Changing parental characteristics and aggregate educational attainment*

Adam Blandin†
Virginia Commonwealth University

Christopher Herrington‡
Virginia Commonwealth University

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Abstract

Between 1968 and 2013, two striking changes occurred in the characteristics of US parents. First, the share of households headed by a single parent grew from 19% to 40%. Second, the share of households in which at least one parent has a four-year college degree grew from 17% to 45%, with most of the increase coming from dual parent households. We first conduct simple accounting exercises to show that each of these trends has substantially impacted the aggregate college graduation rate. We then construct a general equilibrium model of intergenerational human capital investment in which households differ by number and education of parents. Consistent with historical data, equilibrium college graduation rates for children from high-education dual-parent households are high and *elastic* with respect to the college wage premium, while graduation rates for children from low-education, single-parent households are low and *inelastic* with respect to the college premium. Our analysis suggests that further increases in the college wage premium would increase college attainment for children from one large class of households (high-education, dual-parent), but not increase rates for children from another large class (low-education, single-parent).

JEL Classification: J12, J24, E24, E6

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†Contact Info: Department of Economics, Virginia Commonwealth University, P.O. Box 844000, Richmond VA 23284-4000. *Email*: ajblandin@vcu.edu

‡Contact Info: Department of Economics, Virginia Commonwealth University, P.O. Box 844000, Richmond VA 23284-4000. *Email*: herringtoncm@vcu.edu

1 Introduction

In 1968 roughly two-thirds of households were headed by two parents without four-year college degrees. Since then, several striking trends have shaped the composition of U.S. households. The share of households headed by a single parent more than doubled from 19% in 1968 to 40% in 2013. Over the same period, the share of households headed by at least one parent with a four-year college degree rose from 17% to 45%, with most degree-holders heading dual parent households. As a result of these simultaneous trends, two-thirds of households with children now belong to one of two extreme types: single-parent without college education (31%) or dual-parent with college education (36%).

This paper quantitatively studies the impact of these changes in household structure on aggregate educational attainment of children born since 1968.[1] We first discuss the empirical evidence in more detail. We then conduct a series of simple accounting exercises to investigate the impact of changing parental characteristics on aggregate education attainment rates, *ceteris paribus*. We show that aggregate college attainment was negatively affected by the increasing share of single parents and positively affected by the increasing educational attainment of parents overall.

We then investigate why educational attainment differs by household type, why attainment rates within household type changed over the past several decades, and how attainment rates within household type may change in the future. To do so we construct a model of intergenerational human capital transmission in which households differ by the number and education of parents.

In the model parents are assigned to either a single- or dual-parent household according to a stochastic matching function. Parents work, consume, and invest in their children's human capital through a combination of market inputs and time investments. Single parent households have less total time available for either market work or investment in children. Additionally, less educated parents will have less productive time (both in market work and investments in children) compared to more educated parents. Household differences in the number of parents, education of parents, and number of children will collectively result in different human capital investments across children.

As childhood ends, new young adults decide whether or not to enroll in college. If

[1]Notably, we take the demographic trends as given and study the implications for household decision making and aggregate educational attainment. Explaining the source of those trends is beyond the scope of our work; however, recent work by Greenwood et al. (2016) explores technological changes in home production, returns to education, and the shrinking gender wage gap as potential causes of the trends in marriage, divorce, and educational attainment.

the individual graduates college, their human capital is "skilled", and commands a different price than the human capital of non-college graduates. College is risky, in the sense that tuition must be paid up front but the student only graduates with a probability less than one. Human capital accumulated in childhood affects the probability of college graduation conditional on enrollment. In addition, human capital may also impact the probability that an individual will become a dual- or single-parent as an adult.

For our baseline case we calibrate our model to key empirical moments in the US in 2013. In particular, we target the share of single-parent households, and the relative education of parents heading these households. Our model predicts that children with single and/or low education parents are less likely to complete college, even though this is not a moment we target in our calibration.

We utilize the model to conduct two quantitative experiments. First, we ask whether the model is consistent with historical changes in educational attainment rates within household type. Specifically, we exogenously decrease the college wage premium from its 2013 level to its 1986 level (the year in which children born in 1968 turn 18). Consistent with the data, we find that college enrollment rates decrease for children from all household types. However, also consistent with the data, we find that college graduation rates only decrease for children from dual parent households. Graduation rates do not decrease much for children from single, low parent households because children from these households already have low graduation probabilities.

Two features of our model generate different college graduation elasticities among children from different household types. First, college attendance is risky in that some individuals spend time and tuition dollars but do not graduate. Athreya and Eberly (2016) develop a model that includes this feature. In their model, graduation depends on college preparedness, which is an exogenous shock at high school graduation. Second, in contrast to that model, our model endogenizes college preparedness because graduation probabilities depend on the human capital a child develops before college. Because different household types invest in childhood human capital at different rates, different household types will therefore have different graduation probabilities. Viewed through this lens, the changes in household composition since 1968 drastically increased the share of households with very low attainment elasticities (low education single parents) and very high attainment elasticities (high education dual parents).

To conclude, we ask how aggregate educational attainment responds to a hypothetical increase in the college wage premium. Analagous to the previous exercise, we find that college attendance rates increase among children from all household types, but that graduation rates rise much more strongly for children from dual parent households. We contrast the results of our model with a "standard" macroeconomic human capital model where households differ only by parent education, not number of parents. We show that in the model without different numbers of parents, college graduation rates increase by a similar amount for children from all household types. In other words, a model that does not feature heterogeneity in number of parents does not feature heterogeneity in the elasticity of educational attainment rates.

