Subjective Survival Beliefs and Ambiguity: The Role of Psychological and Cognitive Factors*

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Preliminary and Incomplete

Abstract

Based on data of the Health and Retirement Study (HRS), we document new facts on subjective survival beliefs by comparing them to their individual-level objective counterparts. Similar to experimental results on probability weighting in prospect theory, we show that survival beliefs can be described as objective probabilities that have undergone an inverse-S shaped transformation. With increasing age biases are driven through increased likelihood insensitivity combined with increased pessimism. Using new psychological measures in the HRS we provide further direct empirical evidence in support of these cognitive and psychological interpretations.

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1 Introduction

Numerous economic decisions such as retirement, consumption and saving decisions require the formation of beliefs about the probability to survive into the future. Yet, predicting the own demise is a very difficult task which is likely prone to mistakes and biases. A growing economic literature inspired by Hamermesh (1985) documents substantial biases between subjective beliefs and their respective objective counterparts[1] and investigates the importance of such biases for economic decisions[2] An important question that emerges from this literature is about the driving forces behind these biases. A good answer to this question would give us some guidance about how to adequately model subjective survival beliefs in economic applications.

This paper argues that increasing cognitive impairments combined with increasing pessimism for elderly people are important drivers for the age-specific patterns of survival belief biases observed in the data. We base our argument on three different elements. First, we use data on subjective survival beliefs from the Health and Retirement Study (HRS) which we compare to estimated objective survival rates at the individual level. Second, we explore newly available measures on psychological attitudes and cognitive strength, respectively weakness, from the HRS. Third, we combine both data by estimating inverse-S-shaped probability weighting functions as specified in the celebrated prospect theory (cf. Wakker (2010) and references therein). We next describe these steps in detail.

In our first step, we determine age-dependent patterns of biases from the HRS data on subjective survival beliefs. In the HRS interviewees are asked about their beliefs to survive from the interview age to some target age whereby this target age is several years ahead. To compare these subjective survival beliefs (SSB) with their objective counterparts, we construct for each interviewee the corresponding individual level objective survival probability (OSP) by using the information on actual HRS mortality and several conditioning

[1] Cf., e.g., Elder (2013), Hamermesh (1985), Ludwig and Zimper (2013b), Peracchi and Perotti (2012).

[2] Cf., e.g., Salm (2010), Rutledge et al. (2014), **?**), **?**), **?**).

variables including mortality trends.[3] We focus on individuals of age 65 and older because we do not have mortality information on younger individuals. Within a given age-group we find that respondents with low OSPs express overestimation whereas respondents with high OSPs express underestimation, resulting in a "flattening out" of SSB compared to the 45-degree line of OSP.[4]: relatively young respondents (younger than age 70) express underestimation whereas relatively old respondents (older than age 75) express overestimation. These biases are large. On average, 65 year old respondents underestimate their survival probabilities by roughly 10 percentage points whereas 85 year old respondents overestimate them by roughly 15 percentage points. Comparing these biases across different age-groups we additionally find that both, the average degree of underestimation and the flatness of the mapping from OSPs into SSBs, increase in age.

As a next step, we explore psychological and cognitive variables from the HRS. These variables measure (dispositional) optimism, (dispositional) pessimism, and cognitive weakness. We find that optimism is decreasing and pessimism is increasing with age, on average.[5] Likewise, our measure of cognitive weakness is strongly increasing with age.

Third, we provide a structural interpretation of these biases through prospect theory (PT) (Kahneman and Tversky 1979; Tversky and Kahneman 1992). One of the key insights of the experimental PT literature is that probability assessments as well as decision weights can be best described by an inverse-S-shaped transformation of additive probabilities rather than by additive probabilities themselves. As our data on SSB is consistent with an (age-dependent) inverse-S-shaped probability weighting function applied to OSP, we construct a PT model of SSB by applying the Prelec (1998) probability weighting func-

[3]To this purpose we adapt the methods used by Khwaja, Sloan, and Chung (2007), Khwaja, Silverman, Sloan, and Wang (2009), and Winter and Wuppermann (2014) to estimate mortality hazard rates at the individual level.

[4]Our results are thus consistent with the so-called "flatness bias" documented in the previous literature Elder (2013), Hamermesh (1985), Ludwig and Zimper (2013b), Peracchi and Perotti (2012)

[5]It may seem that optimism is just the opposite of pessimism, psychologists measure both phenomena separately. We further explore the differences in Section 4.

tion to OSP. The **?** function features two parameters, one reflecting relative pessimism of respondents, the other measuring likelihood insensitivity. Likelihood insensitivity stands for a cognitive impairment according to which people tend to flatten out the 'true' likelihoods of events that are neither impossible nor certain (an extreme case of such flattening-out are 50-50 probability assessments of all uncertain events and their complements). We fit this PT model to the HRS data on SSB to trace out age-specific parameters for relative pessimism and likelihood insensitivity. We find that relative pessimism and likelihood insensitivity are both increasing with age.

Finally, we combine the HRS data on direct psychological and cognitive measures with the PT model. Specifically, we analyze the extent towards which psychological and cognitive factors are associated with individuals' biases in survival assessments. To this end, we parameterize the coefficients of the non-linear inverse-S-shaped probability weighting functions as linearly dependent on the respective psychological and cognitive measures. Our estimates confirm that psychological variables and cognition are important drivers of survival misperceptions. Beyond a base bias at age 65—which captures the effects of an initial lack of probabilistic sophistication (or cognitive weakness) as well as, possibly, a 50-50 bias in survival perceptions[6]—, our point estimates show that increased pessimism leads to a downward adjustment and increased optimism to an upward adjustment of survival beliefs relative to the objective probabilities. Furthermore, increasing cognitive weakness leads to an increasing upward bias of survival beliefs.

We assign a quantitative role to all effects through a decomposition analysis over the life-cycle. At age 65, almost the entire bias of 10 percentage points is attributable to the base bias. The effects of age dependent psychological variables induce a smaller bias (i.e., optimism dominates) and of cognitive weaknesses a stronger one, but those two effects just offset each other. At age 85 about a third of the observed bias is due to each factor, the base bias, cognitive weakness and psychological effects. We also consider a linear

[6]We cannot decompose the base bias into the effects of an initial cognitive weakness and a 50-50 bias, see, e.g., Bruine de Bruin et al. (2000) for the latter.

approximation of inverse S-shaped probability weighting functions which is often employed in theoretical models because of its parsimonious representation of biased beliefs (Abdellaoui et al. 2011).[7] Statistically, the linear model performs like the non-linear specification.[8] Our results for the linear model confirm our findings for the non-linear specification.

It is unclear how additive probabilities of expected utility theory (EUT) could adequately reflect these dynamics of psychological and cognitive factors. For example, the SSBs of a standard EUT Bayesian learner would converge to the OSPs instead of exhibiting age-specific biases (cf. Ludwig and Zimper (2013a)). Even if cognitive impairments are introduced in Bayesian learning models in the form of 'slow' learning, one would still obtain convergence of SBBs to OSPs. Our analysis therefore suggests that economic applications based on survival beliefs might improve their realistic appeal if they are cast within PT rather than within EUT. Such modeling choice, however, does not come cheap as PT maximization problems (typically) violate the dynamic consistency of the EUT framework.[9]

The remainder of this paper is organized as follows. Section 2 presents the main stylized facts on survival belief biases. Section 3 provides a structural interpretation of these biases through prospect theory. Section 4 looks at the direct psychological measures elicited in the HRS. Section 5 presents empirical evidence on the relationship between psychological and cognitive variables, on the one hand, and biases in survival beliefs, on the other hand. Finally, Section 6 concludes. Separate appendices contain additional information on the data.

[7] Also see Wakker (2010) and references therein. For an axiomatic foundation of neo-additive probability measures within Choquet expected utility theory (Gilboa 1987; Schmeidler 1989) see Chateauneuf et al. (2007).

[8] The values of the Akaike and the Schwartz Bayesian information criteria (?; ?) are lower for the linear model but the confidence bands overlap.

[9] We refer the interested reader to the analysis of life-cycle maximization problems under Choquet expected utility with Bayesian learning in ?) and under rank-dependent utility in Groneck et al. (2016).

2 Age Patterns of Biases in Survival Beliefs

2.1 Data

In our analyses we use the Health and Retirement Study (HRS) which is a national representative panel study. Individuals are interviewed on a biennial basis. Interviews of the first wave were conducted in 1992. In subsequent waves, more cohorts were added in order to keep the sample representative. Interviewees are individuals older than 50 and their spouses regardless of age. An overview on the survey, its waves and interview cohorts is displayed in Appendix B.

