A Bird in the Hand is Worth Two in the Grave

Risk Aversion and Life-Cycle Savings

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Motivation

- Question: How does risk aversion impact life-cycle saving and portfolio choice?
- First answer: Depends on the risks considered
  - Labor income risk: $\nearrow$
  - Financial return risk: depends on IES
  - Mortality risk: $\searrow$
- With multiple risks: ambiguous
  $\Rightarrow$ Need quantitative analysis
- Focus on risk aversion + income, financial and mortality risks

Modelling approach

- Kreps-Porteus recursive preferences:
  - Epstein-Zin (1989)
  - Risk-sensitive: Hansen and Sargent (1995) in their work on robustness
  - Allow us to vary risk aversion without changing IES
- Quantitative life-cycle model with incomplete markets
- Partial equilibrium analysis
- Calibrated to U.S. data
- ... and in particular to value of a statistical life: Viscusi and Aldy (2003) for a review

Main results

- Higher risk aversion
  - Decreases life-cycle savings
  - Decreases participation in the stock market
  - Decreases the conditional share in stock
- With mortality risk, give up homotheticity of Epstein-Zin

  $\rightarrow$ intuition: we cannot "scale" death.
- Risk-sensitive and Epstein-Zin qualitatively similar and quantitatively close

Literature

Risk aversion	... *increases savings*	... *decreases savings*
Income risk	e.g., BCL	
Investment risk	Kihlstrom and Mirman (1974) and BCL if IES< 1	Kihlstrom and Mirman (1974) and BCL if IES> 1
Mortality risk	**HPSA if IES** < 1	Bommier (2006, 2013), BCL, Drouhin (2015), HPSA if IES> 1
All three risks	**Gomes and Michaelides (2005, 2008)**,... ▸ more	This paper

- BCL: Bommier, Chassagnon, and LeGrand (2012)
- HPSA: Hugonnier, Pelgrin, and Saint-Amour (2012)

Relationship between risk aversion and savings (1/2)

Simple framework (see Bommier, Chassagnon, LeGrand, 2012)

- Consumption-saving problem with 2 periods: 0 and 1; 2 states in period 1: G and B
- Saving s_B (resp. s_G) if B (resp. G) for sure
- Saving s^* if uncertain future (B or G)

Role of risk aversion:

- $s^* =$ convex combination of s_B and s_G
- Weight on s_B increases with risk aversion

$\Rightarrow$ the more risk averse, the more important bad state realizations

Relationship between risk aversion and savings (2/2)

- Income risk
  - Bad state = low income
  - $s_B > s_G$
  - Risk aversion *increases* savings.
- Mortality risk
  - Bad state = living for one period only
  - saving = bet on living 2 periods
  - $s_B < s_G$
  - Risk aversion *decreases* savings.
- Investment risk: depends on IES ▸ Show

$\Rightarrow$ All three risks, ambiguous relationship $\rightarrow$ quantitative exercise

Back of the envelope calculation (1/2)

Magnitudes of income vs. mortality risks?

- Income risk from a lifecycle perspective
  - Lifecycle labor income = per period labor incomes discounted to age 20 at the risk-free rate

  $\Rightarrow$ With our calibration, average lifetime labor income of $\$1.1$ million with a standard deviation of $\$0.8$ million

  $\Rightarrow$ Income risk $\approx \$0.8$ million

Back of the envelope calculation (2/2)

Magnitudes of income vs. mortality risks?

- Mortality risk.
  - Life expectancy at age $20 = 58.5$ years with a standard deviation of 14.5 years.

  $\Rightarrow$ Mortality risk ≈ 14.5 years.
  - Using the value of a statistical life, one year alive $\approx \$\,186$ k (VSL$=\$\,6.5$m at 45).

  $\Rightarrow$ Mortality risk $\approx \$\,2.7$ millions.