This paper contributes to a large literature on intergenerational human capital investments in dynamic general equilibrium models. Becker and Tomes (1979, 1986) and Barro and Becker (1989) established the early theoretical constructs upon which many later papers built. Aiyagari, Greenwood and Seshadri (2002) considered the time and goods investments that parents make in their children in environments with missing markets for insurance and childcare. De La Croix and Doepke (2003), Moav (2005), and Bar et al. (2015) examine channels through which inequality can affect human capital investments in children and subsequently impact aggregate growth. Fernandez and Rogerson (2001) and Fernández (2003) quantify how changes in assortative mating over time can affect inequality through differential human capital investments.

Within this literature our paper is most closely related to several papers which explicitly model human capital investments for children within single- and dual-parent households. Greenwood, Guner and Knowles (2003) build a model with marriage, divorce, endogenous fertility, and intergenerational human capital investments. They use their model to study the impact of child tax credits and mandating child support payments from fathers. Gayle, Golan and Soytas (2015) study how racial differences in wage rates, marriage patterns, and divorce rates can generate a black-white achievement gap via endogenous differences in intergenerational human capital investment. Abbott (2016) focuses largely on estimating key elasticities in the human capital production function - namely the input elasticities between a mother's and father's time, and the elasticity between time and goods. In a quantitative exercise, he also uses the model to decompose the sources of the existing wage gap between children raised in single- and dual-parent households. A recent survey of macroeconomic approaches to issues in family economics by Greenwood, Guner and Vandenbroucke (Forthcoming) also presents a model of human capital investment with single mothers and dual parent households. They discuss differences in education investment by different household types and the resultant productivity gaps for children raised in these households. Relative to these papers, we use a similar framework to study a novel question: how have changes in parental characteristics impacted the *level* of college graduation rates and the *elasticity* of graduation rates to the college wage premium?

The remainder of the paper proceeds as follows. Section 2 reviews the empirical trends in household structure and educational attainment of parents. Section 3 constructs the model we use to assess the impact of these changes. Section 4 discusses the quantitative results, Section 5 reviews extensions and robustness exercises beyond the benchmark model. Section 6 extends our earlier accounting exercise to consider projections of future educational attainment over the next several decades given the continued trends in household composition, and Section 7 concludes.

2 Empirical Evidence

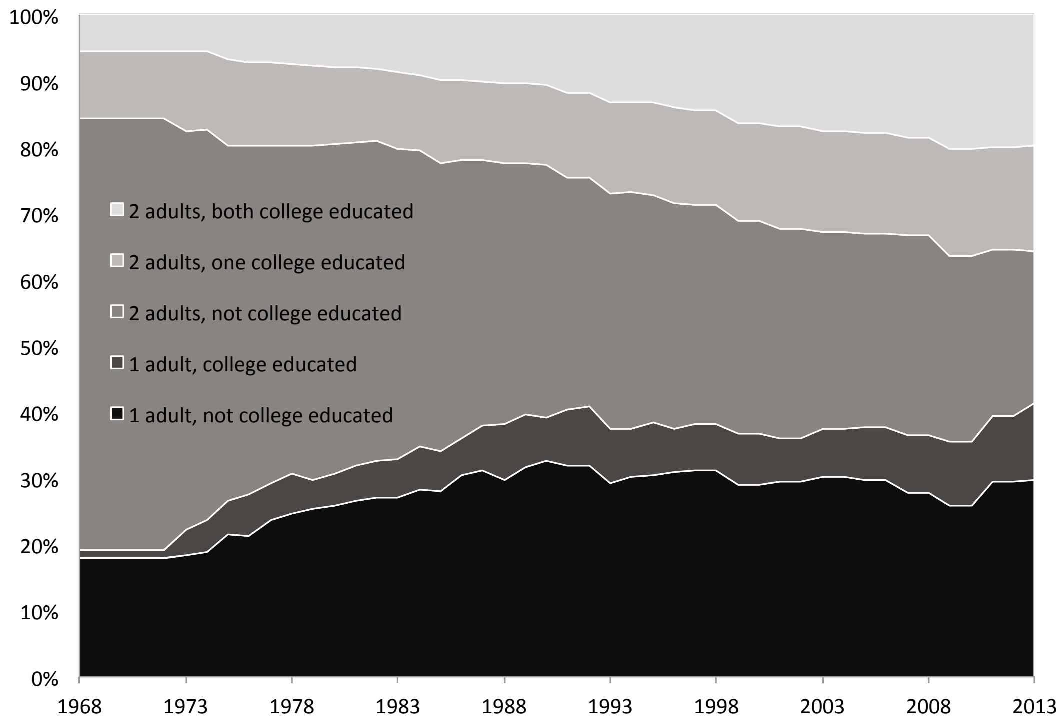
To motivate the quantitative analysis that follows, we first document several facts about the changing composition and educational attainment of US households. Additionally, we conduct two simple accounting exercises to establish how changes in household composition affected aggregate educational attainment of the next generation children, all else equal. These accounting exercises will serves as a baseline for comparison with quantitative model predictions in Section 4.

