[
U'(\theta) = \sum_{t=0}^{T} \beta^t P_t(\theta) \frac{\varphi'_t(\theta)}{\varphi_t(\theta)} l_t(\theta) v'(l_t(\theta)) + \sum_{t=0}^{T} \beta^t P'_t(\theta) \left[ u(c_t(\theta)) - v(l_t(\theta)) \right]
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U(\theta) &\geq W_s(\theta)
\end{align*}
\]
A Cost Minimization Problem
Planning Problem

\[
\min \int \sum_{t=0}^{T} \frac{P_t(\theta)}{(1+r)^t} \left[ c_t(\theta) - \varphi_t(\theta) l_t(\theta) \right] dF(\theta)
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s.t.

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U(\theta) = \sum_{t=0}^{T} \beta^t P_t(\theta) \left[ u(c_t(\theta)) - v(l_t(\theta)) \right]
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\]

\[
U(\theta) \geq W_s(\theta)
\]

status-quo welfare for each \( \theta \)
Properties of the Efficient Allocations

• Next, we investigate some properties of efficient allocations

• What margins should be distorted and why?

• Note that distortions ≠ taxes necessarily

• But are informative statistics about efficient allocations
Distortions

• Intra-temporal distortion: distorting labor supply margin

\[ 1 - \tau_{\text{lab}} = \frac{v'(l_t(\theta))}{\varphi_t(\theta) u'(c_t(\theta))} \]

• Inter-temporal distortion: distorting “annuity margin”

\[ 1 - \tau_{\text{annu}} = \frac{u'(c_t(\theta))}{\beta(1 + r)u'(c_{t+1}(\theta))} \]
Distortions

• Intra-temporal distortion: distorting labor supply margin

$$1 - \tau_{\text{labor}} = \frac{v'(l_t(\theta))}{\varphi_t(\theta) u'(c_t(\theta))}$$

• Inter-temporal distortion: distorting MRS b/w $c_t$ and $c_{t+1}$

$$1 - \tau_{\text{annuity}} = \frac{u'(c_t(\theta))}{\beta(1 + r)u'(c_{t+1}(\theta))}$$
Intra-temporal Distortions

- Mirrlees-Diamond-Saez formula (Standard)

\[
\frac{\tau_{\text{labor}}}{1 - \tau_{\text{labor}}} = \left( \frac{1}{\epsilon(\theta)} + 1 \right) \frac{1 - F(\theta)}{\theta f(\theta)} g(\theta)
\]
Intra-temporal Distortions

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Behavioral response: captured by elasticity of labor supply
Intra-temporal Distortions

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\]

Tail trade-off: taxing type \( \theta \):

- reduces output in proportion to \( \theta f(\theta) \),
- but relaxes incentive constraints for all types above
Intra-temporal Distortions

- Mirrlees-Diamond-Saez formula (Standard)

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\frac{\tau_{\text{labor}}}{1 - \tau_{\text{labor}}} = \left( \frac{1}{\epsilon(\theta)} + 1 \right) \frac{1 - F(\theta)}{\theta f(\theta)} g(\theta)
\]

Social value of resource extraction from type \( \theta \) and above

\[
g_t(\theta) = \int_{\theta}^{\bar{\theta}} \frac{u'(c(\theta))}{u'(c_0(\theta'))} \left[ 1 - \frac{u'(c_0(\theta'))}{\lambda} \right] \frac{dF(\theta')}{1 - F(\theta)}
\]
Inter-temporal Distortions

- Annuity margin (New)

\[ 1 - \tau_{\text{annuity}}(\theta) = \frac{u'(c_t(\theta))}{\beta(1 + r)u'(c_{t+1}(\theta))} = 1 - \frac{p'_{t+1}(\theta)}{p_{t+1}(\theta)} \frac{1 - F(\theta)}{f(\theta)} g(\theta) \]

Intuition: for higher ability future consumption has higher weight
Inter-temporal Distortions