Both for our descriptive analyses as well as our regression analyses our sample comprises waves $8-12$, i.e. years $2006-2014$. For the estimation of the individual-level objective survival probabilities (OSPs) we use waves $4-12$ of the HRS and data of the Human Mortality Database (HMD). For further details on sample selection again see Appendix B.

2.2 Subjective Survival Beliefs

In the HRS an interviewee i of age h is asked about her SSB to live to at least a certain target age m, which we denote as $SSB_{i,h,m}$. We focus on individuals in the survey of age 65 and older. This sample restriction is due to the fact that the data set does not allow us to estimate OSPs for ages less than 65 with details provided in Subsection 2.3 below. The assignment of target age $m(h)$ to interview age h for our sample is provided in Table 1.

2.3 Objective Survival Probabilities

To study survival misconceptions at the individual level our first objective is to assign to each individual in the sample its respective objective survival probability (OSP). Using aggregate data from (cohort) life-tables for this purpose—as, e.g., in Ludwig and Zimper (2013b), **?**), Perozek (2008) and Peracchi and Perotti (2012)—, is ill-suited because individual (objective) survival rates generally deviate from sample averages. To instead estimate

Table 1: Interview Age h and Target Age $m(h)$

Interview age h	Target Age $m(h)$
65-69	80
70-74	85
75-79	90
80-84	95
85-89	100

Source: HRS (2015), waves 2006-2012.

the objective probability on the individual level by adapting the methods described in Winter and Wuppermann (2014),Khwaja, Silverman, Sloan, and Wang (2009) and Khwaja, Sloan, and Chung (2007). We accordingly employ a duration model to estimate hazard rates conditional on several individual-level characteristics.

Among standard variables such as age, socio-economic status, health behavior, etc., the set of explanatory variables includes predicted average OSPs in order to capture time-trends of mortality hazards. We extract the time trend from a decomposition of cross-sectional survival rates into a time dependent indicator and age-specific factors following a **?**) procedure. For a detailed description of our estimation approach we refer to Appendix**??** and for the Lee-Carter approach to Appendix C.1.

We estimate the relationship between individual level observable variables and mortality using a hazard function given by

$$\lambda(t|\mathbf{x}_i') = \lambda_0(t)\exp(\mathbf{x}_i'\beta) \tag{1}$$

where time to failure t is the number of years to death. $\lambda_0(t)$ is the baseline hazard for which we choose the specification given by the Weibull hazard model[10]. This allows us to model duration dependence, i.e., the fact that

[10] A specification of the hazard function that allows for unobserved heterogeneity may be preferable. However, when we tried to estimate the individual OSPs with a specification of the hazard function that allows for unobserved heterogeneity we faced serious convergence

mortality rates are an increasing function of age. Accordingly, we impose the structure

$$\lambda_0(t) = \alpha t^{\alpha-1} \tag{2}$$

that allows for $\alpha > 1$ (capturing positive duration dependence). $\exp(\mathbf{x}'\beta)$ is the proportional hazard. In our estimation, survivors are treated as censored and we estimate function (1) by maximum likelihood.

The objective survival probabilities (OSPs) for all prediction horizons t and each individual i of interview age h are given by (cf., e.g., **?**, **?**)):

$$OSP_{i,h}(t) = \exp\left[-\exp(\mathbf{x}_i'\beta)t^{\alpha}\right] \tag{3}$$

From this we can also construct the OSP until target age (with horizon $t = m(h) - h$), $OSP_{i,h,m(h)}$, which we assign to the respective $SSB_{i,h,m(h)}$ of individual i.

2.4 Biases in Subjective Survival Beliefs

Our following descriptive analysis compares the subjective individual survival beliefs from the survey data with our individual measures of OSPs. First, we replicate the results of previous literature—e.g., Elder (2013, Hamermesh (1985, Ludwig and Zimper (2013b, Peracchi and Perotti (2012)—on the age patterns of survival beliefs in Figure 1. In contrast to that previous literature, we calculate average OSPs with our individual measures instead of average (cohort) life-tables. The step function in the figure is due to the change in assignment of interview and target age, cf. Table 1. Our findings confirm the well-established "flatness bias": At ages prior to age 70, individuals on average underestimate whereas for ages above age 75 they overestimate their probabilities to survive.

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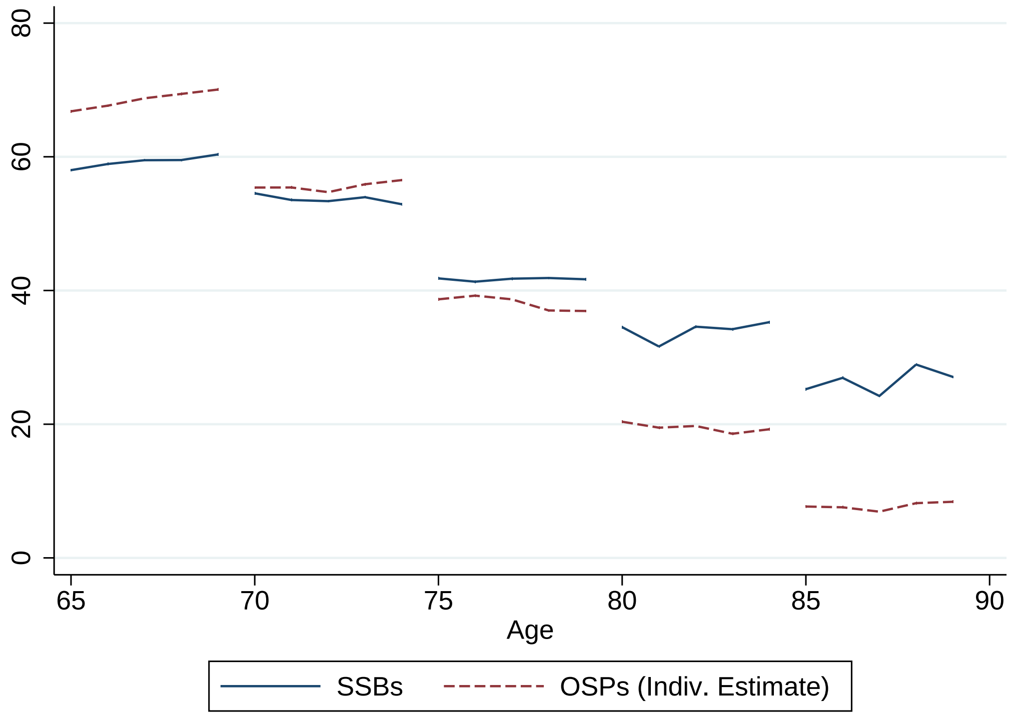
problems in many of our bootstrap iterations. Thus, we compared the results of the first bootstrap of a specification allowing for unobserved heterogeneity with our specification in the paper. Coefficient estimates and the estimate for duration dependence are very similar. Additionally, we compared the descriptive statistics for both specifications which are very similar as well. Hence, we are confident that our results are not significantly affected by ignoring unobserved heterogeneity in our specification of the harzard function.

Figure 1: "Flatness Effect"



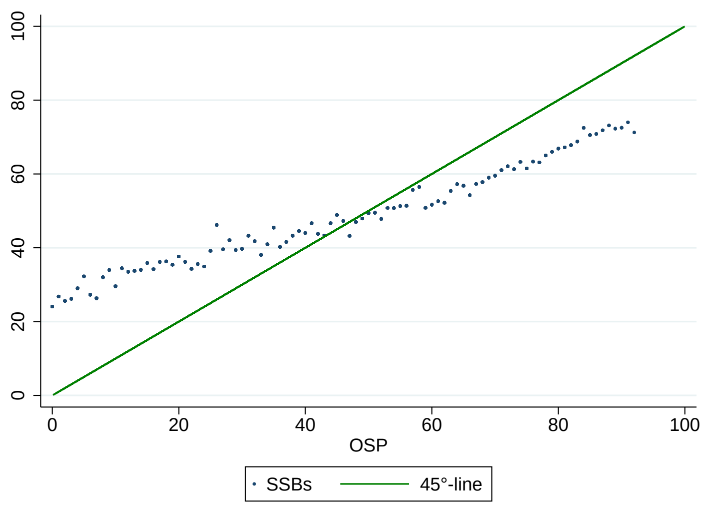
Notes: Unconditional subjective survival probabilities to survive to different target ages. The solid blue line are subjective survival beliefs, the dashed red line are the corresponding objective survival rates estimated with (**??**). Subjective survival beliefs are elicited in the HRS only for a combination of the age at interview of the individual (which is shown on the abscissa) and a corresponding target age, cf. Table 1. The step function follows from changes in the interview age/target age assignment.