$\Rightarrow$ Back of the envelope calculation: Mortality risk $\gg$ income risk

$\Rightarrow$ Impact of risk aversion should be dominated by mortality risk

Endowments

- Working age $t = 1$, retirement age $t = T_R$, max age $t = T_M$
- Mortality risk: survival probabilities $(p_{t+1|t})_t$
- Labor income $(1 \leq t < T_R)$

$$
\begin{aligned}
y_t^L =& y_0 \exp(\mu_t + \pi_t + \varepsilon_t^y) \\
\pi_t =& \rho \pi_{t-1} + \varepsilon_t^\pi \\
\varepsilon_t^y \overset{iid}{\sim}& \mathcal{N}\left(0, \sigma_y^2\right), \quad \varepsilon_t^\pi \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\pi^2\right)
\end{aligned}
$$

- Social security pension income $(T_R \leq t \leq T_M)$, y^R

Asset markets

- Bond: risk-free gross return R^f
- Stock: risky gross return

  $$\ln R_t^s = \ln\left(R^f + \nu\right) + \varepsilon_t^R, \quad \varepsilon_t^R \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_R^2\right)$$

- ε_t^R correlated with both labor income shocks with $\kappa_{R,y}$ and $\kappa_{R,\pi}$
- No short-selling
- Stock-market participation cost, $F \geq 0$, paid once in life

Choices and constraints

- Choices $\{c_t, s_t, b_t, \eta_t\}$
- Constraints

$$c_t + b_t + s_t + F1_{\eta_t=1}1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R^s_t s_{t-1},$$

$$y_t = \begin{cases} y^L_t & \text{if } t < t_R, \\ y^R & \text{else,} \end{cases}$$

$$s_t = 0 \text{ if } \eta_t = 0,$$

$$c_t > 0, \quad b_t \geq 0, \quad s_t \geq 0.$$

and bequests are $w_t = R^f b_{t-1} + R^s_t s_{t-1}$.

Preferences (1/2)

- Felicity (alive) from consumption: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$
- Felicity (dead) from bequests:

$$v(w) = -v_0 + \frac{\theta}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]$$

- Kreps-Porteus recursive preferences

▸ General recursion

$$U_t^A = (1-\beta)u(c_t)$$
$$+\beta\Phi^{-1}\left(p_{t+1|t}\mathbb{E}_t\left[\Phi\left(U_{t+1}^A\right)\right] + (1-p_{t+1|t})\mathbb{E}_t\left[\Phi\left(U_{t+1}^D\right)\right]\right)$$
$$U_t^D = (1-\beta)\,v(w_t) + \beta v\,(0)$$

Preferences (2/2)

Why is v_0 important?

- difference between being alive consuming 1 unit and being dead without leaving bequest
- strongly connected to the value of life
- cannot be set to zero without a loss of generality (and a strong constraint on value of life)
- does not "go away" with non-additive preferences
- (does not affect choices in case of additive preferences)

$$U_t^A = (1-\beta)u(c_t) + \beta p_{t+1|t}\mathbb{E}_t\left[U_{t+1}^A\right] - \beta(1-p_{t+1|t})v_0$$
$$+ (1-p_{t+1|t})\beta\mathbb{E}_t[(1-\beta)\frac{\theta}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]]$$

Epstein-Zin and risk-sensitive preferences (1/2)

- Both Kreps-Porteus
- Epstein-Zin preferences (EZ)

$$\Phi(u) = \frac{1}{1-\gamma}(1+(1-\sigma)u)^{\frac{1-\gamma}{1-\sigma}} - \frac{1}{1-\gamma}, \qquad \text{if } \gamma, \sigma \neq 1$$

- Risk-sensitive preferences (RS)

$$\Phi(u) = -\frac{1}{k}\left(\exp(-ku) - 1\right) \qquad \text{if } k \neq 0$$

- Limit cases ($k = 0$, $\gamma = 1$, $\sigma = 1$) by continuity
- Coincide if
  - $\gamma = \sigma$ and $k = 0$ $\Rightarrow$ additively separable case
  - $\sigma = 1$

Epstein-Zin and risk-sensitive preferences (2/2)

- EZ: homothetic but not monotone (with respect to FSD)
- RS: non-homothetic but monotone.