- Annuity margin (New)

\[ 1 - \tau_{\text{annuity}}(\theta) = \frac{u'(c_t(\theta))}{\beta(1 + r)u'(c_{t+1}(\theta))} = 1 - \frac{p'_{t+1}(\theta)}{p_{t+1}(\theta)} \frac{1 - F(\theta)}{f(\theta)} g(\theta) \]

\( p'_{t+1}(\theta) > 0 \Rightarrow \text{annuity is “taxed”} \)
• Annuity margin (New)

\[ 1 - \tau_{\text{annuity}}(\theta) = \frac{u'(c_t(\theta))}{\beta(1+r)u'(c_{t+1}(\theta))} = 1 - \frac{p'_{t+1}(\theta)}{p_{t+1}(\theta)} \frac{1 - F(\theta)}{f(\theta)} g(\theta) \]

\[ p'_{t+1}(\theta) > 0 \Rightarrow \text{under-insurance is optimal} \]
Inter-temporal Distortions

- Annuity margin (New)

\[
1 - \tau_\text{annuity}(\theta) = \frac{u'(c_t(\theta))}{\beta(1+r)u'(c_{t+1}(\theta))} = 1 - \frac{p'_{t+1}(\theta)}{p_{t+1}(\theta)} \frac{1 - F(\theta)}{f(\theta)} g(\theta)
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\[p'_{t+1}(\theta) > 0 \Rightarrow \text{under-insurance is optimal}\]

- Intuition: for higher ability future consumption has higher weight
Implementation: Finding Optimal Taxes

• So far we only talked about distortions
  ○ these are properties of allocations
  ○ they are not tax functions
Implementation: Finding Optimal Taxes

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• Tax function: a map between a tax base and tax obligations
Implementation: Finding Optimal Taxes

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  - these are properties of allocations
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- Tax function: a map between a tax base and tax obligations

- We propose a set of taxes
  - A nonlinear tax (subsidy) on assets: \( T_{a,t} ((1 + r)a_t) \)
  - A nonlinear tax on labor earnings: \( T_{y,t} (y_t) \)
  - A type-independent retirement transfer: \( S_t \)
Implementation: Finding Optimal Taxes

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- Tax function: a map between a tax base and tax obligations

- We propose a set of taxes
  - A nonlinear tax (subsidy) on assets: $T_{a,t}((1 + r)a_t)$
  - A nonlinear tax on labor earnings: $T_{y,t}(y_t)$
  - A type-independent retirement transfer: $S_t$

- We can solve these tax functions numerically
Calibration

1. Parametrize and estimate earning ability $\varphi_t(\theta)$

2. Parametrize and calibrate model of mortality $P_t(\theta)$

3. Parametrize and calibrate government policy – to US status-quo

4. Parametrize and calibrate preference and technology
Calibration

1. Parametrize and estimate earning ability $\varphi_t(\theta)$

2. Parametrize and calibrate model of mortality $P_t(\theta)$

3. Parametrize and calibrate government policy – to US status-quo

4. Parametrize and calibrate preference and technology

- Do 1, 2 and 3 independent of the model
- Use the model to do 4
Earning Ability Profiles

- Use labor income per hour as proxy for working ability (PSID)

- Assume

  \[ \log \varphi_t(\theta) = \log \theta + \log \tilde{\varphi}_t \]

  with

  \[ \log \tilde{\varphi}_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 \]

- \( \theta \) has Pareto-Lognormal distribution w/ parameters \((\mu_\theta, \sigma_\theta, a_\theta)\)

  \[ a_\theta = 3 \text{ is tail parameter } \rightarrow \text{standard} \]
  \[ \sigma_\theta = 0.6 \text{ is variance parameter } \rightarrow \text{variance of log wage in CPS} \]
  \[ \mu_\theta = -1/a_\theta \text{ is location parameter} \]
• Assume Gompertz force of mortality hazard

\[ \lambda_t(\theta) = \frac{m_0}{\theta m_1} \left( \exp(m_2 t) / m_2 - 1 \right) \]

and

\[ P_t(\theta) = \exp(-\lambda_t(\theta)) \]