Our primary data source is the Panel Study of Income Dynamics (PSID), a nationally representative longitudinal data set that has followed families since 1968. The PSID is well-suited for our investigation because it allows us to observe the number of adults in a household over time, and it contains information on educational attainment of both parents and children. Throughout this section and the remainder of the paper, we will refer to "dual parent" households as those with two adults present, whether married or cohabiting. Single parents are those in which the data report a single adult in the household, even if a second parent takes an active role in raising children.

2.1 Changes in parent characteristics since 1968

We consider households that may differ along two dimensions: the number of parents and parental education. There have been substantial changes over time along both margins. Among households with children, we calculate that the share headed by a single parent more than doubled between 1968 and 2013, from 19% to 40%.

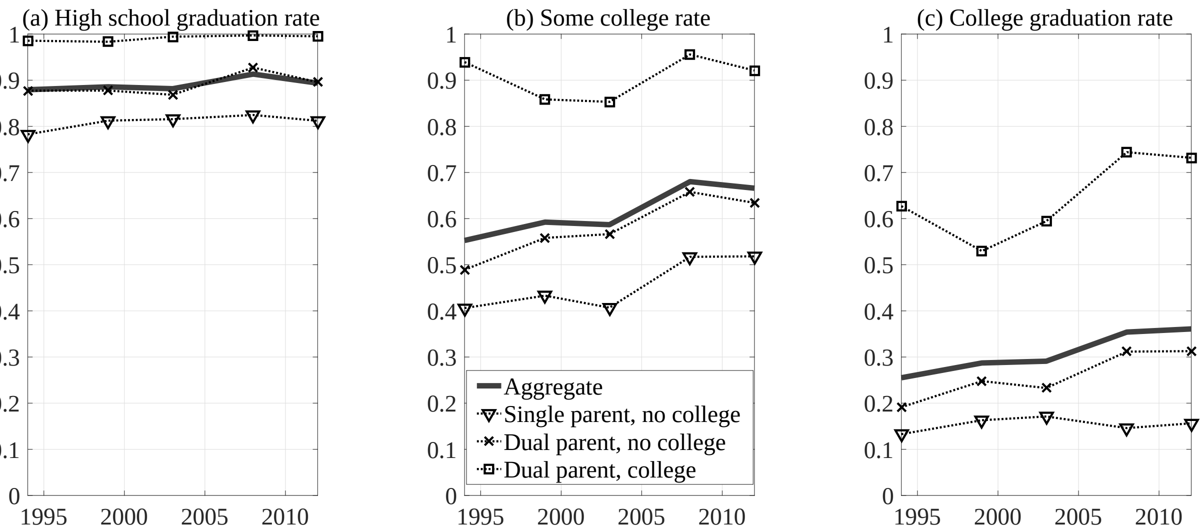
Figure 1: Composition of U.S. households with children, by number of adults and education



Over the same time period, the share of households with at least one college educated parent also more than doubled from about 17% to more than 45%. However, this overall growth in educational attainment was primarily among dual-parent households. For single-parents, the share with a college degree grew from 6% in 1968 to almost 23% in 2013, an increase of 17 percentage points. Yet among dual parent households, the share with at least one college educated parent grew from 19% to 60%, an increase of 41 percentage points. In other words, single parent households have become less educated on average compared to dual parent households, despite overall rising educational attainment among both groups.

Finally, examining households by both number and education of parents, we observe that a growing share of households with children are headed by either non-college educated single parents or dual parents with college education. Figure 1 shows that less than 18% of households with children in 1968 were headed by a single adult without a college degree, and by 2013 this share had grown to nearly 31%. At the other end of the spectrum, we see that in 1968 less than 16% of households had dual parents, at least one of whom was college educated. By 2013 this share more than doubled to 36%. This means that more than two-thirds of households in 2013 were one of two extreme types: single-parent without college education, or dual parent with college education. The fact that households have

Figure 2: Educational attainment at age 26 by parent characteristics



grown increasingly dissimilar along these two dimensions means that parental investments in children—both time and market inputs—may have also grown more disparate.[2] As a result, educational attainment rates would be expected to differ by parent characteristics, and may have also changed over time within household types. We examine these possibilities next.

2.2 Educational attainment of children within household types

To demonstrate how parent characteristics may affect the educational attainment of children, we first calculate the share of children who graduate high school (at least 12 years of education), have some college (at least 13 years), and graduate college (at least 16 years). Figure 2 shows these figures for the aggregate population and three household types: single parents without college education, dual parents without college education, and dual parents where at least one holds a college degree.[3] We restrict attention to children born since 1968 in order to have consistent observations of parent characteristics for the duration of childhood. We measure the educational attainment of children at age 26, which allows us to view educational attainment trends from 1994 through 2013. We compute averages across 5-year groups in order to smooth year-to-year fluctuations arising from the relatively small samples of the disaggregated household types. In the case of children whose parents marry or divorce during their childhood, we categorize them as being in a single parent household

[2]Given that fertility is correlated with household characteristics such as income and education, one might also be interested in calculating the shares of *children* raised in households of each type, rather than the shares of households of each type. We have done both, and the figures are very similar, so we omit the latter to save space.