⇒ Not monotone, what does that mean? ▸ Numerical Example

- RS: the only KP preferences that are monotone and disentangle risk aversion from IES
  - Working paper by Bommier and LeGrand (2014), work in progress by Bommier, Kochov, and LeGrand (2016)
- In our setting:
  - Homotheticity has to be given up, because of value of life.
  - Non-monotonicity little impact

Value of a statistical life

- Standard definition (see Johansson 2002): Marginal rate of substitution between survival rate and consumption

$$VSL_t = \frac{\frac{\partial U_t^A}{\partial p_{t+1|t}}}{\frac{\partial U_t^A}{\partial c_t}}$$

  $\Rightarrow$ how much consumption to give up for increasing the likelihood to live one more year

- Viscusi and Aldy (2003) for empirical estimates

Computation

- Reformulate model
  - Cash-at-hand, $x_t = R^f b_{t-1} + R^s_t s_{t-1} + y_t$
  - Total savings, a_t, and share in stock $\alpha \in [0, 1]$
- Persistent productivity, π_t: continuous state variable
- State space (x_t, π_t, η_t, t)
- Not differentiable
- Standard VFI very long $\rightarrow$ calibration hardly feasible.

$\Rightarrow$ Refinement of VFI

$\Rightarrow$ Use 3D cubic B-spline to interpolate expected continuation value

- Calibration: consider 3 agents: add, EZ, RS

Calibration of preferences

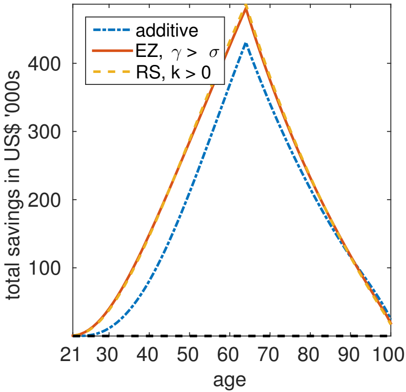
Parameter	Value	Source/ counterpart/ target
Inverse IES, σ	2.0	
Exog. endowment, $\hat{w}$	1.5	
Discount factor, β	*0.96*	$Assets_{45}^{add}$ = US$ 100'000
Life-death gap, v_0	*30.0*	VSL_{45}^{add} = US$ 6.5m
Bequest motive, θ	20.0	$Bequests_{85}^{add}$
Risk aversion, EZ, γ	3.0	
Risk aversion, RS, k	*0.08*	$Assets_{45}^{RS} = Assets_{45}^{EZ}$

Parameterization of endowments and asset markets

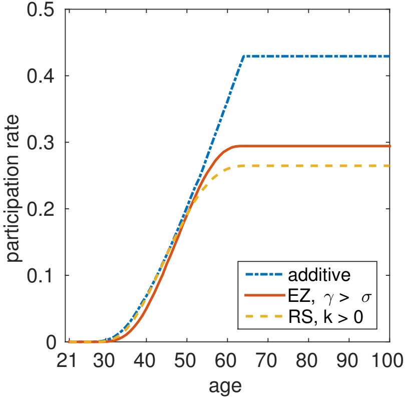
Parameter	Value	Source/ counterpart/ target
Working age, retirement age, maximum age		21, 65, 100
Survival rates, $p_{t+1\|t}$	$\{p_{t+1\|t}\}_1^T$	U.S. mortality 2007, HMD
Age productivity, μ_t	$\{\mu_t\}_1^T$	Earnings profiles 2007, PSID
Average wage, y_0	$21\,756$ USD	Net compensation 2007, SSA
Pensions, y_R	0.3	Replacement rate, preliminary
Autocorrelation, ρ	0.95	Storesletten, et al. (2004)
Var. persistent shocks, σ_π^2	0.03	Storesletten, et al. (2004)
Correlation with stock, $\kappa_{R,\pi}$	0.15	Gomes and Michaelides (2005)
Var. transitory shocks, σ_y^2	0.00	Preliminary
Inheritance, w_0	0.0	Preliminary
Gross risk-free return, R^f	1.01	Bond return, Shiller data
Equity premium, ν	0.02	Preliminary
Stock volatility, σ_R	0.18	Shiller data
Participation cost, F	0.2	Preliminary

