A Bird in the Hand is Worth Two in the Grave

Risk Aversion and Life-Cycle Savings[*]

Antoine Bommier   Daniel Harenberg   François Le Grand[†]

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Abstract

We explore the role of risk aversion on life-cycle savings and portfolio choices. We consider a setup where agents are endowed with recursive preferences, enabling us to disentangle risk aversion and intertemporal elasticity of substitution. Agents face mortality, income, and investment risks, and assign a positive value to being alive. In this framework, the overall impact of risk aversion is theoretically ambiguous, as shown by Bommier, Chassagnon and LeGrand (2012). We carefully calibrate this life-cycle model, with particular attention to the value of a statistical life, and find clear-cut results: greater risk aversion implies smaller (not larger!) savings and safer investment strategies. The impact of mortality risk therefore dominates the other ones.

Keywords: recursive utility, life-cycle model, risk aversion, saving choices, portfolio choices.

JEL codes: .

1 Introduction

There are many sources of risk in life, including mortality, health, income, and investment risks, which impact both future resources and the ability to derive satisfaction out of them. As it was already emphasized by Fisher (1930), a theory of savings has therefore to embed at theory of

behavior under risk. The economic literature provides however a rather blurred view on how risk aversion may influence life-cycle consumption-saving strategies. The first reason is that the literature has long focused on the additively separable model that makes it impossible to study the role of risk aversion in isolation. The second reason is that the few studies that use frameworks that allow to disentangle risk aversion from intertemporal elasticity of substitution end up providing mix messages. Gomes and Michaelides (2005, 2008) –who consider mortality, income and investment risks– explain that risk aversion should increase savings. Hugonnier, Pelgrin and Saint-Amour (2012) (henceforth, HPSA) include health risks in addition to the aforementioned risks, with source dependent risk aversion. They explain that mortality risk aversion increases or decreases savings depending on whether the intertemporal elasticity of substitution is larger or smaller than one. This contrasts with the papers of Bommier (2006, 2013) and Drouhin (2015), focusing on mortality risk exclusively and in which risk aversion is found to always increase time discounting and thus to decrease savings. Bommier, Chassagnon and LeGrand (2012) (henceforth, BCL) discuss how risk aversion impacts savings in simple two-period models where different sources of risk are separately considered. Simple dominance arguments lead to conclude that risk aversion has a positive impact in some cases (e.g., with income risk) and a negative impact in other cases (e.g., with mortality risks). The least that we can say is that these several articles fail to provide a clear and unified picture. In Table 1, we gather these contradictory findings about the role of risk aversion on savings.

The objective of this paper is to help clarifying this question: how does risk aversion impact saving and life-cycle financial strategies? This is a question of importance as it may hep understanding individual behaviors and, in particular, the low level of retirement savings that are empirically observed. In the main part of this paper, we address the question with models that make it possible to disentangle risk aversion, in particular with Epstein and Zin (1989) and risk-sensitive preferences. In line with Gomes and Michaelides, we consider mortality, income, and investment risks but leave health risk aside.[1]

When these different sources of risk are considered simultaneously, we know from the theoretical contribution of BCL that the overall impact of risk aversion will be the result of several opposing effects. The intuition underlying these opposing effects can be stated as follows. Risk aversion basically involves greater concern for bad state realizations. Thus, the impact of an increase in risk aversion on savings depends on whether an agent would save more or less if anticipating a bad state realization for sure. The effects are of different signs and magnitudes, depending on whether

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[1]Despite of being of natural relevance, we choose not to model health risk. One of the reasons is that health being much more difficult to quantify (compared to monetary variables or to survival), it is difficult to find precise estimates on the willingness-to-pay to improve health.

a bad state realization is death, a low income or a low financial rate of return. When the three risks about income, mortality and investment are simultaneously at play, there is therefore no hope to derive a general simple conclusion that would hold for all risk and preference parameters. The conclusion has to rely on a quantitative analysis, based on carefully calibrated life-cycle models. In particular, as mortality risk is taken into account, it is necessary to use models which predict a reasonable value of life.

Our findings are simple and interestingly contradict some "conventional wisdom": we indeed find that greater risk aversion leads to save less (and not to save more!) and to opt for safer investment strategies (with lower stock market participation and a larger share of savings invested in bonds). In fact, when using models that provide a value of life consistent with empirical estimates, it is found that mortality is the main source of uncertainty on lifetime utility. This simply reflects that an early death has a much more dramatic impact on welfare than a (permanent) shock on income risk, or a low return on savings. As a consequence, the effects discussed in Bommier (2006, 2013) and Drouhin (2015), which solely focus on mortality risk, dominate the others. Labor income risk does generate a prudence effect (with more savings) which is magnified by risk aversion, but this contributes quite little compared to the effect of mortality risk. The impact of asset return risks are also visible, but are also of lower magnitude compared to those of mortality risk.

