

Finite Horizon Life-cycle Horizon Learning

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What I do

Develop a new model of bounded rationality

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- Finite Horizon Learning, *within* a Life-cycle model

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Develop a new model of bounded rationality

- Finite Horizon Learning, *within* a Life-cycle model
- Simulate social security policy changes and recessions

Why it matters

- Extend adaptive learning literature into a new class of models

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- Show rational expectations equilibrium is stable under learning

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- Develop new framework for modeling announced/surprise changes

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- Extend adaptive learning literature into a new class of models
- Show rational expectations equilibrium is stable under learning
- Develop new framework for modeling announced/surprise changes
- Learning dynamics propagate recession shock; introduce overshooting for announced policy changes

Outline

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions

Expectations

Two main approaches to modeling expectations

Expectations

Two main approaches to modeling expectations

- Rational Expectations

Expectations

Two main approaches to modeling expectations

- Rational Expectations
- Adaptive Learning
 - Sargent (1993), Evans and Honkapohja (2001)

Adaptive Learning

- Reduced form adaptive learning
 - Evans and Honkapohja (2001) and Bullard and Mitra (2002)

Adaptive Learning

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 - Evans and Honkapohja (2001) and Bullard and Mitra (2002)
- Micro-foundations

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- Micro-foundations
 - Euler-equation learning (Honkapohja, Mitra, and Evans (2002), Evans and Honkapohja (2006))
 - Infinite Horizon Learning (Marcet and Sargent (1989), Preston (2005), Bullard and Russell (1999))

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 - Infinite Horizon Learning (Marcet and Sargent (1989), Preston (2005), Bullard and Russell (1999))
 - Finite Horizon Learning (Branch, Evans, and McGough (2013))

Finite Horizon Learning

Finite Horizon Learning appealing assumption

- Real life forecasts are over a finite horizon

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- Allows agents to respond to announced policy (Evans et al. (2009), Mitra and Evans (2013), Gasteiger and Zhang (2014), Caprioli (2015))

Finite Horizon Learning

Finite Horizon Learning appealing assumption

- Real life forecasts are over a finite horizon
- Allows agents to respond to announced policy (Evans et al. (2009), Mitra and Evans (2013), Gasteiger and Zhang (2014), Caprioli (2015))
- *Somewhat* similar in spirit to short-planning horizon literature
 - Park and Feigenbaum (2017), Caliendo and Aadland (2007), Woodford (2019), Findley and Caliendo (2019), Findley and Cottle Hunt (2019)

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Model Summary

- Households
- Government
- Firms
- Competitive Markets

Model Summary

- Households
 - Work and pay taxes; retire and receive social security
 - Choose savings and consumption to maximize utility
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Model Summary

- Households
 - Work and pay taxes; retire and receive social security
 - Choose savings and consumption to maximize utility
- Government
 - Taxes workers, pays retirement benefits, issues bonds
- Firms
 - Turn labor and capital into output
- Competitive Markets
 - Determine prices of labor, capital, bonds, and output

a few details

formal definition

equations

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Expectations: Adaptive Learning

New Model: Finite Horizon Life-cycle Learning

- Agents combine limited structural knowledge of macroeconomy with full knowledge of government policy
- as in Evans, Honkapohja, and Mitra (2009, 2013)

Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length H

Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length H
- Agents forecast prices using adaptive expectations

Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length H
- Agents forecast prices using adaptive expectations
- Decisions are optimal, conditional on expected future savings

HRS expectations table

Finite Horizon Life-cycle Learning

Agents forecast wages, (w), the gross interest rate (R) and government bonds (b) adaptively:

Finite Horizon Life-cycle Learning

Agents forecast wages, (w), the gross interest rate (R) and government bonds (b) adaptively:

$$w_{t+1}^e = \gamma w_t + (1 - \gamma) w_t^e$$

with a gain parameter $\gamma \in (0, 1)$.

similar equations with same gain for interest rate and bonds

Finite Horizon Life-cycle Learning

also forecast a terminal asset holding

$$a_{t,terminal}^{j,e} = \gamma a_{t-1}^j + (1 - \gamma) a_{t-1,terminal}^{j,e}$$

for $j = 1, \dots, J - 1$

$a_{t,terminal}^{j,e}$ is amount of assets an agent expects to hold at the end of age j .

