Optimal Public Debt with Life Cycle Motives

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***The views herein are the authors' and not necessarily those of the BLS, US DOL, Board of Governors or their staffs.*

Motivation

Q: What level of debt should the Government hold?

Government Debt

- Welfare Costs:
    - Crowds out capital $\Rightarrow$ lower output
    - Financed by distortionary taxes
- Welfare Benefits (financial liquidity):
    - $\Uparrow$ return to savings $\Rightarrow$ reduces cost of holding precautionary savings

Aiyagari & McGrattan (1998)

- Incomplete markets, infinitely lived
- Optimal debt $= \frac{2}{3}$ of output
- Ignores life cycle
    - Agents transition through different phases of life cycle

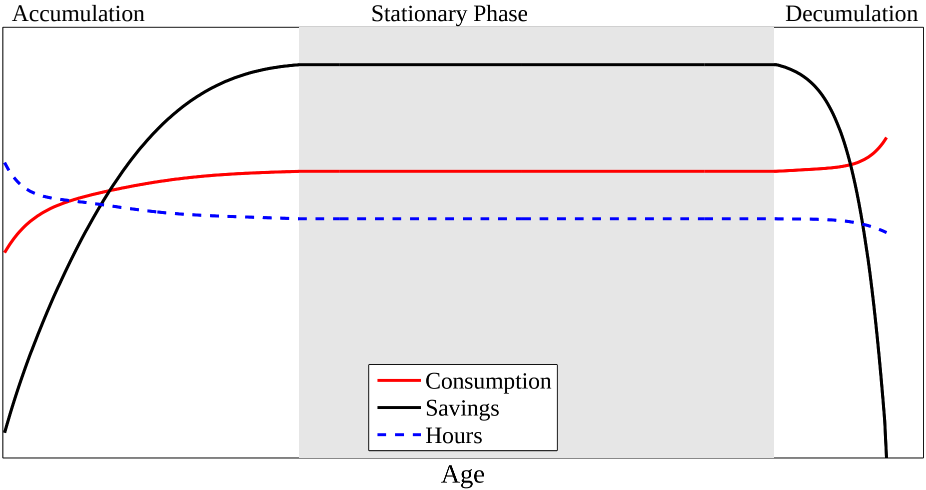
This Paper

Question: What is optimal level of gov't debt in life cycle model?

Effect of Life Cycle on Optimal Pubic Debt

- Large effect on optimal public debt
  - Life cycle model: savings = 160% of output
  - Infinitely lived agent model: debt = 87% of output
- Welfare of adopting misspecified optimal tax policy: CEV = 3.5%
- **Different policies due to different phases of life cycle**

Mechanism: Example (I)



- Life cycle all three phases; Infinitely lived only one phase
- Changing prices has different effects

Mechanism: Example (II)

Affect of Gov't Debt on Factor Prices:

- Decreases Government Debt (increases Gov't. savings)
- Crowds in **Productive Capital**
- Interest rate $\Downarrow$
- Wage $\Uparrow$

Infinitely Lived Agent Model

- Only stationary phase
- Lower interest rate decreases liquidity

Life Cycle Model

- Accumulation, Stationary, Decumulation Phases
- Higher wage more accommodative during accumulation phase

Literature

Effects of government debt with incomplete markets

1. Steady State
    - Aiyagari & McGrattan (1998) - optimal debt large
    - Floden (2001) - if transfers below optimal then $\Uparrow$ gov't debt
    - Dyrda & Pedroni (2015) - if taxes optimized then less debt optimal
    - Winter & Roehrs (2015) - skewed wealth leads to gov't savings being optimal
2. Transition
    - Dydra & Pedron (2015); Winter and Roehrs (2015); Desbonnet & Weitzenblum (2012): Considerable welfare costs in transition

Previous analysis of question done with infinitely lived agent model

Outline

1. Introduction
2. Life cycle Model with Public Debt
3. Calibration
4. Results
5. Conclusion

Life cycle Model with Public Debt

Overview of Model

- General Equilibrium incomplete markets model
- Overlapping generations of heterogenous agents
- Idiosyncratic uninsurable shocks:
  - Agent's labor productivity
  - Unemployment spells
  - Mortality
- Labor is supplied elastically
- Agents choose when to retire
- Social Security and UI programs modeled similar to U.S.