[3]We omit the data for single parents with college degrees because of small sample size.

if they ever report being in a single parent household. Similarly, we categorize children into non-college educated households if at anytime during childhood their parent(s) do not hold a college degree.[4]

The first key point to observe in Figure 2 is that there are large level differences in average educational attainment rates by parent characteristics from 1994 through 2013. Furthermore, these differences increase with each subsequent education level. Panel (a) shows that children from dual parent households are nearly certain to graduate high school if at least one parent has a college degree. By comparison, having dual parents without a college degree lowers the probability of high school graduation by about 10 percentage points. Having a single parent without a college degree lowers the probability even further, by almost another 10 percentage points. Moving to panel (b), we observe the share of children with some college and more stark differences emerge. On average from 1994–2013, about 90% of children from dual parent, college-educated households attend some college. By contrast, rates for children with non-college dual parents are nearly 30 percentage points lower, and rates for children with non-college educated single parents are more than 40 percentage points lower. Finally, we consider college graduation rates in panel (c) and find that the average college graduation rate for children from dual parent, college-educated households is approximately 65%. Yet for children with non-college dual parents this drops to about 25%, a level difference of about 40 percentage points. For children with non-college educated single parents it drops to about 15%, a level difference of about 50 percentage points.

The second key takeaway from Figure 2 is that growth rates of higher educational attainment have also differed widely by parent characteristics. Panel (a) shows that high school graduation rates grew by only a couple of percentage points during 1994–2013 across all parent characteristics. Similarly, Panel (b) shows no trend growth in college attendance rates of children from dual-parent college educated households. However, college attendance rates among households with non-college educated parents (both single and dual) each increased by more than 10 percentage points over this time. In Panel (c) a different story emerges for college graduation rates. The aggregate college graduation rate grew by 10 percentage points, from 26% to 36%. Children from both non-college and college educated dual parent parent households experienced similar growth of about 10 percentage points. However, children from households with single non-college parents saw no growth in college graduation over this period (despite the previously mentioned growth in college attendance among this group).

[4]Alternatively, we have also categorized children according to the parent number and education that they experienced for the largest part of their childhood. The results are consistent under both assumptions.

Table 1: Impact of parental composition on aggregate educational attainment, 1994-2013.

	High school grad rate			Some college grad rate			College grad rate		
	1994	2013	Δ	1994	2013	Δ	1994	2013	Δ
Actual	.88	.89	.01	.55	.67	.12	.26	.36	.10
Single parent share fixed	.88	.90	.02	.55	.69	.14	.26	.40	.14
College parent share fixed	.88	.88	.00	.55	.64	.09	.26	.33	.07

To summarize, we find ample evidence that educational attainment of children differs depending on the number and education of their parents. For high school graduation, the difference is a matter of levels rather than changes over time. However, for some college and college attainment, we observe level differences and trend growth rates that both differ by parent characteristics. Acknowledging that overall educational attainment and the composition of household types have both been changing over time, we next proceed to conduct simple accounting exercises that decompose the relative contributions of each.

2.3 Accounting exercises

Given the evidence of the previous section, we would expect the trend of increasing single-parenthood to depress educational attainment of the next-generation children, all else equal. By contrast, we would expect the trend of greater college attainment among parents to boost educational attainment of their children. To provide a baseline for the relative magnitudes of these opposing effects, we conduct two simple accounting exercises. First, we hold the share of single-parents fixed at the 1968 level and, given the average educational attainment rates by household type from the previous section, compute the share of children who would be expected to complete high school, some college, and college. Comparing this counterfactual to the data quantifies the effect from single-parenthood alone. Second, we hold the share of parents with college degrees fixed and, given the changes over time in marital status, again compute the share of children who would be expected to complete high school, some college, and college. Comparing this counterfactual to the data quantifies the effect from the secular increase in college attainment among parents.

Table 1 shows the results of these two counterfactual accounting exercises, along with the actual attainment rates for high school, some college, and college. We compare the educational attainment of 26-year-olds in 1994 (those born in 1968) to 2013 (those born in 1987). Several results are notable. First, because the high school graduation rate was relatively high and stable during this time period across household types, holding either the share of single parents or college educated parents fixed has only minor impact on high school completion rates. However, the impacts on college attendance and completion rates are more significant.

In actuality, the aggregate share of 26-year-olds with some college increased by 12 percentage points from 1994–2013. Absent the increase in single parents, this growth would have been two percentage points higher, a 16% increase. On the other hand, if college attainment had not been increasing among parents, this growth would have been three percentage points lower, a 25% decrease compared to the data.

Turning attention to college completion rates, we see even larger differences. The data shows a 10 percentage point increase in college attainment among 26-year-olds, from 26% in 1994 to 36% in 2013. Holding the single parent share fixed, this increase would have 14 percentage points, or 40% larger. On the other hand, holding the share of college educated parents fixed implies that college attainment among their children would have only grown by seven percentage points, which is a 30% decrease compared to the data.

From a pure accounting perspective, these results make clear that changes over time in single parenthood and parents' education both have the potential for large effects on educational attainment of the next generation of children. What this exercise lacks, however, is any explanation for why children's educational attainment differs by parent characteristics, why children's educational attainment has changed over time within household types, and what other aspects of the human capital accumulation process may amplify or suppress the effects of parent characteristics. To address these issues, we next build a overlapping generations model in which parental investments affect their children's educational attainment. With the model in hand we have the capacity to address the issues above, as well as make forward-looking projections of educational attainment over the next several decades, given recent trends in parental characteristics of current children.