Next we take a new perspective for which individual-level data are needed. We take the same data but instead of computing averages over age we average over OSPs, i.e., for each OSP we compute the average SSB. Figure 2 shows the corresponding results by plotting average SSPs against average OSPs. If SSPs were aligned along the 45-degree line, then there would not be any biases. However, we observe a very systematic pattern of misconception: Individuals with low OSPs on average overestimate whereas those with high OSPs on average underestimate their survival chances.

The two perspectives on the data taken in the respective figures 1 and 2 are suggestive of a very simple explanation for the observed biases across age.

Suppose that individuals were to always resolve any uncertainty about their survival chances in a 50-50 manner Bruine de Bruin, Fischhoff, and Halpern-Felsher (2000), i.e., their response would be a weighted average of a fifty percent chance of survival and the actual OSP. Observe that the intersection of the average SSB with the 45-degree line in Figure 2 is at an average OSP of about 50 percent lending support to this hypothesis.[11] Such a bias could explain the pattern of Figure 2. Furthermore, young respondents in our data have OSPs above 50 percent. If they were to apply such a simple heuristic then they would on average underestimate their chances to survive. Old respondents, on the other hand, on average have OSPs less than 50 percent.[12] Under such a heuristic they would accordingly overestimate their OSPs on average. Hence, such a 50-50 bias could simultaneously explain the pattern of Figure 1.

Figure 2: Objective Survival Probabilities and Subjective Survival Beliefs



Notes: SSB over OSP. For the figure we discretize OSP in 100 points and calculate average SSB for each point such that one blue dot represents average SSB for each OSP value.

We next argue that there is more information content in the data giving rise

[11] In fact, it is slightly less than 50 percent, see below.

[12] Recall from Table 1 that the target age is several years ahead of the interview age.

to alternative interpretations. To this purpose we repeat the previous analysis for different age-groups. In Figure 3 we display the result of Figure 2 and additionally for each target age group, cf. Table 1. The figure suggests that the flatness of SSBs against OSPs gets stronger with increasing age—compare, e.g., age group 65-69 with age group 80-84. In addition, the intersection with the 45-degree line moves down, from about 50 percent for age group 65-69 to about 40 percent for age group 80-84. Therefore, the average tendency for underestimation increases across age groups.

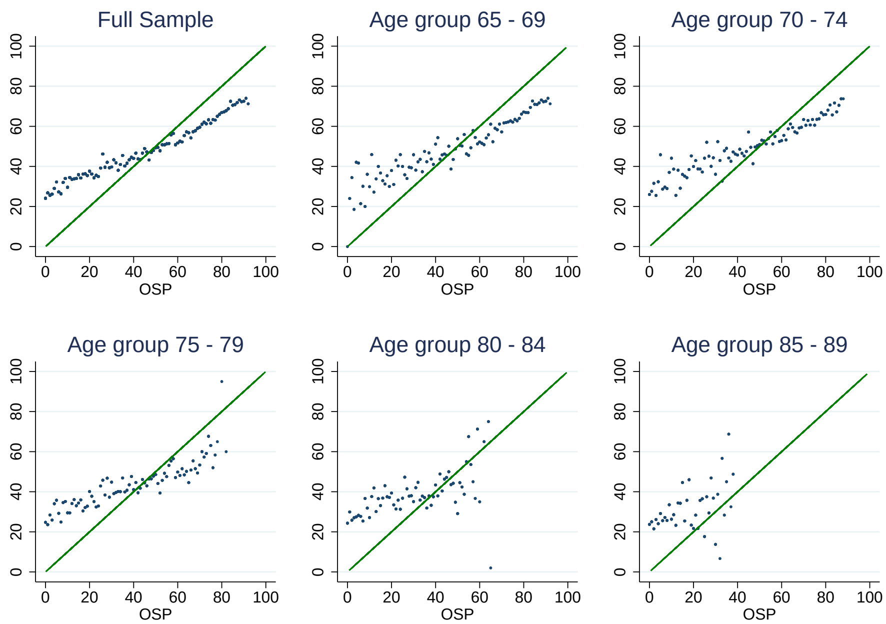
Our next aim is to explain these observations—the flatness itself as well as the increasing flatness and the increasing tendency to underestimate—by use of age-group specific probability weighting functions from prospect theory. We subsequently show that this gives rise to cognitive and psychological interpretations of the data rather than simple 50-50 heuristics.

Before moving on to these theoretical foundations and the following analyses by use of psychological data, a number of cautionary remarks are in order. First, we lack data for the elderly respondents in our sample because there are no high objective survival probabilities for these age-groups. Hence, our estimates of probability weighting functions will be prone to censoring of the data. Second, survival chances are bounded from below by zero and from above by one so that respondents with very high (low) objective survival probabilities cannot overestimate (underestimate) their survival chances by much. In consequence the observed average overestimation/underestimation might be—at least in part—influenced by this truncation of the data. Importantly, our use of psychological variables in our reduced form regressions to explain the observed biases in Section 5 addresses both concerns.

3 Interpreting Biases through Prospect Theory

As a generalization of rank dependent utility theories (pioneered by Quiggin 1981, 1982), modern prospect theory (PT) has developed into a comprehen-

Figure 3: Objective Survival Probabilities and Subjective Survival Beliefs by Age Groups



Notes: SSB over OSP. For the figure we discretize OSP in 100 points and calculate average SSB for each point such that one blue dot represents average SSB for each OSP value. The age-group panel focus on different target ages according to the question in the HRS, cf. Table 1.

sive decision theoretic framework that combines empirical insights (starting with Kahneman and Tversky 1979) with theoretical results about integration with respect to non-additive probability measures (cf. the Choquet expected utility theories of Schmeidler 1989 and of Gilboa 1987). This section models subjective survival beliefs through a probability weighting function applied to objective survival probabilities. Out of the many aspects of PT, our model of biases in survival beliefs is thus related to the experimental PT literature which shows that neither subjective probability assessments nor decision weights can be described as additive probabilities.[13]

[13]The typical finding of the so-called two stage approach is that subjective probability assessments resemble inverse S-shaped transformations of additive probabilities whereby

3.1 The Prelec Probability Weighting Function

To capture the cognitive dimension of likelihood insensitivity, on the one hand, and the psychological dispositions of optimism/pessimism, on the other hand, we adopt the non-linear probability weighting function (PWF) suggested by Prelec (1998). Thereby we allow for a flexible parametrization which allows the functional form to vary across interview age, cf. Table 1, in order to match the age-group specific bias patterns displayed in Figure 3. The objective probability of individual i to survive from interview age h to some age $t > h$, $OSP_{i,h,t}$, is transformed by the Prelec function into the corresponding subjective survival belief, $SSB_{i,h,t}$ as follows:

$$SSB_{i,h,t} = \left(\exp\left(-\left(-\ln\left(OSP_{i,h,t}\right)\right)^{\xi_h}\right)\right)^{\theta_h} + \epsilon_{i,h,t}. \tag{4}$$

Here, $\epsilon_{i,h,t}$ is an error term and $\theta_h \geq 0$ and $\xi_h \geq 0$ are parameters specific to the interview age. These two parameters control the elevation and the curvature of the function which can be interpreted as measures of pessimism/optimism and likelihood insensitivity, respectively.