Production

- Representative Firm:
  - Large number of firms
  - Sell consumption good
  - Perfectly competitive product market

- Technology:
  - Cobb-Douglas: $Y = K^{\zeta} L^{1-\zeta}$
  - No aggregate uncertainty

- Resource Constraint: $C + (K' - (1-\delta)K) + G = Y$

Demographics

- J overlapping generations
- s_j probability of living to $j+1$ given one is alive in j
- Remaining assets are accidental bequests (Tr_t).
- If still alive agents die with certainty at age J
- Agents retire at endogenously determined age (J_{ret}), irreversible
  - $J_{ret} \in [\underline{J}_{ret}, \overline{J}_{ret}]$
- Population growth = g_n

Labor Earnings (I)

Earnings: $y_{ij} = we_{ij}h_{ij}(1 - \bar{h}_{ij})$

- Labor productivity, e_{ij}
- Choice of hours, $h_{ij} \in [0, 1]$
- Unemployment shocks, $\bar{h}_{ij}$

Labor Productivity: $\log(e_{ij}) = \theta_j + \alpha_i + \epsilon_{ij} + \nu_{ij}$

- Age-profile: $\{\theta_j\}_{j=1}^{\bar{J}_{ret}}$
- Idiosyncratic type: $\alpha_i \overset{iid}{\sim} \mathcal{N}(0, \sigma_\alpha^2)$
- Transitory shock: $\epsilon_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$
- Persistent shock: $\nu_{ij+1} = \rho\nu_{ij} + \eta_{ij+1}$

$$\eta_{ij+1} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\nu^2)$$

$$v_{i1} = 0$$

Labor Earnings (II)

Earnings: $y_{ij} = we_{ij}h_{ij}(1 - \bar{h}_{ij})$

- Labor productivity, e_{ij}
- Choice of hours, $h_{ij} \in [0, 1]$
- Unemployment shocks, $\bar{h}_{ij}$

Unemployment Shock: $\bar{h}_{i,j}$

- Fraction of period unemployed
  - Either 0 or d_j
  - Probability of non zero: p_j
  - Probability and duration are age specific
- Receive unemployment benefits
  - $b_{ui}(we_{ij})$

Asset Markets

Incomplete Asset Markets:

- Incomplete w.r.t. idiosyncratic productivity risk, unemployment risk, mortality risk
- Agents save using non-contingent bond, $a \geq 0$
- Before tax rate of return, r

Market Clearing: $A = K + B$

- Supply = Aggregate Savings
- Demand = Productive Capital (K) + Gov't Debt (B)

Government Policy

Budget Constraint:

$$G + UI + rB = (B' - B) + \Upsilon_y$$

1. G: Consumes in an unproductive sector
2. UI: Pays insurance when unemployed
3. B: Borrows or saves at interest r
4. Υ_y : Finances with progressive income taxation

Self Financing Programs:

5. Runs Social Security Program
6. Distributes accidental bequests

Social Security

Overview:

- Finances SS with a flat tax on labor income τ^{ss}
- Half payed by employer (up to cap)
- Pays benefit b_i^{ss} based on
    - Past income AIME: x_i
    - Age of retirement: J_{ret}

Detail

Competitive Equilibrium

1. Agents optimize utility s.t. budget constraint
2. Prices set by marginal product of capital and labor
3. Social Security budget clears
4. General Government budget clears
5. Capital and labor market clear
6. Stationary distribution of individuals over state space
    - Accounting for GDP growth: g

▸ Dynamic Programming

Calibration

Firm

Production: $Y = K^{\zeta} N^{1-\zeta}$

Notation	Parameter	Value	Source
Capital Share	ζ	.36	CKK
Depreciation	δ	.0833	$\frac{I}{Y} = 25.5\%$
Growth	g	0.02	

Demographics

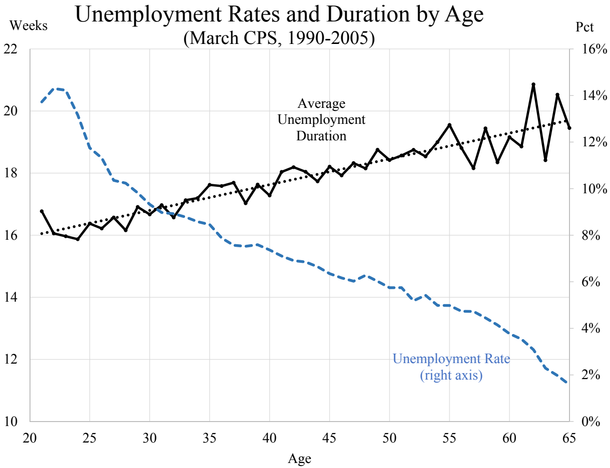
- Agents enter the model at age 20
- s_j - Bell and Miller (2002)
- Remaining agents die with certainty age $100(J)$
- Population growth: $g_n = 1.1\%$

Idiosyncratic Labor Productivity

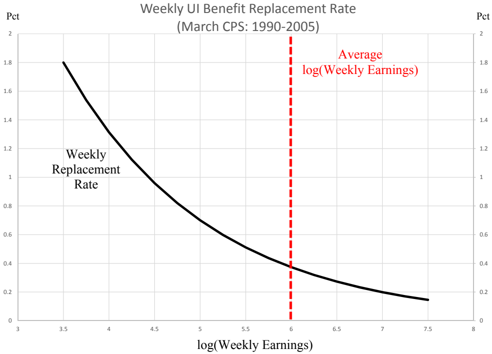
Labor Productivity: $\log(e_{ij}) = \theta_j + \alpha_i + \nu_{ij} + \epsilon_{ij}$