3 The model economy

In this section we construct a dynamic model of intergenerational human capital transmission that incorporates both single and dual parent households. The key ingredients of the model are as follows.

Time is discrete. Individuals live for J periods. There are four stages of life: early childhood (periods $j = 1, ..., J_{lc} - 1$), late childhood (periods $j = J_{lc}, ..., J_{ea} - 1$), early adulthood ($j = J_{ea}, ..., J_{la} - 1$), and late adulthood ($j = J_{la}, ..., J$).

During early childhood individuals grow up in their parents' household and accumulate human capital passively via investments by their parents. When children transition to late childhood they leave their parents' household, start their own single household, and decide whether or not to enroll in college. Individuals in late childhood who are enrolled in college pay a monetary and utility cost. Students graduate with a probability that is increasing in their human capital from early childhood. Students who enroll in college but do not graduate receive a small increase in their human capital; those who do graduate receive a larger increase. Students who graduate also receive a different skill price during their career than non-college graduates. As soon as individuals complete their education (either because they graduate or because they drop out), they begin working.

When children transition to early adulthood they become parents and receive stochastic draws for their parent type (dual or single), as well as the initial human capital of their child. Parents are altruistic. Parents split their time endowment between market work and time investment in their children, and split earnings from market work between consumption and market investments in their children. Heterogeneity in (i) parental human capital, (ii) education, (iii) net assets, (iv) number of parents, and (v) child human capital give rise to different investments by parents. Finally, parents transition to late adulthood at the same time their children transition to late childhood. Late adults have no ties with their children.

3.1 Early Childhood

Children are born into a household with their parents and remain in that household until they exit early childhood. Within a household, all children are born at the same time and are treated identically by their parents.

The state of an early child is the state of the household that child is born into: a vector (H, K, S, P, h), where H is the human capital level of the child's parents; K is the net asset level of parents; $S \in \{G, NG\}$ indicates whether the parents are college graduates (G) or not graduates (NG); $P \in \{1, 2\}$ is the number of parents the child has; and h is the child's human capital. A child's initial human capital endowment at birth is stochastic and given by $h \in \{h_1, h_2\}$. Throughout the paper we use upper case letters to denote state variables of individuals who are not in school, and lower case letters to denote state variables for individuals in school (i.e., all individuals in early childhood, plus individuals in late childhood who are in college).

A child's state vector is constant throughout early childhood with the exception of parental assets K and their own human capital, h.[5] Children are completely passive until the

[5] Note that this assumes away divorce and parental human capital accumulation, among other things. See Greenwood, Guner and Knowles (2003) and Gayle, Golan and Soytas (2015) for papers which explicitly study the role of divorce for intergenerational human capital transmission.

end of early childhood; altruistic parents make investment decisions by considering how those investments will affect their children during late childhood. The evolution of h throughout early childhood is determined by the investment decisions of the child's parents. Specifically, an early child's human capital h evolves according to:

$$h' = F_j\left(m, i, H, h\right) \tag{1}$$

where the function F_j is increasing in market investments m, time investment i, parental human capital H, and the child's existing human capital h.[6] The age subscript on the human capital production function allows the function to change as the child ages.

At the end of the final period of early childhood, $J_{lc} - 1$, individuals receive a bequest from their parent(s) and decide whether or not to enroll in college. Formally, their decision problem is

$$V_{J_{lc}-1}(\tilde{h}, \tilde{k}) = \max\left\{V_{J_{lc}}(\tilde{h}, \tilde{k}, g)\ ,\ V_{J_{lc}}(\tilde{h}, \tilde{k}, ng)\right\}\ , \tag{2}$$

where $\tilde{h}$ is the child's human capital at the end of period $J_{lc} - 1$, $\tilde{k}$ is the child's assets from the bequest at the end of period $J_{lc} - 1$, and $s \in \{g, ng\}$ denotes whether the student will begin the next period going (g) or not going (ng) to college. Throughout the paper we use a $\tilde{}$ symbol to denote end-of-period variables.

3.2 Late Childhood

Individuals in late childhood are described by the state vector (h, k, s) if enrolled in college or (H, K, S) if not enrolled. College graduation requires four years, so individuals in late childhood who remain enrolled in college from period J_{lc} through period $J_{lc} + 3$ graduate college. Individuals who are enrolled in college this period accumulate additional human capital according to the function $F_g(h)$ and remain enrolled next period with probability $\gamma(h)$, which is is increasing in h. Conversely, with probability $1 - \gamma(h)$ individuals exit college, and once a student exits college they cannot re-enroll. Individuals not enrolled in college inelastically supply one unit of labor to market work. The wage rate that workers receive, $\omega_S H$, depends on whether they have graduated college (G) or not (NG), and on the worker's human capital H.

[6] Recall that we have assumed that parents cannot invest more in one child than another.