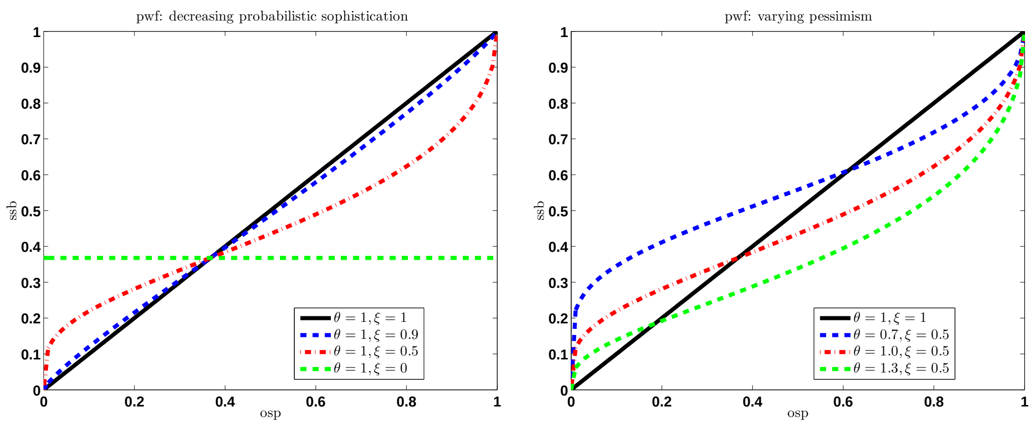
Before using this function in the context of survival belief formation, it is instructive to illustrate the role of these parameters. To this purpose we drop subscript h for now and simply speak of ξ, θ as parameters mapping objective probabilities $o = OSP_{i,h,t}$ into subjective beliefs $s = SSB_{i,h,t}$ according to the functional form in (4). For $\xi = \theta = 1$, the function coincides with the 45-degree line. An increase of ξ above one will then lead to a S-shaped pattern, a decrease below one to an inverse-S-shape. Given the patterns in the data shown in Figure 2, $\xi \leq 1$ is the relevant parametrization in our context. Furthermore, holding θ constant at one, then for any $\xi \neq 1$ it is straightforward to show, cf. Appendix A.1, that the intersection with the 45-degree line is at objective probability $o = \exp(1)$. The lower ξ the more pronounced is the inverse-S-shape of the figure. We illustrate this in Panel (a) of Figure 4 where we

these assessments undergo in turn an inverse S-shaped transformation (with an emphasis on pessimism) when becoming non-additive decision weights (cf., e.g., Fox and Tversky 1998, **?**), Kilka and Weber 2001, Wakker 2004, and **?**)).

decrease ξ from one to zero. In the limit where $\xi = 0$, the curve is flat. Hence, ξ can be interpreted as a measure of likelihood insensitivity and, for given θ, the closer ξ is to one, the less pronounced is this insensitivity. Next, as we illustrate in Panel (b) of Figure 4, decreasing θ leads to an upward shift of the PWF whereas increasing it to a downward shift. Accordingly, θ can be interpreted as a measure of relative pessimism whereby a higher value of θ means higher pessimism. Finally, notice that unless $\theta = 1$ (or $\xi = 1$) the two parameters interact. This can be seen in Panel (b) of Figure 4 where varying the pessimism parameter θ simultaneously affects the shape of the probability weighting function.

Figure 4: Pessimism and Probabilistic Sophistication in Stylized PWF

(a) Varying ξ (b) Varying θ



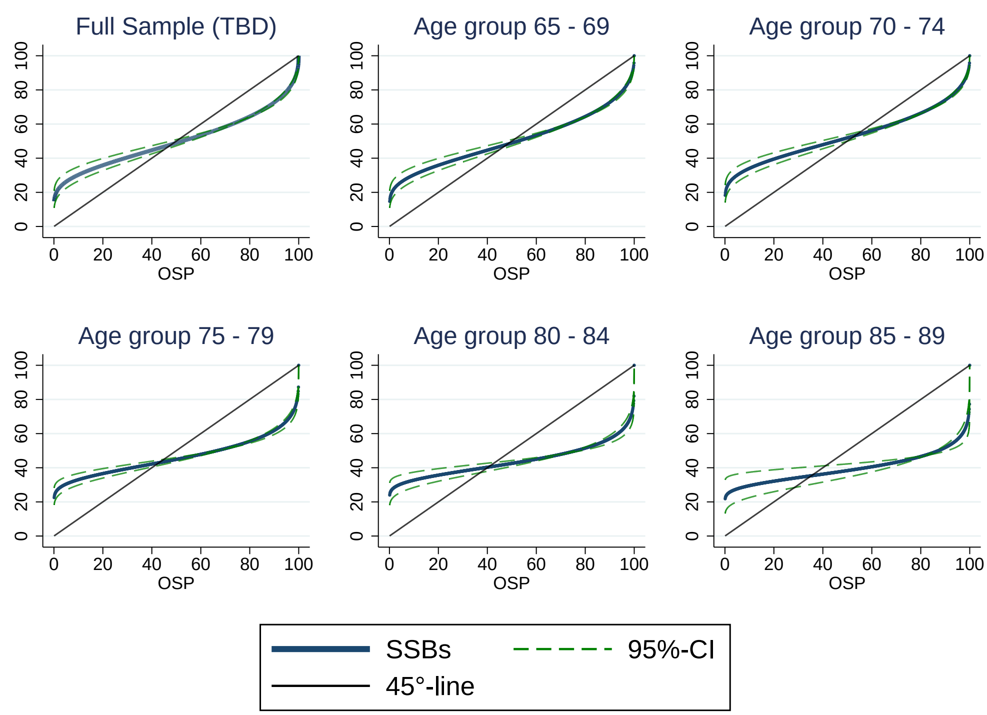
Notes: Stylized Prelec (1998) probability weighting functions. The left panel shows the impact of likelihood insensitivity, ξ, for $\theta = 1$ and $\xi \in [0, 0.5, 0.9, 1]$. The right panel shows the impact of pessimism for $\xi = 0.5$ and $\theta \in [0.7, 1, 1.3]$.

3.2 Estimated Shape of PWF: The Importance of Age

We next estimate parameters ξ_h, θ_h in the PWF 4 to match the data of Figure 4. We restrict these parameters to be the same for each interview age h assigned with a specific target age $m(h)$, i.e., we let $\xi_h = \bar{\xi}_{m(h)}$ and $\theta_h = \bar{\theta}_{m(h)}$. To identify these parameters we minimize the Euclidean distance between predicted and reported subjective survival beliefs for each individual in group $m(h)$.

Figure 5 shows predicted probability weighting functions. For the fitted values of the full sample displayed in the upper left panel we observe a quite symmetric weighting function intersecting the 45-degree line close to 0.5. As already suggested by the pattern in Figure 3, the age-specific weighting functions depicted in the other panels in Figure 5 reveal two facts: First, the functions get flatter with increasing age and second, the intersection with the 45-degree line is at lower values for older ages—it is at about 55 percent for age group 65-69 and at about 40 percent for age group 80-84.

Figure 5: Estimated Probability Weighting Functions



Notes: Estimated Prelec probability weighting functions for the full sample (upper left panel) and for different age-groups rotating clockwise in ascending order. Parameters estimated with non-linear least squares.

Figure 19 depicts the parameter estimates $\xi_{m(h)}, \theta_{m(h)}$ with the corresponding 95% confidence intervals. Standard errors are bootstrapped and confidence intervals are computed using the percentile method.[14] ,[15] According to these results, probability weighting functions get increasingly flatter with increasing age. Using the definition of Wakker (2010) such an increasing flatness may also be termed an increasing likelihood insensitivity (=lack of probabilistic sophistication) because it reflects that the information content of the objective probabilities decreases. We also observe that the intersection with the 45-degree line moves down. Again employing the terminology of Wakker (2010) this suggests that average pessimism is increasing with age.

Finally, we investigate whether a linear specification performs better than the non-linear specification a la **?**). We thereby relate to the theory of non-additive probability measures in the form of neo-additive capacities **?**). Assuming that there is always a positive objective probability to survive or to die, hence that $OSP_{i,h,m(h)} \in (0,1)$, the neo-additive capacity writes as

$$SSB_{i,h,m(h)} = (1 - \xi^l_{m(h)})(1 - \theta^l_{m(h)}) + \xi^l_{m(h)} OSP_{i,h,m(h)} \tag{5}$$

where $\xi^l_{m(h)} \in [0,1]$, $\theta^l_{m(h)} \in [0,1]$ are parameters that represent the analogues to parameters $\xi_{m(h)}$, $\theta_{m(h)}$ from the non-linear specification in (4).