Notation	Parameter	Value	Source
Persistence Shock	σ_ν^2	0.017	Kaplan (2012)
Persistence	ρ	0.958	Kaplan (2012)
Ability	σ_α^2	0.065	Kaplan (2012)
Transitory Shock	σ_ϵ^2	0.081	Kaplan (2012)
Age Profile	$\{\theta_j\}_{j=1}^{\bar{J}_{ret}}$		Kaplan (2012)

Unemployment



Unemployment Insurance



- Base Benefit: $b_{ui}(we) = rr(we)we\ h_{\text{average}}\ \overline{h}$
- Replacement rate: $rr(we) = \phi_{ui,0} \ln(we)^{\phi_{ui,1}}$
- $b_{ui} \in [.13 \times \text{avg. earnings} \times \overline{h}, 1.1 \times \text{avg. earnings} \times \overline{h}]$

Preferences

Preferences: $u(c) + v(h, \overline{h}) = \frac{c^{1-\gamma}}{1-\gamma} - \chi_1 \frac{((1-\overline{h})^{\xi} h)^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} - \chi_2 \mathbb{1}(j < J_{ret})$

Notation	**Parameter**	**Value**	**Source**
Conditional Discount	β	1.0	$\frac{K}{Y} = 2.7$
Risk aversion	γ	2.2	Kaplan (2012)
Frisch Elasticity	σ	0.41	Kaplan (2012)
Utility during unemployment	ξ	0	Kaplan (2012)
Disutility to Labor	χ_1	70.0	Avg. $h_j = \frac{1}{3}$
Fixed Cost to Working	χ_2	1.105	70% retire by J_{nr}

Government

Income tax function: $T(\tilde{y}_t; \tau_0, \tau_1, \tau_2) = \tau_0(\tilde{y}_t - (\tilde{y}_t{}^{-\tau_1} + \tau_2)^{-\frac{1}{\tau_1}})$

Notation	**Parameter**	**Value**	**Source**
Avg. Tax	τ_0	.258	Gouveia & Strauss (1994)
Progressiveness	τ_1	.768	Gouveia & Strauss (1994)
Progressiveness	τ_2	8.99	Balance budget
Gov't Consumption	$\frac{G}{Y}$	15.5%	Data
Debt to GDP	$\frac{B}{Y}$	$\frac{2}{3}$	Aiyagari & McGrattan (1998)
UI	$\phi_{ui,0}$	0.38	March CPS
UI	$\phi_{ui,1}$	-0.80	March CPS

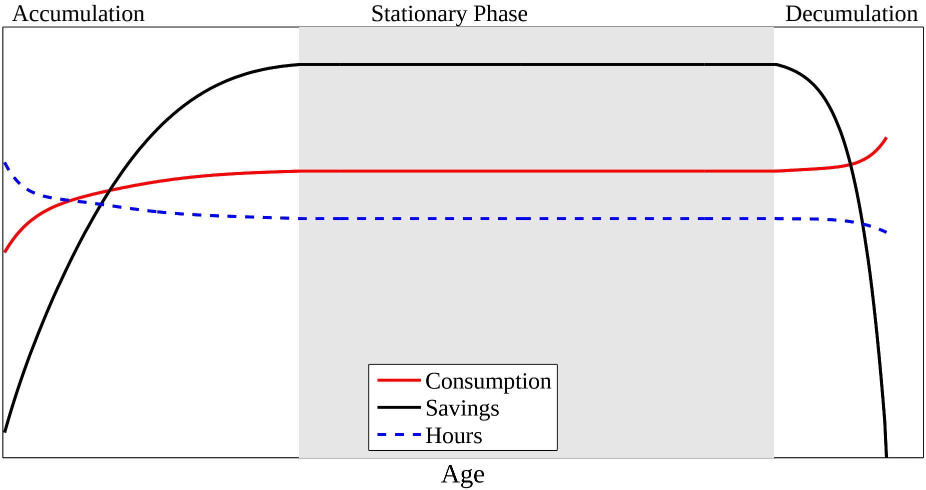
▸ Social Security

Results

Outline:

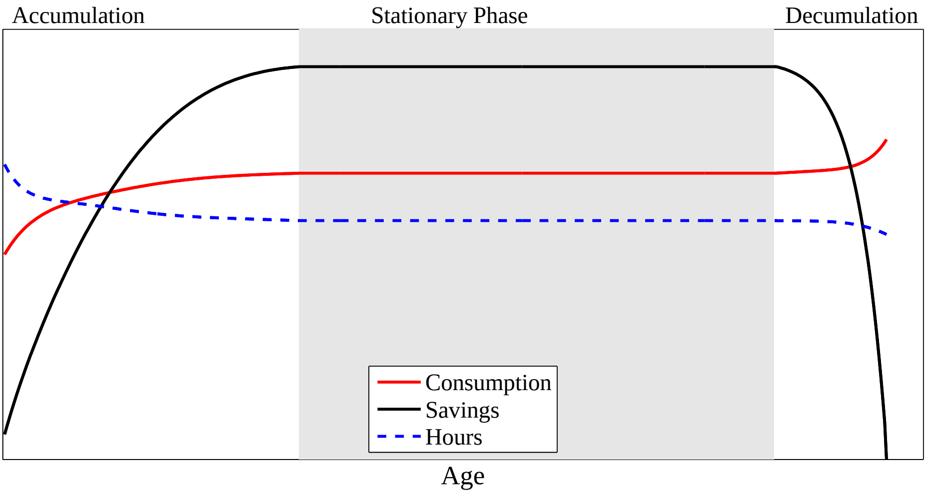
1. Illustrative Example
2. Social Welfare Function
3. Optimal Policy
4. Welfare Effects
5. Decompose Mechanisms
6. Transfer Programs & Borrowing Constraints
7. Sensitivity to Social Welfare Function