In a second part of the paper we explain why our results differ from those of Gomes and Michaelides (2005, 2008) and HPSA. The explanation relates to two channels. First, Gomes and Michaelides (2005, 2008), as well as many others that use Epstein-Zin preferences with mortality risk, consider specifications which implicitly assume a negative value of life. This leads to inverse the impact of risk aversion. Indeed, if long life is considered as an adverse realization, greater risk aversion leads to put a larger weight on the utility derived in case of long life, and therefore leads to save more. The results of Gomes and Michaelides (2005, 2008) are then opposed to ours, but this is imputable to the negative value of life of the agents in their economy. HPSA assume a positive value of life, but in the case where the intertemporal elasticity of substitution is smaller than one, their model assumes that consumption and survival are substitute rather than complementary. In other words, the marginal utility of consumption at a given age decreases with the probability of being alive at that age, so that mortality makes agents more patient, instead of making them more impatient. When survival at a given age becomes almost impossible, marginal utility of consumption tends to infinity, which leads the agent to keep a lot of resources to increase consumption at this old age. This increase in patience due to mortality is magnified by risk aversion. The result is contrary to ours, but again related to an unintuitive aspect of preferences that are used.

The rest of the paper is organized as follows. We present our setup in Section 2. We specify

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	Risk aversion increases savings	Risk aversion decreases savings
Income risk only	BCL.	
Investment risk only	Kihlstrom and Mirman (1974) and BCL if IES< 1.	Kihlstrom and Mirman (1974) and BCL if IES > 1.
Mortality risk only	*HPSA if IES < 1.*	Bommier (2006, 2013), BCL, Drouhin (2015), HPSA if IES > 1.
Income, investment, and mortality risk	*Gomes and Michaelides (2005, 2008).*	This paper.

Note: Models in *italics* have results that are inconsistent with ours for reasons that are explained in the introduction and with further details in Section 6. All others provide a consistent view. BCL stands for Bommier, Chassagnon LeGrand (2012), HPSA for Hugonnier, Pelgrin and Saint-Amour (2012) and IES for "intertemporal elasticity of substitution". HPSA also consider cases where simultaneous risks are at play, using several source dependent coefficients of risk aversion. The corresponding results cannot be reported in the above table.

Table 1: Impact of risk aversion on savings

utility functions in Section 3. We describe our calibration in Section 4 and present the related results in Section 5. In Section 6, we discuss how our results compare to the literature and we conclude in Section 7.

2 The setup

We consider a partial equilibrium economy populated by an agent endowed with recursive preferences and facing several risks: a mortality risk, an income risk and an investment risk through risky financial returns. The agent may save through bonds and a risky asset (similar to a stock). We are interested in studying her lifecycle portfolio allocation and in particular the impact on saving choices of some preference feature, such as risk aversion and the value of life. Time is discrete and

indexed by $t = 0, 1, \ldots$ The period between two dates is one year. As it is standard, t refers to the agent's adult age and is equal to her age minus 20. The initial date $t = 0$ corresponds to age 20. There is a single consumption good, whose price serves as a numeraire. We now describe the setup, starting with risks faced by the agent.

Mortality risk. The agent faces a mortality risk, which is assumed to be independent of any other risk in the economy. While alive at date $t - 1$, the agent faces the probability $p_{t|t-1}$ to be still alive at date t. Thus, $1 - p_{t+1|t}$ denotes the probability of dying at the beginning of period t. The agent is alive at date 0, so that we have $p_{0|-1} = 1$. We denote by $m_{t|0}$ (resp. $p_{t|0}$) the probability of living exactly (resp. at least) until date t. These probabilities relate to each other as follows:

$$m_{t|0} = (1 - p_{t+1|t}) \prod_{k=1}^{t} p_{k|k-1} \quad \text{and} \quad m_{0|0} = 1 - p_{1|0},$$

$$p_{t|0} = \prod_{k=1}^{t} p_{k|k-1} \quad \text{and} \quad p_{0|-1} = 1.$$

The agent will die for sure at some age and there exists a maximal date T_m, such that the probability to live after T_m is 0: $p_{T_m+1|T_m} = 0$.