Lifecycle profiles *without* mortality risk

- Only labor income and asset return risks ▸ Re-calibration



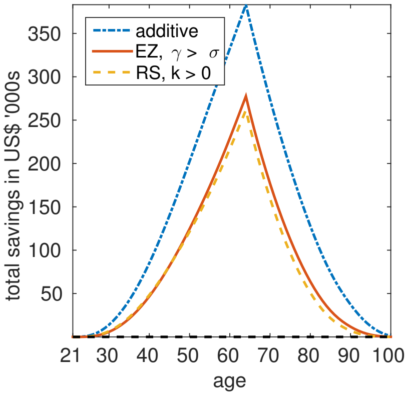
Total savings



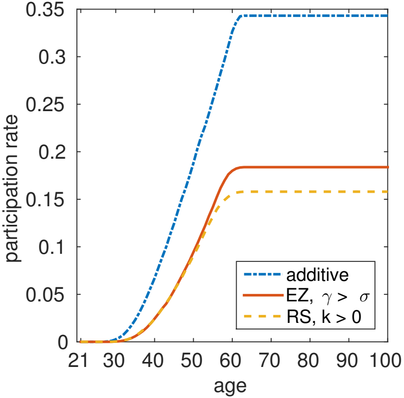
Stock market participation

Lifecycle profiles *with* mortality risk (1/3)

- Baseline with all risks



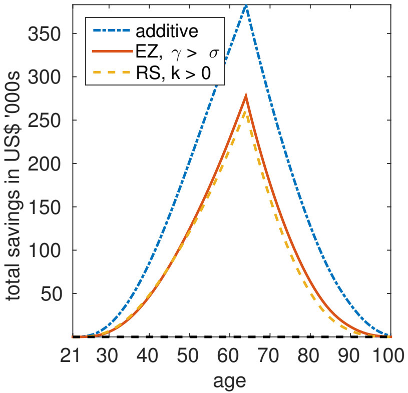
Total savings



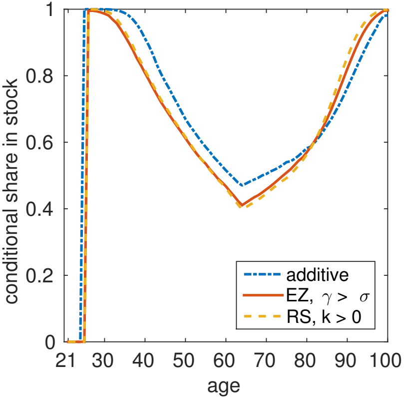
Stock market participation

Lifecycle profiles *with* mortality risk (2/3)

- Baseline with all risks



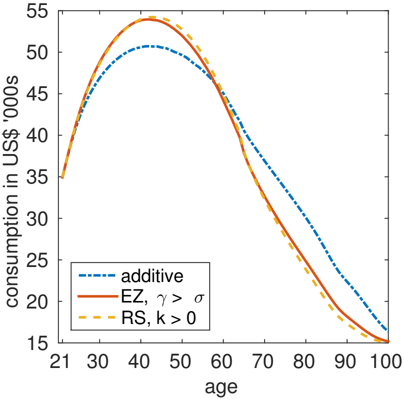
Total savings



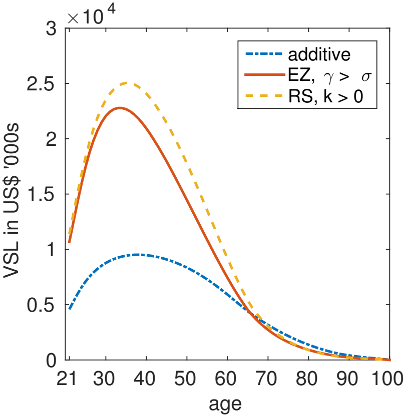
Conditional Share in Stock

Lifecycle profiles *with* Mortality risk (3/3)

- Baseline with all risks



Consumption



Value of a Statistical Life

Typical Epstein-Zin specification

- Many different variants, e.g. ▸ GM 2005. See ▸ Literature Overview.