Illustrative Example



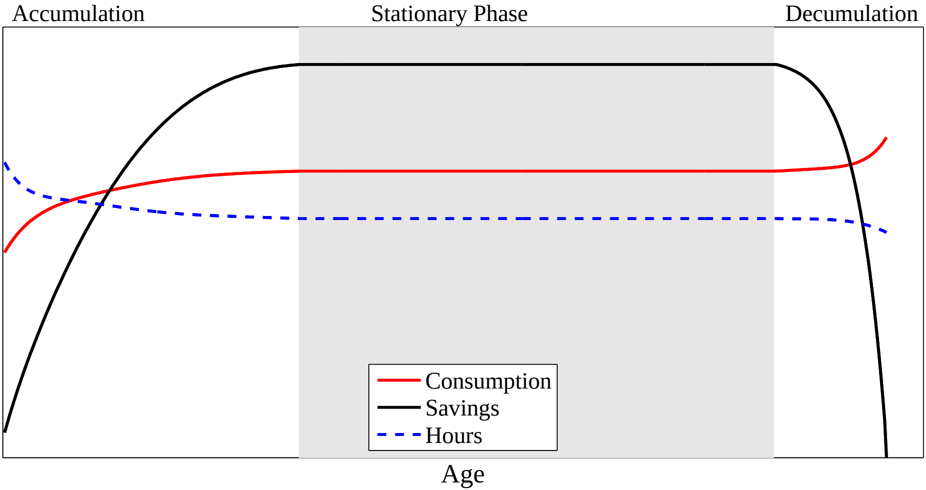
- Infinitely lived: only stationary
- Life cycle: three phases

Accumulation Phase



- Accumulating assets
- Labor income more important

Stationary Phase



- May not exist (shorter) in life cycle model
- Only phase in infinitely lived

Effect of Government Debt

Comparative Static: Holding less debt

- Less crowd-out $\rightarrow$ more productive capital
  - Higher wage, $w = (1-\alpha)(K/L)^{\alpha}$
  - Lower interest rate $r = \alpha(K/L)^{\alpha-1} - \delta$
- During *accumulation phase*:
  - Labor earnings is majority of income
  - Higher wage increases income
  - Life cycle only
- During *stationary phase*:
  - Lower interest rate decreases interest income
  - Accumulate fewer total assets (less liquid)
  - Less emphasis in life cycle model

Computational Experiment

Choose B to maximize social welfare function:

$$S(v, \lambda) \equiv \max_{B} E_0 v_0(a, \epsilon, x; B) \tag{1}$$

Utilitarian SWF: maximizing expected utility of newborn

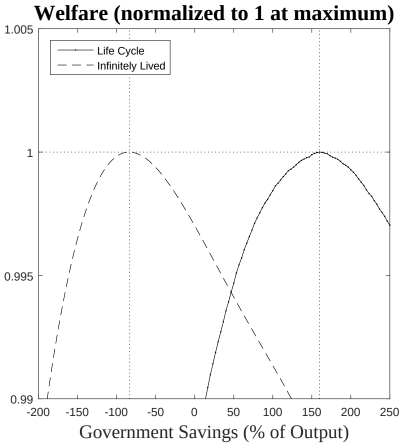
- Adjust taxes to clear budgets
  - τ_{ss} to satisfy Social Security budget
  - τ_0 to clear government general budget (G held fixed)

Experiment 1

Experiment 1: Optimal Policy

- Compute optimal policy in life cycle model
- Compute optimal policy in infinitely lived agent analogue

Experiment 1: Optimal Policy



Optimal Policy:

- Life cycle - savings = 160% of output
- Infinitely lived - debt = 87% of output

Welfare Decomposition

Experiment 2: Welfare Decomposition

- Consumption equivalence (CEV)
  - Optimal (160% savings) vs optimal from infinitely lived (87% debt)
- Decompose into:
  1. **Level effect:** difference in aggregate consumption
  2. **Insurance effect:** difference in volatility of consumption paths
  3. **Redistribution effect:** difference in cross-sectional spread
  4. **Labor effect:** difference in consumption-labor substitution