- \( m_1 \) which determines ability gradient
- \( m_2 \) determines overall age pattern of mortality
- \( m_0 \) is location parameter

• Use SSA’s male mortality for 1940 birth cohort

• Use Waldron (2013) death rates (for ages 67-71)
Death Rates by Lifetime Earning Deciles

Mortality Rate by Income Decile, ages 67 to 71

- Data - Waldron (2013)
- Model
Status-quo Government Policies

- Government collects three types of taxes
  - non-linear progressive tax on taxable income – we use
    \[ T(y) = y - \phi y^{1-\tau}, \]
    the HSV tax function (\(\tau = 0.151, \phi = 4.74\))
  - FICA payroll tax – we use SSA’s tax rates
  - linear consumption tax – McDaniel (2007)

- there is also a social security and Medicare benefit
  - we use SSA’s benefit formula
  - 3% of GDP, paid equally to all retirees
Preferences

- Utility over consumption and hours

\[ u(c) - v(l) = \log(c) - \psi \frac{l^{1+\frac{1}{\epsilon}}}{1 + \frac{1}{\epsilon}} \]

- We choose \( \epsilon = 0.5 \)

- \( \psi \) and \( \beta \) are chosen to match aggregate moments.
## Parameters Chosen Outside the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values/source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>maximum age</td>
<td>75 (100 y/o)</td>
</tr>
<tr>
<td>$R$</td>
<td>retirement age</td>
<td>40 (65 y/o)</td>
</tr>
<tr>
<td>$n$</td>
<td>population growth rate</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of labor supply</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Productivity</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\theta, a_\theta, \mu_\theta$</td>
<td>PLN parameters</td>
<td>0.5,3,-0.33</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>return on capital/assets</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Government policies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{ss}, \tau_{med}, \tau_c$</td>
<td>tax rates</td>
<td>0.124,0.029,0.055</td>
</tr>
<tr>
<td>$G$</td>
<td>government expenditure</td>
<td>$0.09 \times GDP$</td>
</tr>
<tr>
<td>$D$</td>
<td>government debt</td>
<td>$0.5 \times GDP$</td>
</tr>
</tbody>
</table>
# Parameters Calibrated Using the Model

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth-income ratio</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Average annual hours</td>
<td>2000</td>
<td>2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values/source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>discount factor</td>
<td>0.981</td>
</tr>
<tr>
<td>( \psi )</td>
<td>weight on leisure</td>
<td>0.74</td>
</tr>
</tbody>
</table>

[Show Distribution of Earnings, Assets]
Optimal Policy Reform

• We can now use our calibrated model to
  ◦ Solve for status-quo allocations
  ◦ Solve for efficient allocations

• Under both set of allocations we can calculate distortions

• The difference between two sets of distortions motivates policy reform

• We can also use the model to compute optimal tax functions
Inter-Temporal Distortions: Annuitization Margin

\[ 1 - \tau_{\text{annuity}} = \frac{u'(c_t(\theta))}{\beta(1 + r)u'(c_{t+1}(\theta))} \]
Intra-Temporal Distortions: Labor Supply Margin

\[ 1 - \tau_{\text{labor}} = \frac{v'(l_t(\theta))}{\varphi_t(\theta) u'(c_t(\theta))} \]
Optimal Asset Taxes (Subsidies)

![Graph of Optimal Asset Taxes (Subsidies)](image)

- **Marginal Tax, 70 yrs old**
- **Marginal Tax, 80 yrs old**
- **Average Tax, 70 yrs old**
- **Average Tax, 80 yrs old**

Assets (x10^6) vs. \(\%\)
Optimal Labor Income Taxes

![Graph showing optimal labor income taxes. The graph compares marginal and average tax rates for different income levels.]
## Aggregate Effects