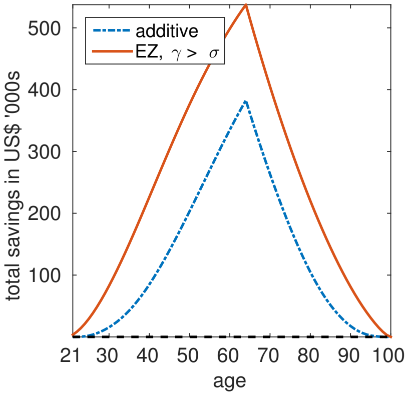
$$\Omega_t = \left((1-\beta) c_t^{1-\sigma} + \beta \left(\mathbb{E}_t \left[p_{t+1|t} \Omega_{t+1}^{1-\gamma} + (1-p_{t+1|t}) \theta w_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}$$

- Bequests explicit and homothetic,
- ... but VSL not necessarily > 0
- In our framework, set $v_0 = -\theta \frac{\hat{w}^{1-\sigma}}{1-\sigma}$ (and $\hat{w} = 0.0$)
- In addition, if no bequests: $\theta = 0$

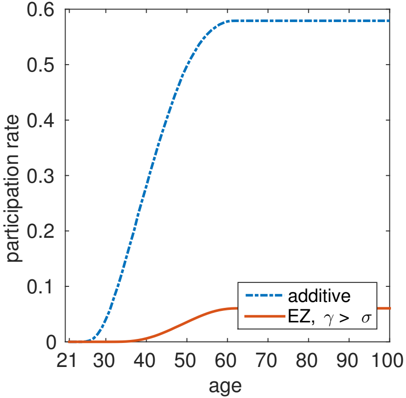
  If $\gamma > 1$: $\frac{\partial \Omega_t}{\partial p_{t+1|t}} < 0 \;\Rightarrow\; VSL < 0$. The term $+(1-p_{t+1|t})(\infty)^{1-\gamma}$ can be added in the recursion, where ∞ =utility of death.

Typical Epstein-Zin specification, $\theta = 0$ (1/2)

- Like baseline with all risks ▸ Recalibration

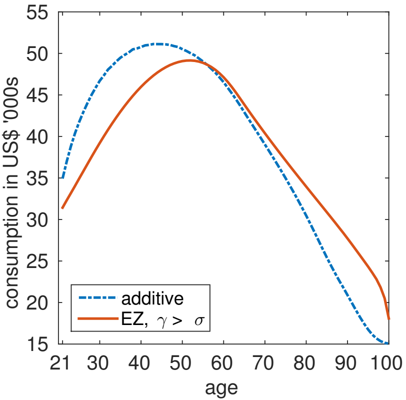


Total savings

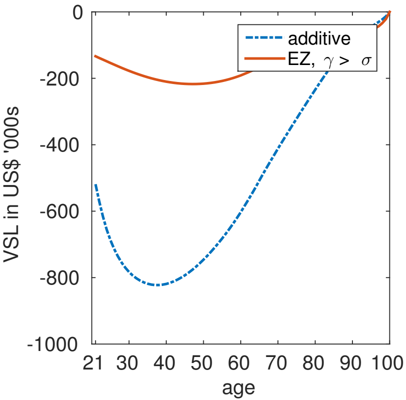


Stock market participation

Typical Epstein-Zin specification, $\theta = 0$ (2/2)



Consumption



Value of a Statistical Life

Conclusion

- Mortality = main risk in life
  - importance of value of life
  - saving = risk-taking behavior
  - Higher risk aversion decreases lifecycle savings
- EZ vs. RS
  - EZ can accommodate positive VSL, but lose homotheticity
  - Typical EZ implementation may yield negative VSL
- Observed low levels of saving may be rational and explained by higher risk-aversion. Alternative explanation to time-inconsistency (e.g., Caliendo and Findley, 2013)
- In paper, also explain the different results of Hugonnier, Pelgrin, and Saint-Amour (2012)

Thank you !

Appendix Table of Contents

Literature

- Epstein-Zin preferences:
  - With bequests: Gomes and Michaelides (2005), Inkman, Lopez, and Michaelides (2011), Horneff, Maurer, and Stamos (2008a, 2008b), Chai, Horneff, Maurer, and Mitchell (2011)
  - Without bequests: Gomes and Michaelides (2008), Gomes, Michaelides, and Polkovnichenko (2009), Fehr and Habermann (2008), Fehr, Habermann, and Kindermann (2008) Fehr, Kallweit, and Kindermann (2013)
- Risk aversion and savings:

  Bommier (2006, 2013), Bommier, Chassagnon, LeGrand (2012), Bhamra and Uppal (2006)
- Value of a statistical life:

  Kaplow (2005), Viscusi and Aldy (2003), Bommier and Villeneuve (2010), Cordoba and Ripoll (2013)