▸ Detail

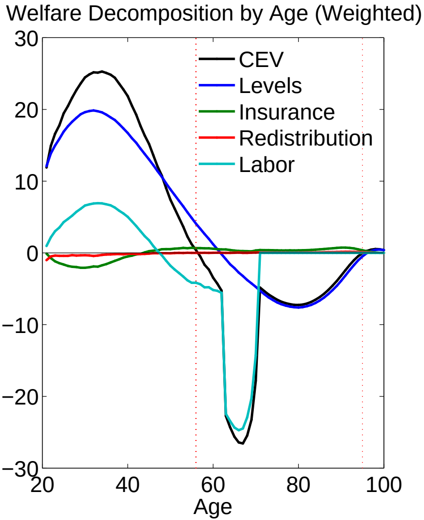
Welfare Decomposition

Welfare Decomposition, ex ante

CEV (% Change)	=	3.47 %
Levels Effect	=	5.62 %
Insurance Effect	=	-0.46 %
Redistribution Effect	=	0.14 %
Labor Disutility Effect	=	-1.72 %

- Optimal policy has strong positive Levels Effect
- Optimal policy somewhat mitigated by labor disutility

▸ Benchmark

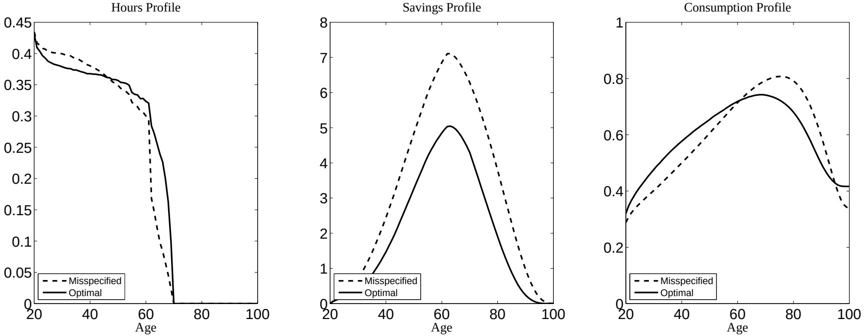


Level Effect:

- Higher wages → more consumption early
- Lower r → less consumption later, work longer

Benchmark

The Effect on Life Cycle Profiles



Optimal policy: More government savings, ↑ wage, ↓ r

Experiment 3

Decompose the Effect of Life Cycle Features:

- Sequentially remove life cycle features
  1. Age-varying aspects
  2. Demographics
  3. Endowment
- Recalibrate each model
- Calculate optimal policy

Models

	Bench.	Less Age-Spec. I	Less Mortality Risk II	Less Pop. Growth III	Extend Life IV	Eliminate Accum. V	Inf. Lived
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

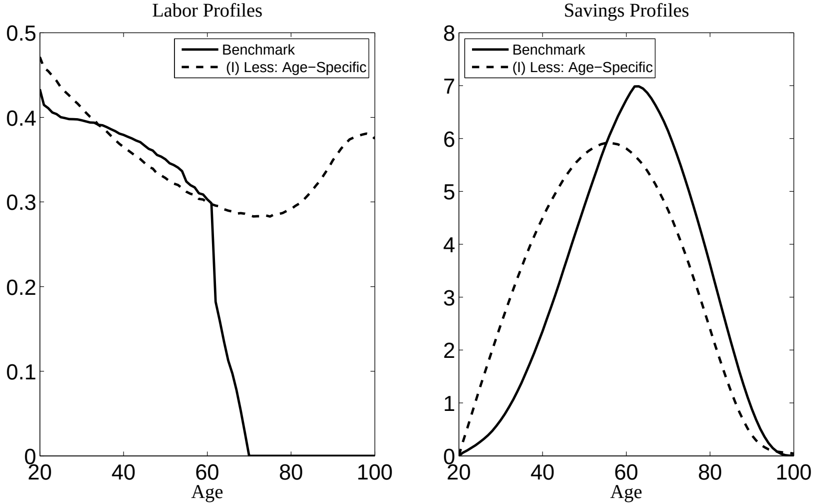
- Age-secific I
- Demographics II-IV
- Endowment V

Optimal Policy (Age-specific)

	Bench.	Less Age-Spec. I	Less Mortality Risk II	Less Pop. Growth III	Extend Life IV	Eliminate Accum. V	Inf. Lived
Optimal (% of GDP)	160%	**173%**	287%	307%	360%	-100%	-87%
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

$\Uparrow$ optimal savings because work throughout whole life

Life cycle Profiles



Competing effects on optimal policy

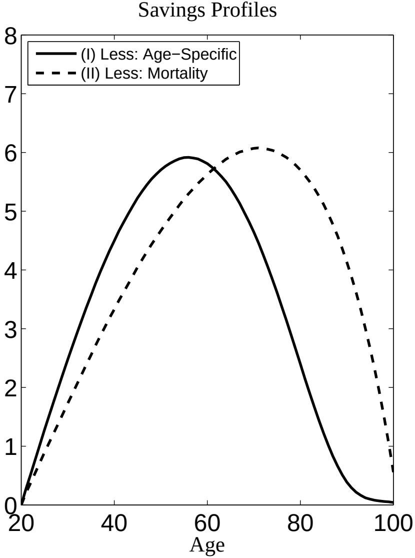
- Wage more important
- Less building time