$a^6 = 0$; agents deplete their savings account at the end of the lifecycle

Finite Horizon Life-cycle Learning

Suppose planning horizon $H = 2$

Finite Horizon Life-cycle Learning

Suppose planning horizon $H = 2$

- Young agent chooses consumption and savings (c^1 and a^1) and plans for the next period (c^2 and a^2) according to:

$$u'(c_t^1) = \beta R_{t,t+1}^e u'(c_{t,t+1}^2)$$

$$u'(c_{t,t+1}^2) = \beta R_{t,t+2}^e u'(R_{t,t+2}^e a_{t,t+2}^2 + y_{t,t+2}^e - a_{t,terminal}^{3,e})$$

where $y_{t,t+2}^e$ is the time t expectation of age $t+2$ income, and $a_{t,terminal}^{3,e}$ is the terminal condition

Finite Horizon Life-cycle Learning

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- Older agents are following similar process choosing consumption and savings according to planning horizon and forecasts

where $y_{t,t+2}^e$ is the time t expectation of age $t + 2$ income, and $a_{t,terminal}^{3,e}$ is the terminal condition

Finite Horizon Life-cycle Learning

For a planning horizon of length H , and J cohorts, there will be $J - H$ terminal conditions and $H(J - H) + \frac{H(H-1)}{2}$ household first order equations.

Together,

- the decisions of households of all ages
- asset market and bond clearing
- expectation equations

create a recursive system that governs the dynamics of the economy.

RE model is stable under Finite Horizon Life-cycle Learning

Parameterization

life-cycle modeled as six decade-long periods

gain parameter $\gamma = 0.93$ set to minimize welfare cost of learning relative to RE

calibration details

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Examples

Social security reform

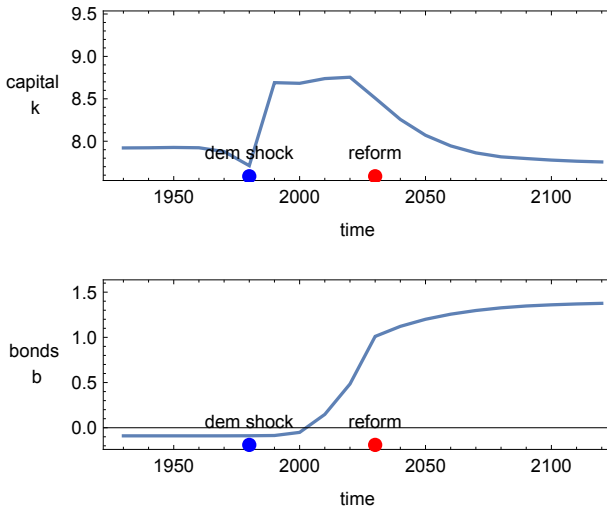
Recession

conclusion

Social Security Reform

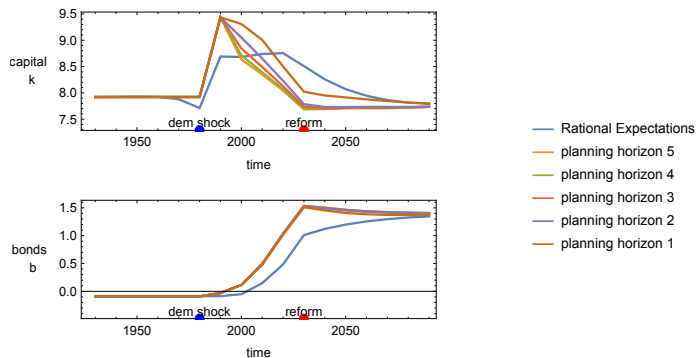
- Demographic change beginning in 1980
- social security tax increase in 2030

Social Security Reform

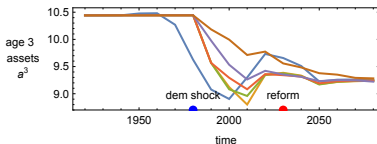
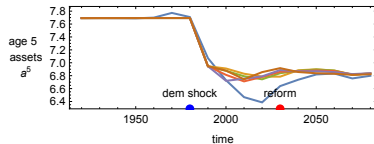
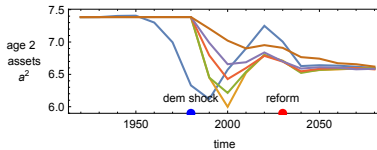
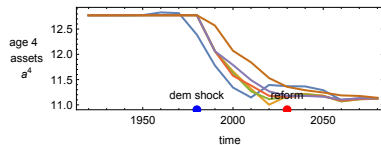
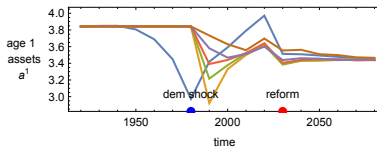