<table>
<thead>
<tr>
<th>Shares of GDP</th>
<th>Status-quo</th>
<th>Reform (efficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>Capital</td>
<td>3.00</td>
<td>3.67</td>
</tr>
<tr>
<td>Government Debt</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td>Net worth</td>
<td>3.53</td>
<td>3.78</td>
</tr>
<tr>
<td>Tax Revenue (Total)</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>Labor income tax</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Capital tax</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Government Transfers (Total)</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td>To retirees</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>To workers</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Asset subsidy</td>
<td>0</td>
<td>0.07</td>
</tr>
</tbody>
</table>

PDV of net transfers to each cohort falls by 9.3%
How Important Are Asset Subsidies?

- What is the best that can be achieved without them?
How Important Are Asset Subsidies?

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- We can include the following restriction in our planning problem

\[ P_t(\theta)u'(c_t) = \beta(1 + r)P_{t+1}(\theta)u'(c_{t+1}) \]

- The resulting allocations cost 0.5% more than status-quo

Implication: IF proper asset subsidies are not in place, phasing out old-age transfers is not a good idea!
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  IF proper asset subsidies are not in place, phasing out old-age transfers is not a good idea!
How Important is Progressivity of Asset Subsidies?

- Progressivity is a consequence of differential mortality

\[ P_t(\theta) u'(c_t) = (1 - \tau_t + 1) \beta (1 + r) P_{t+1}(\theta) u'(c_{t+1}) \]

and find optimal \( \tau_t \)'s

- The resulting allocations cost 3% less than status-quo i.e., one third of the cost saving, relative to fully optimal

Implication: differential mortality matters for optimal policy!
How Important is Progressivity of Asset Subsidies?

- Progressivity is a consequence of differential mortality
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Conclusion

• This paper has two main contributions:

1. It develops a methodology to study optimal policy reform that does not rely on an arbitrary social welfare function allows separation of efficiency gains from redistribution

2. It points to a novel reason for subsidizing assets To correct for in-efficiencies due to imperfect annuity markets
Conclusion

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2. It points to a novel reason for subsidizing assets To correct for in-efficiencies due to imperfect annuity markets

• Contrast to asset subsidies in the current US system asset subsidies should not stop at retirement asset subsidies must be progressive
Distribution of Wealth

![Graph showing the distribution of wealth with two curves: Model (Gini = 0.61386) and SCF Data (Gini = 0.78).]
A Cost Minimization Problem

- We start by writing objective in terms of allocations only

- From individual budget constraint PDV of Net Transfers is equal to

\[
\min \int \sum_{t=0}^{T} \frac{P_t(\theta)}{(1 + r)^t} \left[ c_t(\theta) - \varphi_t(\theta) l_t(\theta) \right] dF(\theta)
\]

for any set of tax and transfers
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Intuition: Static Model

\[ c = \varphi(\theta)l - T \]
A Cost Minimization Problem

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for any set of tax and transfers

Intuition: Static Model

$$c - \varphi(\theta) l = -T$$
A Cost Minimization Problem

• For any set of policies, let \( \{ c_t(\theta), l_t(\theta) \} \) individual choices

• Let \( U(\theta) \) be utility associated with this allocation
A Cost Minimization Problem

- For any set of policies, let \( \{c_t(\theta), l_t(\theta)\} \) individual choices

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- Then

\[
U'(\theta) = \sum_{t=0}^{T} \beta^t P_t(\theta) \frac{\varphi'_t(\theta) l_t(\theta)}{\varphi_t(\theta)} \nu'(l_t(\theta)) + \sum_{t=0}^{T} \beta^t P'_t(\theta) [u(c_t(\theta)) - \nu(l_t(\theta))] 
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\]