Labor income risk. The agent is supposed to exogenously retire at the age of 65, which corresponds to $T_R = 45$. In retirement, the agent gets a constant income y_R, which is therefore riskless from any date perspective. When active, the agent earns at any date $t < T_R$ a risky income y_t, which is affected by the combination of two shocks, a persistent one π_t and a transitory one u_t:

$$y_t = y_0 \exp \left(\mu_t + \pi_t + \varepsilon_t^y \right), \tag{1}$$

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t^\pi, \tag{2}$$

where the two independent processes $(\varepsilon_{t\,t}^\pi)_{t \geq 0}$ and $(\varepsilon_{t\,t}^y)_{\geq 0}$ are IID and normally distributed with mean 0 and respective variance σ_π^2 and σ_y^2. The quantity y_0 in (1) is the constant riskless component of income, while $(\mu_t)_{t \geq 0}$ is a deterministic process that contributes to fit the wage process to the data and in particular the humped-shape pattern of income during active age. We will discuss this later on in Section 4. The parameter ρ in (2) drives the persistence of the process π and will be assumed to be very close to 1.

Financial risk and security markets. The agent has the opportunity to save through either a riskless one period asset (similar to a T-Bill) and a risky asset (similar to a stock). The bond is a security of price 1 which pays R^f as a riskless gross return in the subsequent period. The rate

of interest R^f is constant and exogenous. The risky asset is similar to the bond except that the gross return R_t^s is risky and time dependent. More precisely, we make the following assumption regarding the evolution of the risky return:

$$\ln R_t^s = \ln R^f + \nu + \varepsilon_t^R,$$

where ν interprets as the average risk premium of stocks over bonds, while the financial risk $(\varepsilon_t^R)_{t \geq 0}$ is an IID normally distributed process with mean 0 and variance σ_R^2. The financial risk is assumed to be possibly correlated to both income shocks (ε_t^π) and (ε_t^y). The correlation with each income process is assumed to be constant and is denoted respectively $\kappa_{R,\pi}$ and $\kappa_{R,y}$.

The participation to the stock market is not free, as for example in Gomes and Michaelides (2008). The agent must pay a fixed cost $F \geq 0$ to participate to the stock market, which may interpret as transactions costs or as an opportunity cost to discover how the stock market works. This is a once-in-a-life cost: if the cost is paid at a given date t, the agent can freely trade stocks at date t and at any date afterwards.

Timing and notations. At the beginning of every period, the agent first learns the realizations of financial and labor shocks and whether she is alive or not. She thus knows the amount of her current savings and, if she is alive, what will be her income over the period to come. More precisely, at any date t, we assume that the agent knows the entire history of all shocks up to date t, which is formalized by the natural filtration $(\mathcal{F}_t)$ generated by the processes (u_t), (v_t) and (ε_t). The alive agent then decides her consumption level c_t, her savings in bonds b_t and in stocks s_t and her stock market participation status η_t (if she has never paid the participation cost before and never participated). The bequest of a dead agent is denoted w_t.

Constraints.

1. If the agent is dead at date t, she bequeaths all her wealth:

$$w_t = R^f b_{t-1} + R_t^s s_{t-1}. \tag{3}$$

   The stock holding s_{t-1} may be null if the agent has never participated to the stock market. Moreover, the total wealth is null if the agent was already dead at date $t-1$, since it has already been bequeathed.

2. If the agent is alive, her resources at the beginning of the period is made of stock and bond payoffs plus the labor income y_t of the period. Resources cover consumption, bond and stock savings. The agent can only invest in stocks if the participation cost has been paid at some date prior to t, i.e., if $\eta_t = 1$. Moreover, the agent may also have to pay the participation cost

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F at date t if she participates at date t in the stock market for the first time, i.e., if $\eta_t = 1$ and $\eta_{t-1} = 0$. The budget constraint at date t of the alive agent can then be expressed as follows:

$$c_t + b_t + s_t 1_{\eta_t=1} + F1_{\eta_t=1} 1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R_t^s s_{t-1}, \tag{4}$$

where $1_{\eta_s=1}$ is an indicator function equal to 1 if $\eta_s = 1$ and 0 otherwise. Moreover, the agent is prevented from short-selling bonds and stocks and her consumption must also be strictly positive. Formally, at date t, we have the following borrowing and consumption positivity constraints:

$$b_t \geq 0 \text{ and } s_t \geq 0, \tag{5}$$

$$c_t > 0. \tag{6}$$

A feasible allocation is a sequence of choices $(c_t, b_t, s_t, \eta_t)_{t \geq 0}$ satisfying the constraints (3)–(6). The set of feasible allocations is denoted $\mathcal{A}$.