Optimal Policy (Demographics II)

	Bench.	Less Age-Spec. I	Less Mortality Risk II	Less Pop. Growth III	Extend Life IV	Eliminate Accum. V	Inf. Lived
Optimal (% of GDP)	160%	173%	**287%**	307%	360%	-100%	-87%
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

$\Uparrow$ optimal savings because agents live to older age

Savings Profiles



→ Removing mortality lengthens accumulation phase

Optimal Policy (Demographics III)

	Bench.	**Less Age-Spec. I**	**Less Mortality Risk II**	**Less Pop. Growth III**	**Extend Life IV**	**Eliminate Accum. V**	**Inf. Lived**
Optimal (% of GDP)	160%	173%	287%	**307%**	360%	-100%	-87%
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

⇑ optimal savings: more old agents affects aggregate dynamics

Increased Population of Old

Elasticity of Private Savings wrt Government Savings

Model II	Model III
-0.923	-0.900

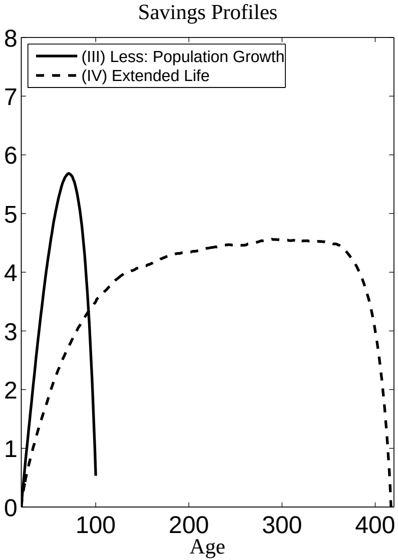
- Young are more responsive to interest rates changes
- Model III compared to II:
  - Fewer young agents
  - Government savings crowds out less private savings
  - Public saving is more productive
  - Government saves more

Optimal Policy (Demographics IV)

	Bench.	**Less Age-Spec. I**	**Less Mortality Risk II**	**Less Pop. Growth III**	**Extend Life IV**	**Eliminate Accum. V**	**Inf. Lived**
Optimal (% of GDP)	160%	173%	287%	307%	**360%**	-100%	-87%
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

$\Uparrow$ optimal savings: extend building period

Savings Profiles



→ Lengthens accumulation phase

Optimal Policy (Endowment)

	Bench.	Less Age-Spec. I	Less Mortality Risk II	Less Pop. Growth III	Extend Life IV	Eliminate Accum. V	Inf. Lived
Optimal (% of GDP)	160%	173%	287%	307%	360%	**-100%**	-87%
Retirement	Yes	No	No	No	No	No	No
Soc. Sec	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	Infinite
Save Endow.	0	0	0	0	0	Avg. IV	Dist.

- Eliminate building phase
- Optimal to hold debt

Takeaways

Why savings optimal in life cycle and debt in infinitely lived?

- In infinitely lived no accumulation phase
  - Link between stationary phase (endowment) and gov't savings/debt
  - Less gov't savings increases agents liquidity
- In life cycle agents experience an accumulation phase
  - More public savings increases wage
  - Particularly helpful during accumulation phase
  - Liquidity not affected until stationary phase

Experiments 4 & 5

(4) Interactions With Government Transfers

- Remove UI and solve for optimal
- Remove Social Security and solve for optimal
- Recalibrate each model
- Very small effect on optimal debt

(5) Interaction With Borrowing Constraint

- Allow for individual borrowing, ad hoc constraint
- Optimal public savings **increases** from 160% to 220%
- Precautionary savings less important when borrowing allowed

Experiment 6

Social Welfare Criteria

- We use ex ante Utilitarian social welfare function
  - Equivalent to welfare weight of 1 for newborn and 0 for others
- What if put different weight on cohorts?

Welfare weights

Allow for welfare weights on each generation $\{\alpha_j\}_{j=20}^{J}$:

$$\sum_{j=20}^{J} \alpha_j E_0[v_j(a_j, \epsilon_j, x_j)] = \sum_{j=20}^{J} \left(\sum_{t=20}^{j} \alpha_t \beta^{j-t} \mu_j \right) E_j[U_j(c_j, h_j, J_j)]$$

- We assumed $\alpha_{j=20} = 1$ and $\alpha = 0$ for other j

Illustrative example

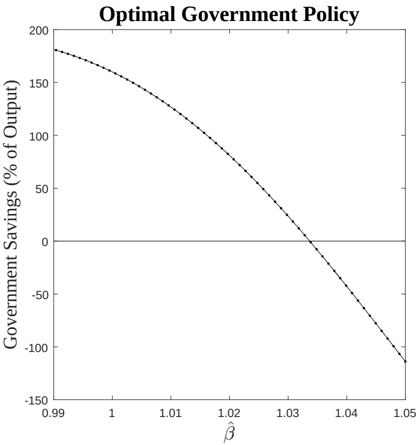
What is relationship between cohorts' weights and optimal policy?