Individuals who are enrolled in college (i.e. $j < J_{lc} + 3$) solve the following problem:

$$V_j(h,k,g) = \max_{c,k'}\{u(c) + \beta\left[\gamma(h)V_{j+1}(h',k',g) + (1-\gamma(h))V_{j+1}(H',K',NG)\right]\} \tag{3}$$

$$s.t. \quad c + \tau + k' = k(1+r) \tag{4}$$

$$h' = F_g(h). \tag{5}$$

Note that if these individuals exit college via $1-\gamma(h)$, then their state variables become capitalized, and $S = NG$ denotes that they are not college graduates. Individuals may also be out of college during late childhood if they chose not to enroll or they graduated college. In any case, once individuals complete all schooling their human capital and schooling status remain constant. Thus, individuals who are not in college during any period before the final period of late childhood ($j < J_{ea} - 1$) face the following consumption-savings decision problem:

$$V_j(H,K,S) = \max_{c,K'}\{u(c) + \beta V_{j+1}(H',K',S)\} \tag{6}$$

$$s.t. \quad c + K' = \omega_S H + K(1+r) \tag{7}$$

Finally, for $j = J_{ea} - 1$ we have

$$V_{J_{ea}-1}(H,K,S) = \max_{c,\tilde{K}}\{u(c) + \beta E_{x'}\left[V_{J_y+1}(x')|\tilde{x}\right]\} \tag{8}$$

$$s.t. \quad c + \tilde{K} = \omega_S H + K(1+r) \tag{9}$$

$$\tilde{x} = (\tilde{H},\tilde{K},\tilde{S})\ . \tag{10}$$

3.3 Transition to Early Adulthood

A child who ends period $J_{ea} - 1$ with state $\tilde{x} = (\tilde{H},\tilde{K},\tilde{S})$ will begin early adulthood the next period with state $x' = (H',K',S',P',h')$. The evolution from state $\tilde{x}$ to x' is stochastic. Specifically, the individual is hit by three shocks at the end of period $J_{ea} - 1$.

First, the individual receives a parenthood shock, $P' \in \{1,2\}$, where $P' = 1$ corresponds to single parenthood and $P' = 2$ corresponds to dual parenthood. The shock has a value of P' with probability $\pi_P(P'|\tilde{x})$.

Second, if the individual is hit by a dual parent shock ($P' = 2$), they are matched with another individual of the same age via the stochastic matching function $Z(\tilde{y}|\tilde{x})$. Two new parents $(\tilde{x},\tilde{y})$ who have been matched become one household defined by a single state vector. The assimilation function $x'_{-h} = a(\tilde{x},\tilde{y})$ combines the state vectors for each individual from the end of period J_a into a single state vector for the household. Specifically, we assume that the new household has human capital $H' = \frac{\tilde{H}_x+\tilde{H}_y}{2}$, assets $K' = \tilde{K}_x + \tilde{K}_y$, and schooling $S = \max\{\tilde{S}_x, \tilde{S}_y\}$. In words, the combined human capital stock of both parents is the average of the two, the combined asset stock is the sum of each parent's assets, and the schooling level is G if either parent graduated college, NG otherwise. If the parent was hit by a single parent shock, then $x'_{-h} = (\tilde{x}, P' = 1)$.

Finally, individuals receive a child human capital shock $h' \in \{h_1, h_2\}$. The shock has a value of h' with probability $\pi_h(h'|x'_{-h})$.

Having detailed the transition from late childhood to early adulthood, we can now explicitly define the expectation term in equation (8), the value function of individuals in late childhood about to enter early adulthood:

$$Prob\,(x'|\tilde{x}, P' = 1) = \pi_h(h'|(\tilde{x}, 1)\mathbf{I}_{\left\{\tilde{x}=x'_{-h}\right\}} \;, \tag{11}$$

$$Prob\,(x'|\tilde{x}, P' = 2) = \int \pi_h(h'|a(\tilde{x}, \tilde{y}))\mathbf{I}_{\left\{a(\tilde{x},\tilde{y})=x'_{-h}\right\}} Z(d\tilde{y}|\tilde{x}) \;, \tag{12}$$

$$Prob\,(x'|\tilde{x}) = \pi_P(1|\tilde{x})Prob\,(x'|\tilde{x}, P' = 1) + \pi_P(2|\tilde{x})Prob\,(x'|\tilde{x}, P' = 2) \;\;, \tag{13}$$

$$E_{x'}\left[V_{J_y+1}(x')|\tilde{x}\right] = \int V_{J_y+1}(x')Prob(x'|\tilde{x})dx' \;. \tag{14}$$

3.4 Early Adulthood

As with early childhood, the state during early adulthood can be fully described by the state of the household: (H, K, S, P, h). In addition to consumption and savings decisions, individuals in early adulthood also invest in the human capital of their children. Parents of age $j < J_{la} - 1$ in state $x = (H, K, S, P, h)$ choose consumption c, net assets next period K', market investment m, and time investment i to maximize the value of remaining lifetime utility:

$$V_j(x) = \max_{c,K',m,i} \quad u(c) + \beta V_{j+1}(x') \tag{15}$$

$$s.t. \quad c + m + K' = \omega_S H(P - i) + K(1 + r) \;; \tag{16}$$

$$h' = F_j\,(m, i, H, h)\;\;; \tag{17}$$

$$x' = (H, K', S, P, h')\;; \tag{18}$$

$$i \leq P. \tag{19}$$

The problem looks slightly different for parents in the final period of early adulthood, $j = J_{la} - 1$:

$$
\begin{aligned}
V_{J_{la}-1}(x) = \max_{c,K',\tilde{k},m,i} \quad & u(c) + \beta V_{J_{la}}(x') + \alpha V_{J_{lc}-1}(\tilde{x}) && (20)\\
s.t. \quad & c + m + K' + \tilde{k} = \omega_S H(P-i) + K(1+r) \ ; && (21)\\
& \tilde{h} = F_{J_{lc}-1}(m, i, H, h) \ ; && (22)\\
& \tilde{x} = (\tilde{h}, \tilde{k}) \ ; && (23)\\
& x' = (H, K', S, P) \ ; && (24)\\
& i \leq P; \quad B \geq 0 \ . && (25)
\end{aligned}
$$