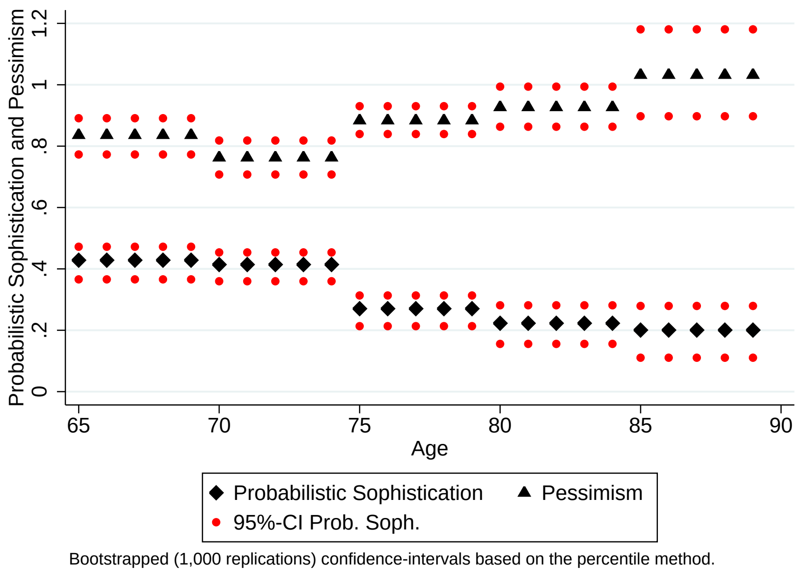
To see this observe that ξ^l controls the slope of the function whereby for $xi^l = 1$ the line in (5) corresponds with the 45-degree line. Therefore, any $\xi^l \in [0,1]$ can be interpreted as a measure of likelihood insensitivity. Likewise, $1 - \theta^l \in [0,1]$ determines the intersection of (5) with the 45-degree line with the intersection moving down when θ^l increases. Accordingly, θ^l can be interpreted as a measure of pessimism. Relative to (4), the particular advan-

[14]Since our data are clustered we perform a cluster bootstrap that samples the clusters with replacement. Thus, in each bootstrap we solve

$$\min_{\bar{\xi}_{m(h)}, \bar{\theta}_{m(h)}} \left\{ \sum_{i=1}^{N^{m(h)}} \left[\epsilon_{i,h,m(h)}\right]^2 \right\}.$$

[15]The percentile method uses the relevant percentiles of the empirical distribution of our bootstrap estimates of the Prelec parameters.

Figure 6: Estimated Prelec Parameters



Notes: Bootstrapped (1000 replications) 95%-confidence intervals are based on percentile method.
Source: Own Calculation based on the HRS.

tage of (5) is the parsimony in the specification which also implies that the measures of probabilistic specification ξ^l and pessimism θ^l are independent of each other.

We test the difference between the non-linear and the linear specifications in (4) and (5) by applying the Akaike and the Schwartz Bayesian information criteria (**?**; **?**) which are the relevant criteria for comparing non-nested models, cf. Cameron and Trivedi (2005). Since both functional forms have the same number of parameters no adjustment for a difference in the degree of freedom is required. Our findings summarized in Table 2 show that the non-linear specification performs generally better than the linear one, with the exception of interview ages 75-79.[16]

We conjecture that the better fit of the non-linear model is a consequence

[16]We show the coefficient estimates $1 - \xi^l_{m(h)}, \theta^l_{m(h)}$ of the linear specification in analogy to Figure 19 in Appendix E.

Table 2: Information Criteria for Non-Linear and Linear Probability Weighting Functions

	AIC			SBC		
Interview age h	Linear	Prelec	Difference	Linear	Prelec	Difference
65 - 69	423.72	419.47	4.25	434.58	430.33	4.24
70 - 74	662.83	662.78	0.05	673.98	673.93	0.05
75 - 79	717.68	719.87	-2.19	728.36	730.55	-2.19
80 - 84	569.82	562.01	7.81	579.68	571.88	7.81
85 - 89	259.87	256.79	3.09	268.52	265.43	3.09

Notes: Linear: Linear PWF. Prelec: Specification of the PWF according to **?**). AIC: Akaike information criterion. SBC: Schwartz Bayesian information criterion. *Source:* Own calculations based on the HRS (2015), waves 2008-2012.

of the natural truncation of objective survival probabilities at 0 and 1, respectively. The linear model postulates that subjective beliefs discontinuously jump from a positive value for an OSP slightly above zero to zero when the OSP equals zero (respectively from a positive value below one for an OSP slightly below on to one when the OSP equals one). Our estimates suggest that this is not an appropriate model of belief formation. In particular, eyeballing of Figure 2 suggests that the SSBs bend towards zero at low values of the respective OSPs. The non-linear model better accommodates this feature of the data.

This behavior of SSBs might be driven by focal point answers at SSBs of 0, 0.5, and 1, respectively. Bunching at these focal points has been documented in the literature, cf. **?**) and references therein. Ideally, we would explicitly model the probability of giving such focal points answers. To investigate their importance in a simplified manner we instead adopt a simpler approach by redoing the analysis from above for a sample in which all observations with focal point answers are excluded. Results, summarized in Appendix E suggest that our findings do not hinge on focal point answers.

We can therefore summarize our quantitative findings on probability weighting functions as follows. There is a strong age dependency in non-linear inverse-S-shaped probability weighting functions in that both the implied measures of relative pessimism as well as likelihood insensitivity are increasing with age. In the next section we explore whether direct psychological measures in the HRS support this cognitive/psychological interpretation of the biases in survival beliefs.

4 Age Patterns of Psychological and Cognitive Measures

In this section we analyze the age pattern of direct cognitive and psychological variables in the HRS. Our aim is to compare these with the indirect measures derived from estimating non-linear probability weighting functions on data on subjective survival beliefs in the previous section.

4.1 Measures

From wave 8 onward the HRS contains measures on optimism and pessimism. Measures on *dispositional optimism (pessimism)* are derived from the same statements as in the well-known Life Orientation Test-Revised (LOT-R).[17,18] Respondents are given various statements regarding a specific latent variable. For most variables they were asked *"please say how much you agree or disagree with the following statements"*. Each statement is rated on a scale from one (*strongly disagree*) to six (*strongly agree*). Average scores are taken to create indices for each psychological construct. Higher values for the psychological variables imply more optimistic, respectively more pessimistic attitudes.[19]

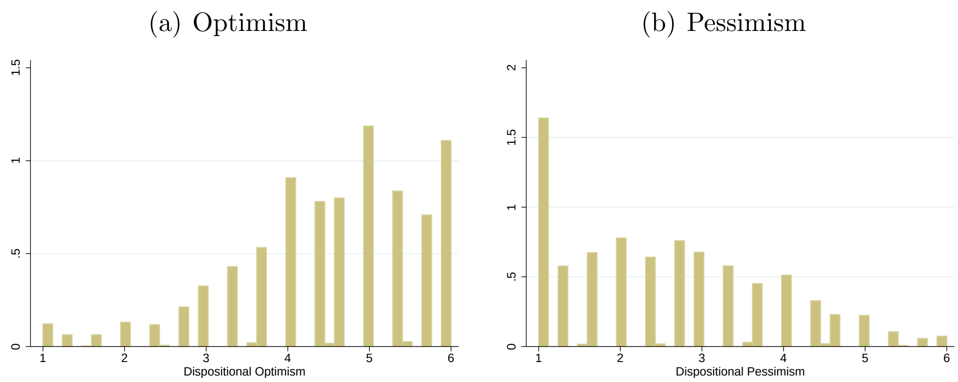
Note that optimism and pessimism are usually measured separately, i.e., respondents are asked questions with negative connotations (pessimism) as well

[17] Such statements are, e.g., "In uncertain times I usually expect the bes".

[18] The Life Orientation Test-Revised questionnaire (LOT-R) was developed to measure dispositional optimism, i.e., a generalized expectation of good outcomes in one's life Scheier and Carver (1987, **?**). Kaniel, Massey, and Robinson (2009) find dispositional optimism as measured with LOT-R to be related to various expectations about events in a labor market setting.

[19] The index score is set to missing if responses on more than half of the respective statements are missing.

Figure 7: Histogram of Optimism and Pessimism



Notes: Histogram of 'optimism' and 'pessimism' variable. Averages of answer pattern where 1 indicates 'strongly disagree' and 6 'strongly agree'.

as with positive connotations (optimism). The reason for separate measures is that these two concepts were found to show some bi-dimensionality Herzberg, Glaesmer, and Hoyer (2006).[20] Figure 7 showing the histograms on both measures in our sample underscores this aspect. Dispositional pessimism shows a clear focal point at index value 1 (="strongly disagree") whereas dispositional optimism apparently has focal point answers at values 4, 5 and 6 whereby the peak is at 5. In our empirical analyses we therefore use separate variables for each concept although in our theoretical analysis we speak of increasing pessimism and decreasing optimism interchangeably.

For a measure corresponding to "likelihood insensitivity" our choice of a proxy variable is motivated by our cognitive interpretation of likelihood insensitivity Wakker (2010). Thus, we include a variable measuring cognitive weakness of the respondent. It is a version of a composite score taken from RAND and combines the results of several cognitive tests. For instance, respondents were asked to recall a list of random words, to count backwards and to name the (Vice) President of the United States. In total there are 35 questions and results are summarized in an ability score. We take RAND's

[20]Some authors neglect the possibility of bi-dimensionality, cf., e.g., Liu, Tsou, and Hammitt (2007).

composite score of cognitive ability as given and create our score of cognitive weakness. For this we subtract the cognitive ability score from the maximal achievable value, i.e., our measure of cognitive weakness is 35 minus cognitive ability. A higher value of the score indicates higher cognitive weakness. An overview of our three measures of psychological/cognitive variables is given in Table 3.