Assuming $\hat{\beta}^j \mu_j \propto \sum_{t=20}^{j} \alpha_t \beta^{j-t} \mu_j$ can rewrite:

$$S_{\hat{\beta}}(v, \lambda) = \max_{B} \sum_{j=20}^{J} \hat{\beta}^j \mu_j E_j \Big[U_j \big(c_j, h_j, J_j; v_j(\cdot\ ; B) \big) \mid \lambda_j(\cdot\ ; B) \Big]$$

- Allows us to reweight each age's stream
- Demonstrates effect of different weights
- Larger $\hat{\beta}$ more weight on older generations

Effect of Cohort Weights



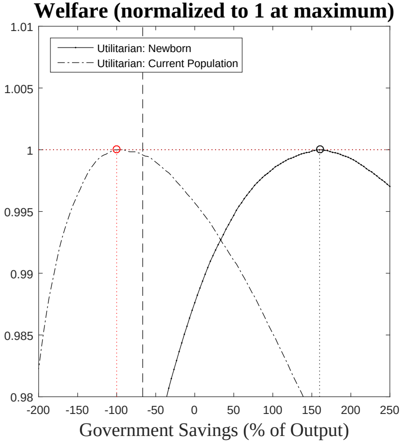
- $\Uparrow$ weights on older less savings (more debt) optimal
- Putting more weight on ages after building phase

Alternative Criteria

- SWF=total expected future utility from population
- $\alpha_j = 1 \forall\ j$

$$\sum_{j=20}^{J} \alpha_j E_0[v_j(a_j, \epsilon_j, x_j)]$$

Equally Weight Population



- Examine population average expected future utility
- Optimal debt is 100% of GDP

Conclusion

- Optimal debt policy is different in life cycle model
- Instead holding debt optimal for government to save
  - Facilitates accumulation phase
  - Stationary phase less important
- Large welfare consequences to ignoring life cycle model
  - Overall conclusion not sensitive to gov't transfers or agents allowed some borrowing

For optimal debt assuming infinitely lived for tractability has large economic consequences

Thank you

Optimal Policy (With Endowment Shock)

	Bench.	Less Age-Spec. I	Less Mortality Risk II	Less Pop. Growth III	Extend Life IV	Savings Endow. V	Hetero. Savings Endow. VI
Optimal (% of GDP)	160%	**173%**	287%	307%	360%	233%	273%
Soc. Sec	Yes	No	No	No	No	No	No
Retirement	Yes	No	No	No	No	No	No
Age H.C.	Yes	No	No	No	No	No	No
Age Unemp	Yes	No	No	No	No	No	No
Mort. Risk	Yes	Yes	No	No	No	No	No
Pop. Growth	Yes	Yes	Yes	No	No	No	No
Life Length	81	81	81	81	400	400	400
Endowment							
Save Endow.	0	0	0	0	0	Avg. IV	Dist.
Idio. Shock	Avg.	Avg.	Avg.	Avg.	Avg.	Avg.	Hetero

Removing age-specific: competing effects

- Exposed more periods to idiosyncratic shock
- No need to accumulate for retirement

Social Security

Benefit Formula: $b^{ss} = [\text{Replacement Rate}] \text{ x } [\text{Past Earnings}(x)]$

(1) Past earnings: x

$$x' = \begin{cases} \frac{y+(j-1)x}{j} & \text{if} \quad j \leq 35, \\ \max\{x, \frac{y+(t-j)x}{j}\} & \text{if} \quad 35 < j < J_{ret}, \\ x & \text{if} \quad j \geq J_{ret}, \end{cases}$$

(2) Replacement rate (piecewise linear)

$$\begin{cases} \tau_{r1} & \text{for} \quad 0 \leq x_R < b_1 \\ \tau_{r2} & \text{for} \quad b_1 \leq x_R < b_2 \\ \tau_{r3} & \text{for} \quad b_2 \leq x_R < b_3 \\ 0 & \text{for} \quad b_3 \leq x_R, \end{cases}$$

(3) Retirement Age Credits/Deductions (b^{ss} adjusted s.t.):

- 64-66: 6.7% reduction per year
- 62-63: 5% reduction per year
- 67-70: 8% increase per year

Back

Dynamic Programming: Worker

$$v_j(a,\epsilon,x) = \max_{c,a',h} \quad \left[u(c,h)\right] + \beta s_j \sum_{\epsilon'} \pi_j(\epsilon'|\epsilon) v_{j+1}(a',\epsilon',x')$$

$$\text{s.t.} \quad \begin{aligned} c + a' &\leq we(\epsilon)h(1-\bar{h}) + (1+r)(a+Tr) - T(h,a,\epsilon) + b_{ui}(we)\bar{h} \\ a' &\geq 0 \\ \varepsilon &\equiv (\theta_j, \alpha_i, \nu_{ij}, \epsilon_{ij}, \bar{h}_{ij}) \end{aligned}$$