One important difference in this problem is that parents are able to provide a one time monetary bequest $\tilde{k} \geq 0$ to their children, who are in their final period of early childhood and about to make decisions about college. Another difference is that in period $J_{la} - 1$ the objective explicitly includes the expected lifetime utility of their children. Of course, this valuation implicitly enters the parent's objective in all years of early adulthood through the continuation term $V_{j+1}(x')$ in equation (15).

Lines (16) and (21) reveal a central distinction between single ($P = 1$) and dual ($P = 2$) parent households: dual parent households have twice the time endowment of single parent households. All else equal, this allows dual parents to spend additional time investing, to spend additional earnings on market investment, to consume more, or some combination of all three.

3.5 Late adulthood

Parents transition to late adulthood and their children transition to late childhood simultaneously. At this point, all ties between parents and children are severed. Therefore, the optimization problem of late adults becomes simpler: parents do not invest ($i = m = 0$), and simply make a consumption/saving decision each period.[7] We can therefore write the optimization problems for all ages $j = J_{la}, ..., J$ during this phase as:

$$
\begin{aligned}
V_j(x) = \max_{c,K'} & \{u(c) + \beta V_{j+1}(x')\} && (26)\\
s.t. \quad & c + K' = \omega_S H + K(1+r) && (27)\\
& x' = (H, K', S) && (28)
\end{aligned}
$$

[7] With no human capital depreciation we might expect a large increase in earnings when parents enter late adulthood. This sounds counterfactual. Introducing human capital depreciation (e.g. for stay-at-home parents) can reduce this increase in earnings. Human capital depreciation for parents may therefore be important to introduce at a later date.

where $V_{J+1}(x') = 0$.

3.6 Production

The single consumption good c is produced by a stand-in firm, which operates a linear technology

$$Y = A(H^f_{NG} + \eta H^f_G) \; , \tag{29}$$

where A is TFP, H^f_{NG} are non-graduate human capital services rented by the firm, and H^f_G are college graduate human capital services rented by the firm. The firm rents human capital services from a competitive labor market at wage rates ω_{NG} and ω_G. The firm sells its output in a competitive market at a price normalized to one and attempts to maximize profit.

3.7 Equilibrium

See Appendix A for details on elements 7 and 8 of our equilibrium definition.

Definition 1 *State variables are a 5-tuple during early childhood and early adulthood, $x = (H, K, S, P, h)$; are a dual at the end of the last period of early childhood, $\tilde{x} = (\tilde{h}, \tilde{k})$; are a triple during early adulthood, either $x = (h, k, s)$ if enrolled or $x = (H, K, S)$ if not enrolled, and are a 4-tuple during late adulthood, $x = (H, K, S, P)$.*
A stationary equilibrium for the model economy is a collection of
college decisions $s(x)$ during the last period of early childhood;
decisions $\{c_j(x), K'_j(x)\}$ during late childhood;
decisions $\{c_j(x), K'_j(x), m_j(x), i_j(x)\}$ during early adulthood;
a bequest decision $\tilde{k}(x)$ during the last period of early adulthood;
aggregate variables $\{C, M, H^f_{NG}, H^f_G\}$;
prices $\{(\omega_{NG}, \omega_G, r)\}$;
parental matching function $Z(\tilde{y}|\tilde{x})$;
household assimilation function $x'_{-h} = a(\tilde{x}, \tilde{y})$;
and measure of households $\Lambda(x) = (\Lambda_j(x))$ that collectively satisfy the following:

1. *College enrollment decisions solve the decision problem defined in Section 3.1.*
2. *Individuals in late childhood solve the decision problems defined in Section 3.2.*
3. *Individuals in early adulthood solve the decision problems defined in Section 3.4.*
4. *Individuals in late adulthood solve the decision problems defined in Section 3.5.*
5. *The wage rate is determined competitively: $\omega_{NG} = A$ and $\omega_G = A(1 + \eta)$.*

6. *The parent market clears:* $\forall x, \Lambda_{J_y}(\tilde{y}(y)) = \int Z(\tilde{y}(y)|\tilde{x}(x))\Lambda_{J_y}(dx)$.

7. *The labor and output markets clear.*

8. *The age vector of distributions* $\Lambda(x)$ *is stationary.*

3.8 Calibration

Work in progress.

4 Quantitative Results

Work in progress.

5 Model Extensions

The benchmark model developed in Section 3 and evaluated in Section 4 obviously abstracted from several potentially important dimensions. In this section we extend that benchmark model to consider alternative assumptions and additional model features. The goal is to determine which, if any, of these alterations is quantitatively important for the answers we provide.

5.1 Alternative assumptions on human capital production

5.2 Financial Assets and Bequests

5.3 Expectations

6 Projections for Future Educational Attainment

Work in progress.