Table 3: Psychological and Cognitive Variables

	Min	Max	Mean	SD	α^*
Psychological Variables					
Dispositional Optimism	1	6	4.53	1.16	0.80
Dispositional Pessimism	1	6	2.60	1.30	0.77
Cognitive Variable					
Cognitive Weakness	0	35	13.50	5.19	n.a.

Notes: * Cronbrach's α. This statistic is a measure for the internal consistency of a psychometric test. As a rule of thumb the α has to be ≤ 0.7 (**?**)

4.2 Age Patterns

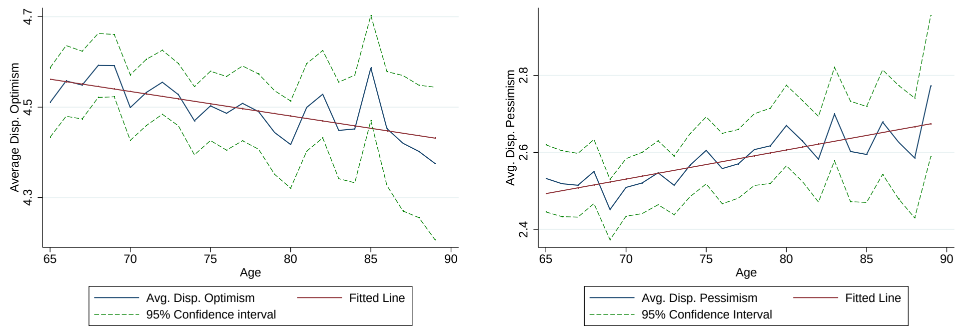
We now display average values of the measures of psychological and cognitive weakness over age, cf. Figures 8 for optimism/pessimism and Figure 9 for cognitive weakness. Optimism decreases by 2.9% and pessimism increases by 12.2% from age 65 to 90. The fact that pessimism increases more strongly than optimism decreases supports the notion of bi-dimensionality of these two measures.[21]

Turning to cognitive weakness the average index value is increasing from 11.8 to 17.9 between ages 65 and 89. Age-dependence is more pronounced for cognitive weakness than for the two psychological measures.

[21] Note that in both regressions $o_h = \beta_0 + \beta_1 h + \varepsilon_h$ with $\hat{\beta}_1 = -0.008$ and $p_h = \beta_0 + \beta_1 h + \varepsilon_h$ with $\hat{\beta}_1 = 0.010$ the coefficient $\hat{\beta}_1$ is significant at the 1.0% significance level.

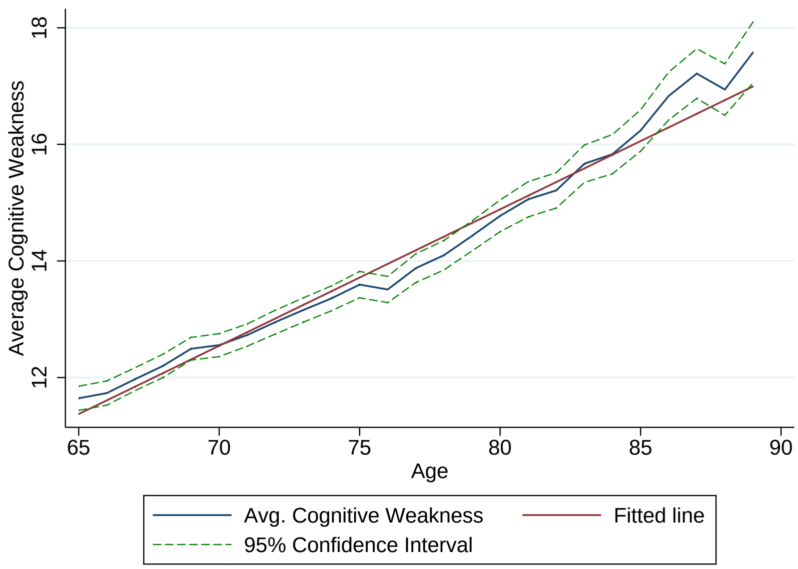
Figure 8: Average Optimism and Pessimism over Age

(a) Optimism (b) Pessimism



Source: Own calculations based on HRS.

Figure 9: Average Cognitive Weakness Score over Age



Source: Own calculations based on HRS.

Hence, the age trends of the direct psychological measures coincide with the indirect measures we derived from estimating non-linear probability weighting functions on the data of subjective survival beliefs. These findings therefore provide support of our psychological interpretation of the biases in subjective beliefs. Our next aim is to investigate this interpretation further through regression analyses.

5 Psychological and Cognitive Factors in Survival Assessments

In this section we go beyond our previous descriptive analyses by investigating the impact of psychological, respectively cognitive, measures on the formation of subjective survival beliefs taking several control variables into account. Because psychological measures are available since 2006 we pool waves 2006-2012 of the data. Psychological variables are only available for half of the sample in each wave. We run pooled OLS regressions with fixed-effects for waves and target age (TA) groups. We consider the psychological/cognitive measures in lags so that we can treat those as weakly exogenous. As the objective survival probabilities are themselves estimates, we implement a two-sample bootstrap procedure to estimate the standard errors for our coefficient estimates, c.f. Appendix D for a detailed description of the procedure.

5.1 Psychological Dispositions, Cognitive Weakness and Subjective Survival Beliefs

We first investigate whether the psychological and cognitive variables are associated with subjective survival beliefs. As we argue in Section 3, inverse-S-shaped probability weighting functions are a reasonable model of biases in subjective survival beliefs. We now consider a parameterized variant of the Prelec (1998) function whereby we postulate that for each individual in the sample i and each age h the implicit measures of cognition and optimism/pessimism from equation (4) are linearly dependent on the cognitive, respectively psychological, variables as follows:

$$\xi_h = \xi_0 + \xi_1 c_{i,h} \tag{6}$$

$$\theta_h = \theta_0 + \theta_1 p_{i,h-2} + \theta_2 o_{i,h-2}. \tag{7}$$

In the above, $c_{i,h-2}$ is our measure of cognitive weakness and $p_{i,h-2}$ is the lag of our measure of pessimism, respectively $o_{i,h-2}$ is the lag of our measure of optimism. We include these measures with lags in order to address potential endogeneity concerns. Using (6) in (4) we start by estimating the following specification on the pooled sample of HRS data:

$$SSB_{i,h,m(h)} = \left(\exp\left(-\left(-\ln\left(OSP_{i,h,m(h)}\right)\right)^{\xi_0+\xi_1 c_{i,h}}\right)\right)^{\theta_0+\theta_1 p_{i,h-2}+\theta_2 o_{i,h-2}} \varepsilon_{i,h,m(h)}. \tag{8}$$

Turning to the parameters of interest in specification (8), we refer back to our analysis of Section 3, in particular to the illustration Figure 4. In light of our discussion there, parameters ξ_0 and θ_0 capture a “baseline bias” in subjective beliefs. This may encompass both a 50-50 bias as well as any psychological biases that are of relevance for the formation of subjective survival beliefs prior to our sample (which starts at age 60). If such a bias exists in terms of psychological predisposition, then $\theta_0 \neq 1$. For a dominance of pessimism we would expect $\theta_0 > 1$, and respectively $\theta_0 \in (0, 1)$ if optimism dominates. Also, if such a bias exists in terms of an initial lack of cognition, or, a general 50-50 bias (our approach does not allow us to disentangle between these explanations), then $\xi_0 \in (0, 1)$.

Furthermore, recalling the illustrative analysis of Figure 4 lowering ξ_h leads to a flatter PWF. Therefore, if cognitive weakness is relevant for the formation of subjective beliefs, we expect that $\xi_1 < 0$. Now, suppose we find that $\xi_0 \in (0, 1)$ and $\xi_1 = 0$. Then the predicted PWF is initially (at age 60) flatter than the 45-degree line but cognition is not a driver of the formation of subjective survival beliefs for ages past 60. Accordingly, such a finding would indicate that the baseline bias is due to a 50-50 bias and not due to a lack of cognition.

Figure 4 also shows that increasing θ_h leads to a lower elevation of the PWF. If pessimism and optimism are relevant for the formation of subjective beliefs, then $\theta_1 > 0$ and $\theta_2 < 0$.