Dynamic Programming: Could Retire

Agents could retire ($j \in [\underline{J}_{ret}, \overline{J}_{ret}]$) but have not:

$$v_j(a,\epsilon,x) = \max_{c,a',h,\mathbb{1}(j=J_{ret})} \left[u(c,h)\right] +$$

$$\beta s_j \sum_{\epsilon'} \pi_j(\epsilon'|\epsilon)(\mathbb{1}(j<J_{ret})v_{j+1}(a',\epsilon',x') + (1-\mathbb{1}(j<J_{ret}))v^{ret}_{j+1}(a',x'))$$

s.t.

$$\begin{aligned} c + a' &\leq (1+r)(a+Tr) - T(a) + b_{ss}(x) && \textbf{if } \mathbf{j} \geq \underline{\mathbf{J}}_{\mathbf{ret}} \\ c + a' &\leq we(\epsilon)h(1-\bar{h}) + (1+r)(a+Tr) - T(h,a,\epsilon) + b_{ui}(we)\bar{h} && \textbf{else} \\ a' &\geq 0 \end{aligned}$$

Dynamic Programming: Retired

$$v_j^{ret}(a,x) = \max_{c,a'} \quad u(c) + \beta s_j v_{j+1}^{ret}(a',x)$$

$$\text{s.t.} \quad \begin{array}{rcl} c + a' & \leq & (1+r)(a+Tr) - T(a) + b_{ss}(x) \\ a' & \geq & 0 \end{array}$$

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Social Security

Parameter	Value	Source
κ_{1a} Year 1 - 3	6.7%	U.S. SS Program
κ_{1b} Year 4 & 5	5%	U.S. SS Program
κ_2	8%	U.S. SS Program
b_1	.21 x Avg Earnings	Huggett and Parra (2010)
b_2	1.29 x Avg Earnings	Huggett and Parra (2010)
b_3	2.42 x Avg Earnings	Huggett and Parra (2010)
τ_{r1}	90%	U.S. SS Program
τ_{r2}	32%	U.S. SS Program
τ_{r3}	15%	U.S. SS Program
τ_{ss}	10.3%	Mrkt Clearing
j_{nr}	66	Data
$\underline{J}_{ret}$	62	U.S. SS Program
$\overline{J}_{ret}$	70	U.S. SS Program

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Decomposition Details

Define Welfare:

$$S = S_c + S_h \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} s_j u(c_j) \right] d\lambda_1 + \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} s_j \varphi(h_j) \right] d\lambda_1$$

CEV Decomposition:

$$(1+\Delta_{CEV}) = (1+\Delta_{level}) \quad (1+\Delta_{insure}) \quad (1+\Delta_{distr}) \quad (1+\Delta_{hours})$$

$$\left(\frac{S^{opt} - S_h}{S_c} \right)^{\frac{1}{1-\sigma}} = \frac{C^{opt}}{C} \quad \frac{\bar{C}^{opt}/\bar{C}}{C^{opt}/C} \quad \frac{(S_c^{opt}/S_c)^{\frac{1}{1-\sigma}}}{\bar{C}^{opt}/\bar{C}} \quad \left(\frac{S^{opt} - S_h}{S_c^{opt}} \right)^{\frac{1}{1-\sigma}}$$

where:

- **Consumption Equivalent:** $(1+\Delta_{CEV})^{1-\sigma} S_c + S_h = S^{opt}$
- **Labor Substitution Effect:** $(1+\Delta_{hours})^{1-\sigma} S_c^{opt} = S_c^{opt} + (S_h^{opt} - S_h)$
- **Certainty Equivalent:** $\bar{C} = \sum_j \mu_j \int \bar{c}(a, \varepsilon, x) d\lambda_1$

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Welfare Decomposition

Welfare Decomposition, ex ante

CEV (% Change)	=	2.33 %
Levels Effect	=	4.36 %
Insurance Effect	=	-0.47 %
Redistribution Effect	=	0.11 %
Labor Disutility Effect	=	-1.59 %

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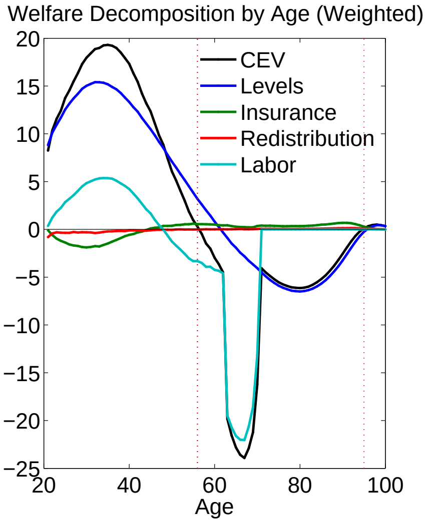

Level Effect:

- Higher wages → more consumption early
- Lower r → less savings and consumption later

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