rational expectations

Social Security Reform



Social Security Reform

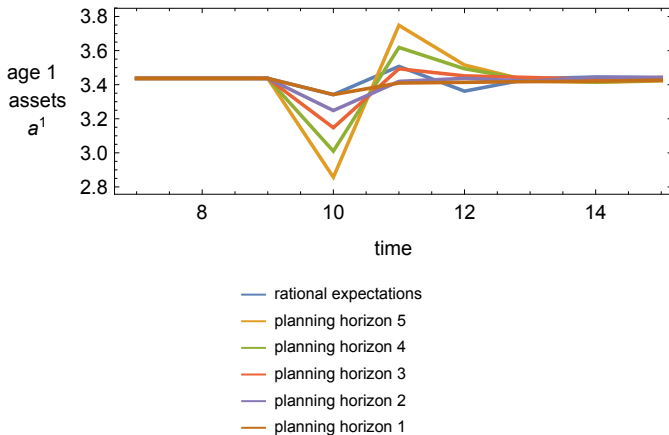


- rational expectations
- planning horizon 5
- planning horizon 4
- planning horizon 3
- planning horizon 2
- planning horizon 1

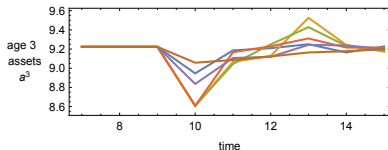
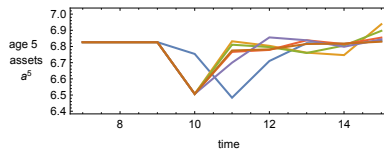
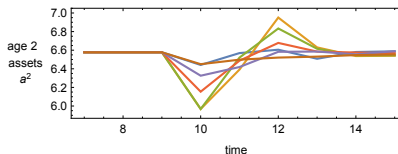
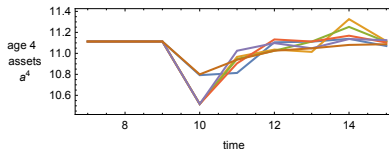
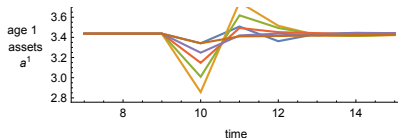
Recession

Surprise, one-period recession, modeled as TPF reduction

Recession: Savings

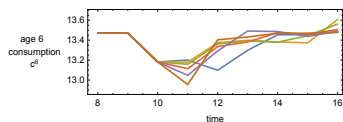
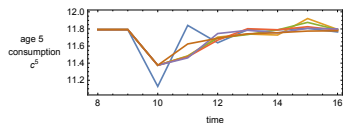
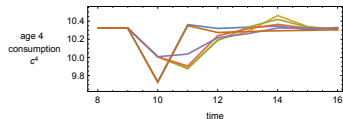
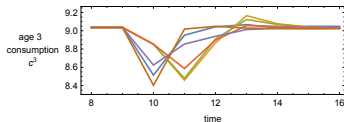
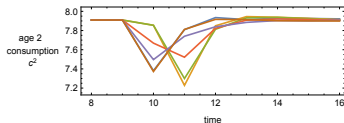
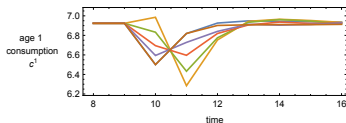


Recession: Savings



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Recession: Consumption



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Welfare Comparison

compares the life-time utility initial steady state with life-time utility in any other period

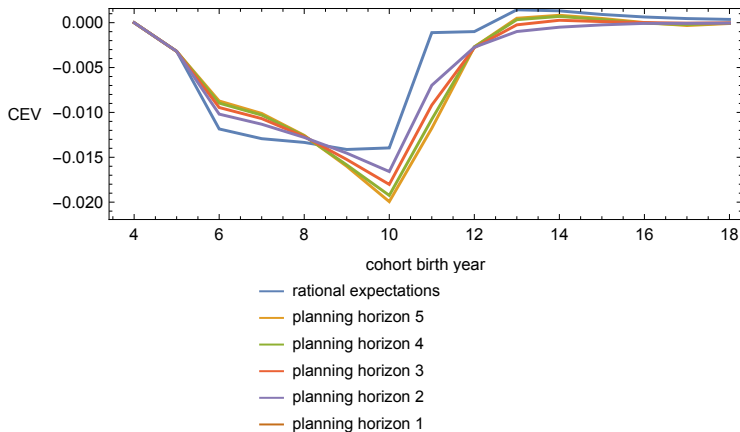
$$\sum_{j=1}^J \beta^{j-1} u(c_{ss}^j(1 + \Delta)) = \sum_{j=1}^J \beta^{j-1} u(c_{t+j-1}^j)$$