7 Conclusion

This paper documented several important trends in US household composition since 1968. The share of single-parent households more than doubled, while at the same time educational attainment of the overall population increased. However, educational gains were realized disproportionately by dual parents as opposed to single parents. By 2013, two-thirds of US households were headed by either single parents without college education, or dual parents with at least one having college education. Few papers in macroeconomics have considered the implications of these changes for children's human capital. We do so in a series of exercises.

First, we conduct a basic accounting exercise to decompose the relative contributions to aggregate educational attainment from: (i) increasing single-parenthood; and (ii) rising parent education overall. We show that the growth of single-parent households depressed college completion rates, while rising parent education boosted college completion rates. The magnitudes of these opposing effects were similar.

Next, we construct an overlapping generations model with intergenerational human capital investment. Households in the model differ by number and education of parents, which affects the time and financial resources available to invest in children. Using the model, we examine why children's educational attainment differs by parent characteristics and why children's educational attainment has changed over time within household types. By comparing quantitative model results against a standard one-parent, one-child (or two-parent, two-child) macroeconomic model, we show that number of parents is an important source of heterogeneity.

Finally, we use the model offer a forward-looking projection of educational attainment for children born since 1988, given the observed characteristics of parents over this time. We anticipate that college completion rates will continue to rise slowly over the next several decades. The rising share of single-parent households will continue depressing overall college completion, but the rising share of dual-parent college educated households is projected to result in more rapid college completion growth.

Having established that single-parenthood is an important feature to include in macroeconomic models of intergenerational human capital investment, we see several fruitful paths for future research. First, there are immediate implications for intergenerational educational and income mobility. The framework developed in this paper can be easily adapted to study changes over time within the US as well as cross-country differences. Second, we recognize that there are are both geographic and racial differences in the rates of single-parenthood, which may contribute to observed gaps (again, both geographic and racial) in income inequality and intergenerational mobility. We plan to apply this model to study these topics in future work.

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APPENDIX

A Equilibrium definition details

This appendix details elements 7 and 8 of our equilibrium definition (see Section 3.7).

A.1 Labor market clearing

The labor market clears if the following conditions hold:

$$H^f_{NG} = \left(\sum_{j=J_{lc}}^{J_{ea}-1} \int_{x:S=NG} H \Lambda_j(dx) \right) + \left(\sum_{j=J_{ea}}^{J} \int_{x:S=NG} H(P - i_j(x)) \Lambda_j(dx) \right) \tag{A1}$$

$$H^f_{G} = \left(\sum_{j=J_{lc}+4}^{J_{ea}-1} \int_{x:S=G} H \Lambda_j(dx) \right) + \left(\sum_{j=J_{ea}}^{J} \int_{x:S=G} H(P - i_j(x)) \Lambda_j(dx) \right) \tag{A2}$$

A.2 Consumption market clearing

The consumption market clears if the following conditions hold:

$$\hat{C} = \sum_{J_{lc}}^{J} \int c_j(x) \Lambda_j(dx) \tag{A3}$$

$$\hat{\tau} = \sum_{J_{lc}}^{J_{lc}+3} \int_{x:S=G} \tau \Lambda_j(dx) \tag{A4}$$

$$\hat{K} = \sum_{J_{lc}}^{J} \int K \Lambda_j(dx) \tag{A5}$$

$$\hat{K'} = \sum_{J_{lc}}^{J} \int K'_j(x) \Lambda_j(dx) \tag{A6}$$

$$\hat{C} + \hat{\tau} + \hat{K'} = Y + \hat{K} \tag{A7}$$

A.3 Stationary distribution

The age vector of distributions $\Lambda(x) = \{\Lambda_j(x)\}_{j=1}^{J}$ is stationary if the following conditions hold:
For $j \in [1, ..., J_{lc} - 1] \cup [J_{lc} + 4, ..., J_{ea} - 1] \cup [J_{ea} + 1, ..., J - 1]$:

$$\Lambda_{j+1}(x') = \int \mathbf{I}_{\{\tilde{x}(x)=x'\}} \Lambda_j(dx). \tag{A8}$$

For $j \in [J_{lc}, ..., J_{lc} + 3]$ if $S' = G$:

$$\Lambda_{j+1}(x') = \int \mathbf{I}_{\{\tilde{x}(x)=x'\}} G(H) \Lambda_j(dx). \tag{A9}$$

For $j \in [J_{lc}, ..., J_{lc} + 3]$ if $S' = NG$:

$$\Lambda_{j+1}(x') = \int_{x:S=NG} \mathbf{I}_{\{\tilde{x}(x)=x'\}} \Lambda_j(dx) + \int_{x:S=G} \mathbf{I}_{\left\{\tilde{x}(x)_{-S}=x'_{-S}\right\}} (1 - G(H)) \Lambda_j(dx). \tag{A10}$$

For $j = J_{ea}$ if $P' = 1$:

$$\Lambda_{j+1}(x') = \int \pi_P(1|\tilde{x}(x)) Prob(x'|\tilde{x}(x), P' = 1) \Lambda_j(dx). \tag{A11}$$

For $j = J_{ea}$ if $P' = 2$:

$$\Lambda_{j+1}(x') = \int \left(\frac{\pi_P(2|\tilde{x}(x))}{2}\right) Prob(x'|\tilde{x}(x), P' = 2) \Lambda_j(dx). \tag{A12}$$

Note: for terms used in the previous two conditions, see Section 3.3.