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Relationship Between Risk Aversion and Savings (3/3)

- Investment risk
  - Bad state = low rate of return
  - If IES< 1
    - Income effect dominates
    - $s_B > s_G$
    - Risk aversion *increases* savings
  - Else if IES> 1
    - Substitution effect dominates
    - $s_B < s_G$
    - Risk aversion *decreases* savings

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General Kreps-Porteus Recursion

- Recursion

$$U_t = (1-\beta)u_t + \beta\Phi^{-1}\left(\mathbb{E}_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi\left(U_{t+1}\right)\right]\right),$$

$$\text{with } u_t = \begin{cases} u(c_t) & \text{if alive at } t \\ v(w_t) & \text{if dead at } t \end{cases}$$

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Numerical Example of Non-Monotonic Preferences

- Consider EZ utility: $V(c_0, \tilde{c}_1) = c_0^{\frac{1}{2}} + (\mathbb{E}[\tilde{c}_1^{-\frac{1}{2}}])^{-1}$.
- Lotteries $i = \ell_1, \ell_2$ paying off (c_0^i, c_d^i) or (c_0^i, c_u^i) $(50\%–50\%)$:

Lottery	c_0^i	c_d^i	c_u^i	$V(c_0^i, c_d^i)$	$V(c_0^i, c_u^i)$
$i = \ell_1$	4	1	7	9.00	21.58
$i = \ell_2$	2	2.5	9	8.97	19.49

$\Rightarrow$ ℓ_1 always pays off more than ℓ_2.

- BUT, ex ante, $V(c_0^{\ell_1}, \tilde{c}_1^{\ell_1}) = 11.91 < 12.15 = V(c_0^{\ell_2}, \tilde{c}_1^{\ell_2})$!

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Implications for consumption-saving problems

- Two states B, G, two periods, constant rate R
- $y_B < y_G$ and $s_B > s_G$
- With monotone preferences: $s_B > s_m^* > s_G$
- With EZ preferences, it may be the case that: $s_{EZ}^* > s_B > s_G$, while saving s_B offers a greater lifetime utility in both states B and G.

Re-calibration Without Mortality

Parameter	Value	Source/ counterpart/ target
Inverse IES, σ	2.0	
Exog. endowment, $\hat{w}$	1.5	
Discount factor, β	*0.96* $\rightarrow$ *0.95*	$Assets_{45}^{add}$ = US$ 100'000
Life-death gap, v_0	*30.0* $\rightarrow$ *30.3*	VSL_{45}^{add} = US$ 6.5m
Bequest motive, θ	20.0	$Bequests_{85}^{add} = ?$
Risk aversion, EZ, γ	$\mathbf{3.0 \rightarrow 7.0}$	
Risk aversion, RS, k	$\mathbf{0.08 \rightarrow 0.58}$	$Assets_{45}^{RS} = Assets_{45}^{EZ}$

▸ Go Back

EZ in Gomes and Michaelides 2005

$$V_t = \left((1 - \beta p_t) c_t^{1-\frac{1}{\varepsilon}} + \beta E_t \left(p_t V_{t+1}^{1-\rho} + (1 - p_t) b \frac{(X_{t+1}/b)^{1-\rho}}{1 - \rho} \right)^{\frac{1-\frac{1}{\varepsilon}}{1-\rho}} \right)^{\frac{1}{1-\frac{1}{\varepsilon}}}$$

- Derivative ambiguous if $\rho > 1$ and $\varepsilon < 1$

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Re-calibration for ‘typical’ EZ Specification

Parameter	Value	Source/ counterpart/ target
Inverse IES, σ	2.0	
Exog. endowment, $\hat{w}$	1.5	
Discount factor, β	0.96	$Assets_{45}^{add}$ = US$ 100'000
Life-death gap, v_0	$\mathbf{30.0 \to 0.0}$	***not targeted***
Bequest motive, θ	$\mathbf{20.0 \to 0.0}$	***exogenous***
Risk aversion, EZ, γ	$\mathbf{3.0 \to 7.0}$	
Risk aversion, RS, k	$\mathbf{0.08 \to 0.71}$	$Assets_{45}^{RS} = Assets_{45}^{EZ}$

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