Δ consumption equivalent variation (CEV)

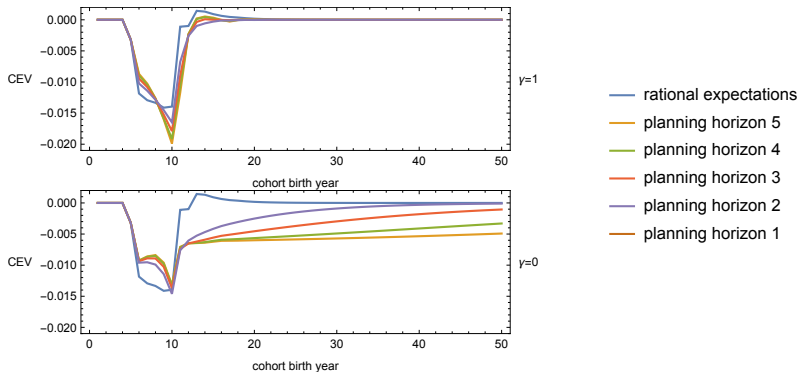
c_{ss}^j is the consumption in the initial steady state

c_{t+j-1}^j is the consumption of an agent age j in time period $t + j - 1$

Recession: CEV



Recession: CEV different gain parameters

[gain parameter selection](#)[gain parameter graph](#)[other examples](#)[conclusion](#)

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New modeling framework

- Embeds finite horizon learning in a lifecycle model
 - E-stability result

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 - Longer planning horizon
 - Respond to announced policy sooner
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- Embeds finite horizon learning in a lifecycle model
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- Trade-off in optimal gain parameter γ
 - Small γ optimal for temporary shocks
 - Large γ optimal for permanent shocks

Next Steps

- Calibrate model (rather than parameterize)
- refine examples (add others?)
- submit paper!

Extensions

Finite Horizon Life-cycle Learning

- Great Recession and fiscal policy
- Unfunded liabilities and explosive debt
- Optimal gain parameter or planning horizon
- Euler-equation learning in life-cycle model

The end

Thank you!

Contribution

- Adaptive Learning

Contribution

- Adaptive Learning
 - New model of Finite Horizon Life-cycle Learning [learning references](#)

Model details

Demographics

- Agents live for J periods and work the first T periods of life
- Population grows at rate n_t
- Demographic change modeled as a one-time reduction in n_t

[back: model summary](#)

Model details: Household Problem

Choose savings a^j (consumption c^j) for each age $j = 1, \dots, J$

$$\max_{a_{t+j-1}^j} E_t^* \sum_{j=1}^J \beta^{j-1} u(c_{t+j-1}^j)$$

E_t^* : time t expectation, $*$ indicates not necessarily rational. $\beta < 1$: discount factor.

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$$\max_{a^j_{t+j-1}} E_t^* \sum_{j=1}^J \beta^{j-1} u(c^j_{t+j-1})$$

$$c^j_{t+j-1} + a^j_{t+j-1} \leq R_{t+j-1} a^{j-1}_{t+j-2} + y^j_{t+j-1}$$

E_t^* : time t expectation, * indicates not necessarily rational. $\beta < 1$: discount factor.

R gross interest rate. y^j (age specific):

gross labor income $((1 - \tau)w$, with tax rate τ and wage w)

or social security benefit (z)

Model details: Government

- Payroll tax: τ_t
- Social Security Benefits: $z_t^j = \phi_t w_{t+T-j}$

ϕ : benefit replacement rate. w_{t+T-j} wage at time of retirement.

tax details

Model details: Government

- Payroll tax: τ_t
- Social Security Benefits: $z_t^j = \phi_t w_{t+T-j}$

Government Debt equation:

$$B_{t+1} = R_t B_t + \sum_{j=T}^J N_{t+1-j} \phi_t w_{t+T-j} - \sum_{j=1}^{T-1} N_{t+1-j} \tau_t w_t$$

ϕ : benefit replacement rate. w_{t+T-j} wage at time of retirement.