Table 4 summarizes our main results. [TBC]

Table 4: The Effects of Cognition and Psychological Measures on Subjective Survival Beliefs

	point estimate	CI-	CI+
Cog.Weak. Intercept (ξ_0)	0.5457	0.4945	0.5922
Cog.Weak. Slope (ξ_1)	-0.0134	-0.0167	-0.0095
Psycho. Intercept (θ_0)	1.0285	0.9482	1.1239
Pessimism Slope (θ_1)	0.0295	0.0171	0.0413
Optimism Slope (θ_2)	-0.0583	-0.0732	-0.0442
AIC	2990.0	2771.4	3198.0
BIC	3025.5	2806.8	3233.4
RSS	726.1	705.6	746.2

Notes: Column 2 shows the point estimates, columns 3 and 4 the respective bounds of 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1000 replications).

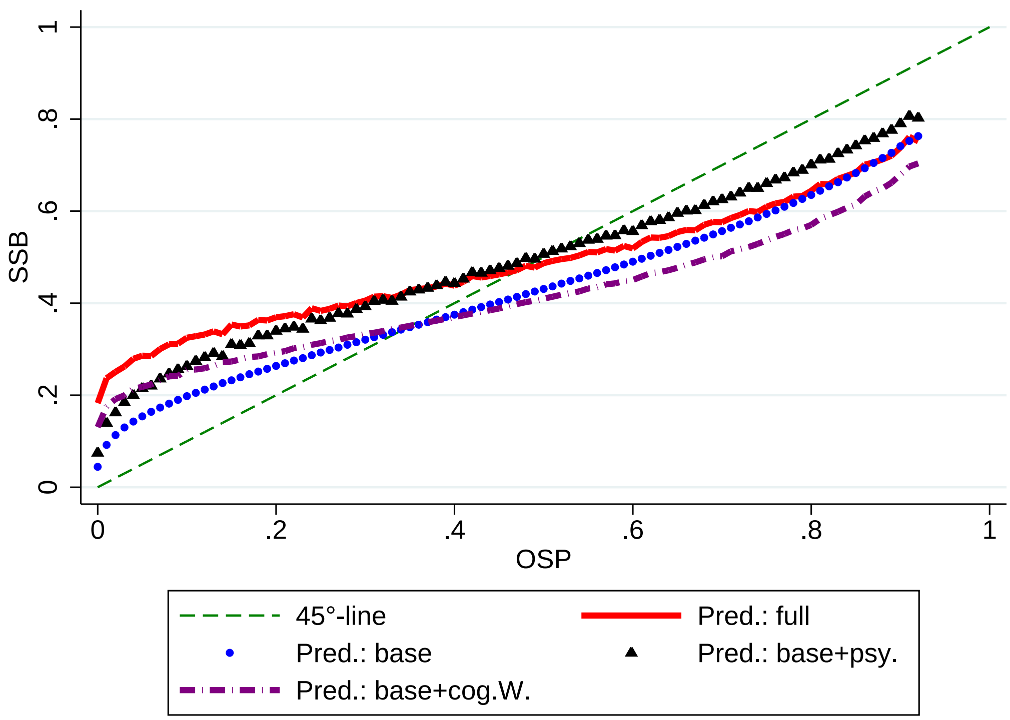
5.1.1 Sensitivity Analysis: Linear Specification

We estimate the following specification:

$$\begin{aligned} SSB_{i,h,m(h)} = \theta_0 + \theta_1 p_{i,h-2} + \theta_2 o_{i,h-2} + \theta_3 c_{i,h-2} \\ + \theta_4(c_{i,h-2} \times o_{i,h-2}) + \theta_5(c_{i,h-2} \times p_{i,h-2}) \\ \theta_6 OSP + \theta_7(c_{i,h-2} OSP) + \epsilon_{i,h} \end{aligned} \quad (9)$$

Table 5 summarizes our main results.

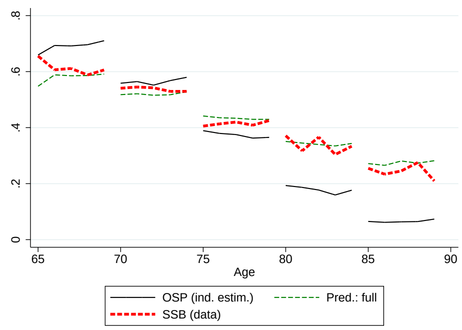
Figure 10: Decomposition of Probability Weighting Function



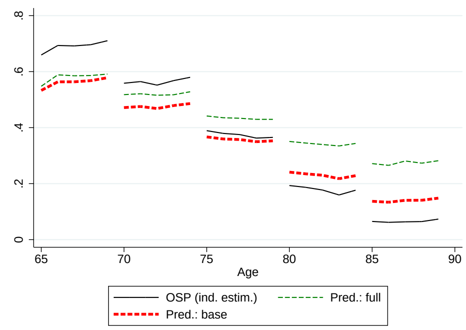
Notes: Probability weighting function according to parameter estimates. "base bias": only axis coefficients of cognitive and psychological measures ($\xi_1 = \theta_1 = \theta_2 = 0$); "base pl. cogn. w.": base bias plus average cognition; ($\theta_1 = \theta_2 = 0$); "base+psy.": base bias plus psychological variables ($\xi_1 = 0$); "full": all effects.
Source: Own calculations based on HRS.

Figure 11: Decomposition over the Life-Cycle

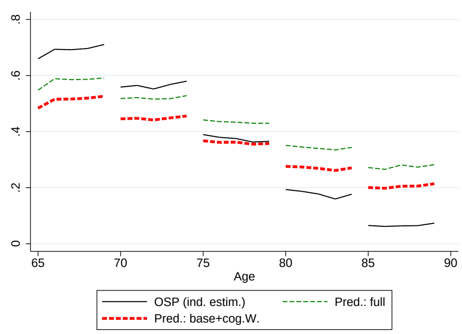
(a) Prediction



(b) Base



(c) Base & Cognitive Weakness



(d) Base & Psych.

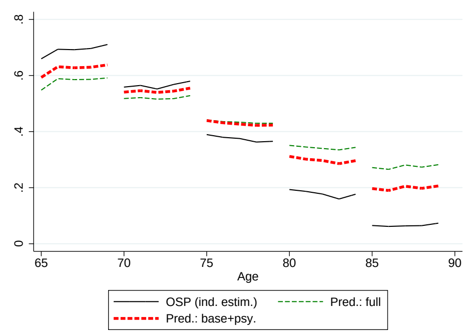
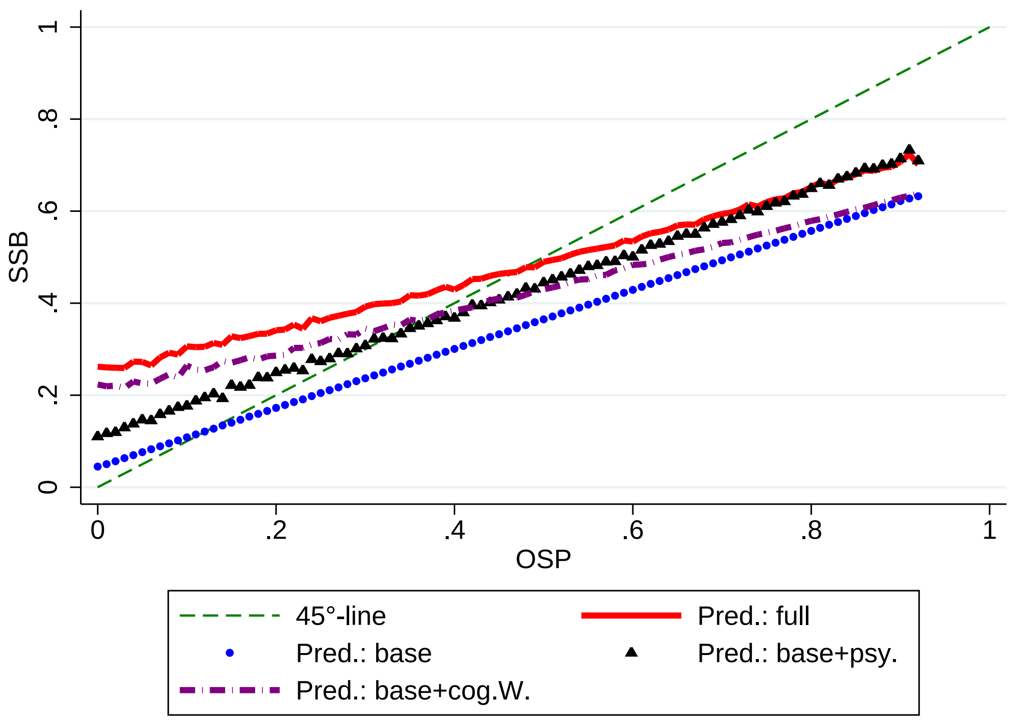


Figure 12: Linear Model: Decomposition of Probability Weighting Function



Notes: Probability weighting function according to parameter estimates. "base bias": only axis coefficients of cognitive and psychological measures ($\xi_1 = \theta_1 = \theta_2 = 0$); "base pl. cogn. w.": base bias plus average cognition; ($\theta_1 = \theta_2 = 0$); "base+psy.": base bias plus psychological variables ($\xi_1 = 0$); "full": all effects.
Source: Own calculations based on HRS.