• This is called implementability constraint
A Cost Minimization Problem

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- Then

\[
U'(\theta) = \sum_{t=0}^{T} \beta^t p_t(\theta) \frac{\phi'_t(\theta) l_t(\theta)}{\phi_t(\theta)} v'(l_t(\theta)) + \sum_{t=0}^{T} \beta^t p'_t(\theta) [u(c_t(\theta)) - v(l_t(\theta))]
\]

- This is called *implementability constraint*

Intuition: Static Model

\[
U(\theta) = \max u(c) - v(l) \text{ s.t. } c = \phi(\theta) l - T(\phi(\theta) l)
\]
A Cost Minimization Problem

- For any set of policies, let \( \{c_t(\theta), l_t(\theta)\} \) individual choices

- Let \( U(\theta) \) be utility associated with this allocation

- Then

\[
U'(\theta) = \sum_{t=0}^{T} \beta^t p_t(\theta) \frac{\varphi'_t(\theta) l_t(\theta)}{\varphi_t(\theta)} v'(l_t(\theta)) + \sum_{t=0}^{T} \beta^t p'_t(\theta) [u(c_t(\theta)) - v(l_t(\theta))]
\]

- This is called implementability constraint

**Intuition: Static Model**

\[
U(\theta) = \max u(c) - v \left( \frac{y}{\varphi(\theta)} \right) \quad \text{s.t.} \quad c = y - T(y)
\]
A Cost Minimization Problem

- For any set of policies, let \( \{c_t(\theta), l_t(\theta)\} \) individual choices

- Let \( U(\theta) \) be utility associated with this allocation

- Then

\[
U'(\theta) = \sum_{t=0}^{T} \beta^t P_t(\theta) \frac{\varphi_t'(\theta) l_t(\theta)}{\varphi_t(\theta)} v'(l_t(\theta)) + \sum_{t=0}^{T} \beta^t P_t'(\theta) [u(c_t(\theta)) - v(l_t(\theta))]
\]

- This is called implementability constraint

Intuition: Static Model

\[
U'(\theta) = \frac{\varphi'(\theta) l(\theta)}{\varphi(\theta)} v'(l(\theta))
\]
Implementation: Finding Optimal Taxes

- We have set of individual FOC’s

\[ P_t(\theta)u'(c_t) = \beta(1 + r)P_{t+1}(\theta)(1 - T'_{a,t+1})u'(c_{t+1}) \]

\[ (1 - T'_{y,t})\varphi_t(\theta)u'(c_t) = v'(l_t) \]

- We also have their budget constraints

- Using these equations we can back-out tax and transfers such that efficient allocations are implemented
Implementation: Finding Optimal Taxes

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\[
P_t(\theta)u'(c_t) = \beta(1 + r)P_{t+1}(\theta)(1 - T'_{a,t+1})u'(c_{t+1})
\]
\[
(1 - T'_{y,t})\varphi_t(\theta)u'(c_t) = \upsilon'(l_t)
\]

- We also have their budget constraints

- Using these equations we can back-out tax and transfers such that efficient allocations are implemented

- Before, doing that we need to calibrate the model
Unconditional Survival Probabilities

![Survival Probabilities Graph]

- $P_t(\theta)$
- Age
- 10th percentile (lifetime income)
- Median
- 90th percentile

Roozbeh Hosseini (UGA)
Earnings Ability Profiles

![Earning Ability Profiles Graph]

- $\varphi_t(\theta)$, 2000$/hour$
- Age
- 10th percentile
- Median
- 90th percentile
Source of Retirement Income

Share of Public Transfers in Retirement Income

100

Data (Poterba (2014))
Status-quo
Efficient

%

1st 2nd 3rd 4th
Income Quartiles
Consumption for pre- and post- Retirement

![Graph showing efficient consumption relative to status-quo](image-url)
Optimal Replacement Ratio

Retirement Transfers relative to Average Lifetime Earning

- Status-quo
- Status-quo (excl. Medicare)
- Efficient

Average Lifetime Earning $\times 10^5$