[tax details](#)

B : total government bonds. R_t : gross interest rate, N_t : number of young at time t , T retirement age

[back: model summary](#)

Model details: Government

$$\tau_t = \tau_t^0 + \tau_t^1(B_t/H_t)$$

- τ_t payroll tax rate
- τ_t^0 base tax rate (e.g. 10%)
- τ_t^1 Leeper tax rate (responds to government debt)
- B_t government debt, H_t working population

Rational Expectations Equilibrium

Definition

Given initial conditions $k_0, b_0, a_{-1}^1, \dots, a_{-1}^{J-1}$, and an initial population $\sum_{j=1}^J (1+n)^{1-j} N_0$ (where N_0 initial cohort of young), a competitive equilibrium is a sequences of functions for the household savings $\{a_t^1, a_t^2, \dots, a_t^J\}_{t=0}^{\infty}$, production plans for the firm, $\{k_t\}_{t=1}^{\infty}$, government bonds $\{b_t\}_{t=1}^{\infty}$, factor prices $\{R_t, w_t\}_{t=0}^{\infty}$, and government policy variables $\{\tau_t^0, \tau_t^1, \phi_t\}_{t=0}^{\infty}$, that satisfy the following conditions:

1. Given factor prices and government policy variables, individuals' decisions solve the household optimization problem
2. Factor prices are derived competitively
3. All markets clear

Rational Expectations Equilibrium

- Households

$$(R_t a_{t-1}^{j-1} + y_t^j - a_t^j)^{-\sigma} = \beta E_t[R_{t+1}(R_{t+1} a_t^j + y_{t+1}^{j+1} - a_{t+1}^{j+1})^{-\sigma}]$$

for $j = 1, \dots, J-1$

- Asset market

$$(k_{t+1} + b_{t+1})(1 + n_t) = \frac{\sum_{j=1}^J N_{t+1-j} a_t^j}{H_t}$$

- Government Debt

$$(1+n_t)b_{t+1} = R_t b_t + \frac{\sum_{j=\tau}^J N_{t+1-j} \phi_t w_{t+\tau-j}}{H_t} - (\tau_t^0 + \tau_t^1 (B_t/H_t)) w_t$$

Model details: Saddle-node bifurcation

Zero, one, or two steady states are possible in the model

- Calibrated to have two steady states
- Parameter change that increases the endogenous social security deficit, drives the steady states closer together
- At a critical value of the relevant parameter, only one steady state exists
- Beyond that, no steady states exist

Numerical analysis (Laitner 1990) of linearized system confirms the high-capital steady state is determinate, the low-capital steady state is explosive

[back](#)[More Stability](#)

Model details: More Stability

Three predetermined variables in the model (k , b , and a^{J-1}) and $J-2$ free variables (a^1, \dots, a^{J-2})

Let λ_i indicate an eigenvalue of the linearized system

- **Determinate** $\lambda_i < 1$ for $i = 1, 2, 3$; the remaining $J - 2$ eigs $\lambda_i > 1$
- **Indeterminate** $\lambda_i < 1$ for more than three, the remaining $\lambda_i > 1$
- **Explosive** $\lambda_i > 1$ for more than $J - 2$ eigs

Note, complex eigs are possible, consider modulus

Model details: E-stability

- Given constant (potentially incorrect) expectations $p^e = (R^e, w^e, b^e, a_{terminal}^{j,e})'$, the learning dynamics of the FHL model asymptotically converge to $p = (R, w, b, a^j)'$

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$$T : \mathbb{R}^{J-H+3} \rightarrow \mathbb{R}^{J-H+3}$$

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- a fixed point of the T map is E-stable if it locally stable under the ODE

$$\frac{dp}{d\tau} = T(p) - p$$

Model details: E-stability

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$$\frac{dp}{d\tau} = T(p) - p$$

- E-stability requires the real parts of the eigenvalues of the derivative matrix $dT < 1$
 - Numerically verified all determinate steady states in the paper are E-stable under FHL learning (at all horizons)

Motivation: short planning horizon

Time horizon	Fraction of Respondents
Next few months	0.18
Next year	0.12
Next few years	0.27
Next 5-10 years	0.31
Longer than 10 years	0.12

Table: Fraction of HRS survey respondents that selected each time horizon in response to the question “in planning your family’s saving and spending, which time period is most important to you?” Table reports mean across waves 1, 4, 5, 6, 7, 8, 11, and 12.

[back](#) Note: In waves 6, 11, and 12 only respondents younger than 65 were asked this question. In all other waves, the full panel of respondents were asked about their financial planning horizon.