Figure 13: Linear Model: Decomposition over the Life-Cycle

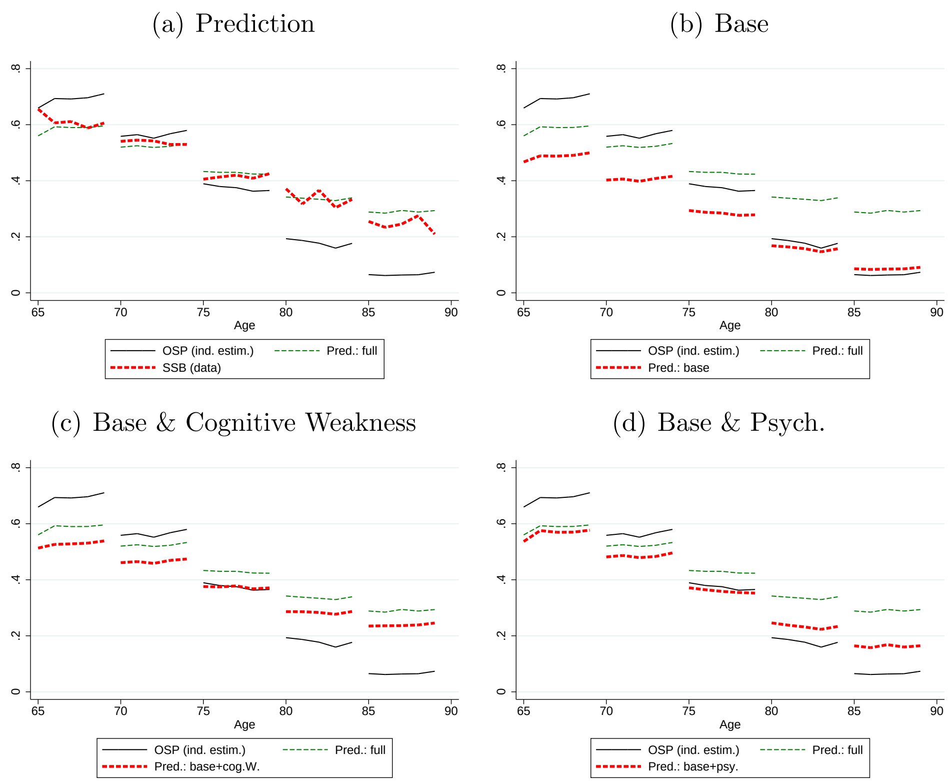


Table 5: Linear Model: The Effects of Cognition and Psychological Measures on Subjective Survival Beliefs

	point estimate	CI-	CI+
Constant	0.0441	-0.0487	0.1441
OSP	0.6415	0.5721	0.6985
Cog. Weak.	0.0112	0.0040	0.0185
OSP × Cog. Weak.	-0.0001	-0.0002	-0.0001
Pessimsim	-0.0163	-0.0316	-0.0015
Optimism	0.0259	0.0114	0.0407
Optimism × Cog. Weak.	-0.0003	-0.0014	0.0008
Pessimsim × Cog. Weak.	-0.0001	-0.0011	0.0010
AIC	2943.5078	2741.4094	3146.1782
BIC	3000.2205	2798.1226	3202.9202
RSS	721.7598	702.1242	741.0155

Notes: Column 2 shows the point estimates, columns 3 and 4 the respective bounds of 95%-confidence intervals (CI- and CI+), which are calculated with the percentile method (1000 replications).

5.2 The Role of Psychological and Cognitive Factors for Survival Biases

Our findings so far confirm that psychological variables have predictive power beyond objective survival rates: pessimists underestimate whereas optimists overestimate their survival probabilities. We also find that cognitive weakness takes away predictive power from the objective survival probabilities and is associated with an increasing upward bias in beliefs. These insights are consistent with our theoretical considerations of Section 2.

Yet, our previous analysis only shows that psychological and cognitive factors impact subjective survival beliefs. This does not mean that psychological variables and cognitive weaknesses are also associated with higher levels of misconception. To investigate this, we now turn to quantile regressions. We rank the data from underestimation to overestimation so that we have strong underestimators at the 10th percentile with $SSB_{i,h,m(h)} << OSP_{i,h,m(h)}$ and strong overestimators at the 90th percentile. For the two extreme percentiles and the median we next study the impact of psychological and cognitive variables on the *difference* between subjective and objective survival probabilities, i.e., on the strength of survival misconception according to the following specification:

$$SSB_{i,h,m(h)} - OSP_{i,h,m(h)} = \beta_0 + \beta_1 OSP_{i,h,m(h)} + \beta_2 p_{i,h-2} + \beta_3 o_{i,h-2} + \beta_4 c_{i,h} + \vec{\beta}_5' \vec{x}_{i,h} + \varepsilon_{i,h}. \quad (10)$$

Since the absolute value of misperception depends on the level of the objective survival probability, we include $OSP_{i,h,m(h)}$ on the right-hand side.[22]

If pessimism is a driver of underestimation and optimism is a driver of overestimation, then pessimism should be more pronounced for the 10th percentile, respectively optimism should be more important at the 90th percentile. We therefore hypothesize that the coefficient on optimism will increase when moving up across the percentiles and the coefficient on pessimism will decrease. As to cognitive weakness, recall from Figure 9 that cognitive weakness is increasing in age and from Figure 2 that overestimation is particularly relevant in older age, hence when cognitive weakness is also higher. Given that both cognitive weakness as well as the extent of overestimation are increasing with age, we conjecture that cognitive weakness is increasingly positively related with biases in survival changes when we move across percentiles from strong underestimators to strong overestimators.

Our results reported in Table 6 confirm our hypotheses. [TBC]

[22]Observe that these quantile regressions address the concerns of biases induced by truncation and censoring, cf. our discussion at the end of Section 2.4.

Table 6: Drivers of Misconception: Results from Quantile Regressions

SSB-OSP	point estimate	CI-	CI+
10thPercentile			
Constant	-1.3236	-3.6362	0.7056
OSP	-77.4857	-79.4506	-74.8909
Cog. Weak.	-0.1257	-0.2076	-0.0349
Pessimism	-1.0341	-1.3243	-0.6521
Optimism	0.7204	0.3825	0.9889
Median			
Constant	1.8679	-3.6604	7.0010
OSP	-31.3842	-35.0754	-28.5811
Cog. Weak.	0.4822	0.2400	0.7508
Pessimism	-2.4665	-3.1401	-1.5820
Optimism	2.6295	1.9636	3.4899
90thPercentile			
Constant	52.7518	45.5359	56.3831
OSP	-69.9088	-73.1349	-65.8222
Cog. Weak.	1.0592	0.8765	1.2755
Pessimism	-0.0234	-0.7309	0.6903
Optimism	1.8437	1.2028	3.0487

TBC

6 Concluding Remarks

This paper compares subjective survival beliefs (SSBs) with objective survival probabilities (OSPs) that we estimate based on individual level characteristics. We establish a two-fold and related strong regularity of survival misperceptions. First, relatively young households in our sample underestimate whereas relatively old households overestimate their chances to survive. Second, households overestimate survival chances with low objective probabilities and underestimate chances with high objective probabilities. Based on this latter finding we estimate inverse-S-shaped probability weighting functions on the data and establish a strong age dependency in the shape of these functions. Our coefficient estimates suggest that implied measures of pessimism and of cognitive weaknesses are increasing with age. Direct psychological and cognitive variables confirm these age patterns.

Based on these descriptive findings, we turn to reduced form regressions. Our results show that psychological and cognitive variables have strong quantitative effects on survival beliefs. [TBC]

Our decomposition analysis also suggests that our findings are consistent with theories of rational learning with psychological biases developed in Ludwig and Zimper (2013a) and **?**). Specifically we show that over age predicted subjective survival beliefs converge to the respective objective survival probabilities when we shut down the effects of psychological variables and the lack of cognition. This is consistent with rational Bayesian learning. The psychological and cognitive factors then superimpose the aforementioned biases. However, with respect to learning dynamics, our findings are only suggestive because we do not develop econometric specifications of learning models and accordingly do not directly test their implications with dynamic panel methods.

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