Choice of gain parameter

- Compute consumption of a single rational agent in the learning model

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- Choose gain parameter that minimizes the welfare cost to learning agent of not using rational expectations to forecast

Choice of gain parameter

- Compute consumption of a single rational agent in the learning model
- Choose gain parameter that minimizes the welfare cost to learning agent of not using rational expectations to forecast
- Optimal gain parameter near $\gamma = 0.93$

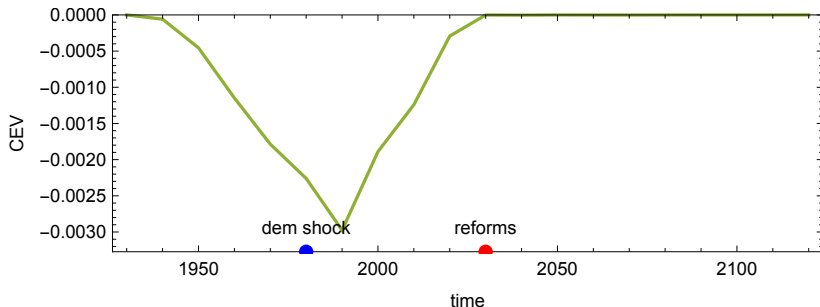
Choice of gain parameter

γ	Minimum CEV	
	Tax increase	Benefit Cut
0.1	-2.56%	-0.51%
0.2	-1.56%	-0.43%
0.3	-1.11%	-0.44%
0.4	-0.84%	-0.41%
0.5	-0.64%	-0.34%
0.6	-0.47%	-0.27%
0.7	-0.39%	-0.23%
0.8	-0.30%	-0.18%
0.9	-0.28%	-0.18%
1	-0.33%	-0.22 %

- Compares the consumption of a single rational agent (in each cohort) living in a world with life-cycle horizon learners
- Learning gain parameter γ chosen to minimize this cost

Choice of gain parameter

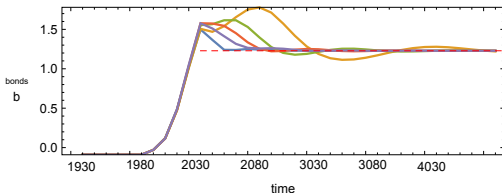
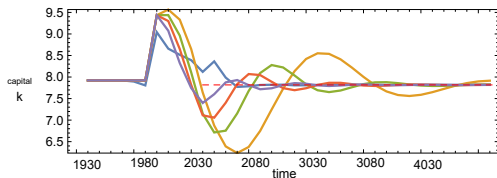
Compute consumption of a single rational agent in the learning model



this experiment is the announced tax increase

Choice of gain parameter

Finite-Horizon Life-cycle Example capital and bond paths:
demographic shock in 1980, tax increase in 2030



- Rational
- Learn $\gamma=0.2$
- Learn $\gamma=0.4$
- Learn $\gamma=0.6$
- Learn $\gamma=0.9$

Calibration

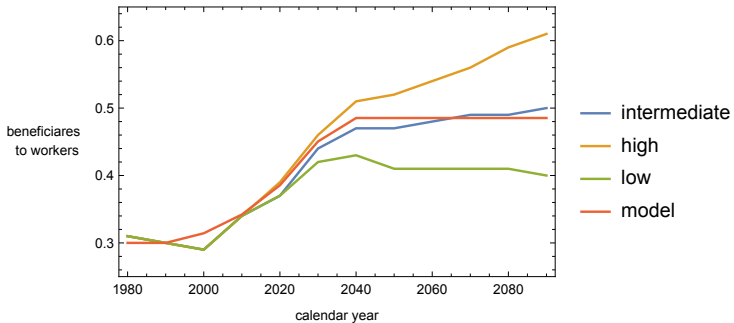
Parameter	Value
J number of periods	6
T retirement date	5
α Capital share of income	$1/3$
β Discount factor	0.995^{10}
σ Inverse elasticity of substitution	1
δ Depreciation	$1 - (1 - 0.10)^{10}$
A TFP factor	10

population growth

go back

Calibration

Population growth rate n is calibrated to match the projected ratio of social security beneficiaries to retirees.



calibration details

References: learning

- Branch, Evans, McGough in *Macroeconomics at the Service of Public Policy* (2013)
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References: Anticipated Fiscal Policy

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- Mitra and Evans, *Journal of Economic Dynamics and Control* (2013)
- Gasteiger and Zhang, *Journal of Economic Dynamics and Control* (2014)
- Caprioli, *Journal of Economic Dynamics and Control* (2015)