Preferences. The agent enjoys instantaneous felicity from current spending, in either consumption or bequest, depending on her survival status. We denote by $u(c_t) : \mathbb{R}^+ \to I$ the instantaneous felicity she gets when being alive and consuming c_t and by $v(w_t)$ the utility she derives when being dead and bequeathing the amount w_t. Preferences are separable over time and future instantaneous utilities are discounted by a factor $\beta \in (0, 1)$ representing the agent's exogenous time preference.

Regarding risk preferences, we consider recursive utilities *à la* Kreps and Porteus (1978). Agents value the certain equivalent of a concave transformation of the future utility stream. More precisely, for an increasing concave function $\Phi : \mathbb{R} \to \mathbb{R}$, the utility U_t at date t expresses as follows:[2]

$$U_t = (1-\beta)u_t + \beta\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi(U_{t+1})\right]\right), \text{ with } u_t = \begin{cases} u(c_t), & \text{if the agent is alive at } t, \\ v(w_t), & \text{if the agent is dead at } t. \end{cases} \tag{7}$$

In the above equation, $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ the conditional expectation operator with respect to the information available at date t. Formally, the information is the filtration $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t \geq 0}$, where $(\mathcal{G}_t)_{t \geq 0}$ is the filtration generated by the independent mortality process.

In such models, if there were no uncertainty, the utility U_t would be independent of the function Φ and the recursion (7) would reduce to $U_t = (1-\beta)u_t + \beta U_{t+1}$. We thus have a possible separation between preferences over certain consumption streams (determined by the functions u, v and the

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[2]Formally speaking, preferences are defined over the set of temporal lotteries, allowing for preferences for late or early uncertainty resolution. See Epstein and Zin (1989) or Wakai (2007) for a formal treatment.

scalar β) and risk preferences driven by the function Φ. A more concave Φ implies lower certainty equivalents $\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}[\Phi(U_{t+1})]\right)$ and therefore greater risk aversion.

Our specification of recursive preferences nests some of the most standard cases, including the additive specification, or the Epstein and Zin (1989) isoelastic specification or the risk-sensitive specification introduced by Hansen and Sargent (1995) in their work on robustness. We will introduce below (see in Section 3) some additional properties that we will be made on the functions u and v and make precise the functions Φ which correspond to these different popular specifications. Our results will make it possible to discuss the impact of these specifications on saving decisions.

Agent's program. We can now write the agent's program that can be expressed recursively by taking advantage of the structure of preferences. We denote as U_t^D the intertemporal utility at date t of a dead agent and U_t^A the one of an alive agent. Regarding the dead agent, note that the instantaneous utility of an agent dead for more than two periods is constant (all bequests take place in the first period after death) and simply equals to $v(0)$. From the recursive formulation (7), we deduce that there is no actual optimization and that the program of a dead agent can then be expressed as:

$$U_t^D(w_t) = (1-\beta)v(w_t) + \beta v(0). \tag{8}$$

For an alive agent, the agent maximizes her intertemporal utility by picking up the proper feasible allocation $(c_t, b_t, s_t, \eta_t)_{t\geq 0}$ in the set $\mathcal{A}$. The utility U^A of the alive agent depends on four state variables: the beginning-of-period holdings in stocks s_{t-1} and bonds b_{t-1}, the permanent shock π_{t-1} of labor income and the stock market participation $\eta_{t-1} \in \{0, 1\}$. The latter is discrete, while the three former ones are continuous. From the recursive formulation (7) and feasibility constraints (3)–(6) and using the fact that the mortality risk is assumed to be independent of other risks, the program of an alive agent at date t can be expressed as follows:

$$U_t^A(s_{t-1}, b_{t-1}, \eta_{t-1}, \pi_{t-1}) = \max_{(c_t, s_t, b_t, \eta_t)\in\mathcal{A}} (1-\beta)u(c_t) \tag{9}$$

$$+ \beta\Phi^{-1}\left(p_{t+1|t}E_t[\Phi(U_{t+1}^A(s_t, b_t, \eta_t, \pi_t))] + (1 - p_{t+1|t})E_t[\Phi(U_{t+1}^D(w_{t+1}))]\right),$$

where $E_t[\cdot]$ is the expectation for an alive agent with respect to the filtration $\mathcal{F}$ (i.e., made of all past shock realizations but the death). Note that we distinguish it from the expectation $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ with respect to the whole information $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t\geq 0}$ (including death information). It should be noted that the program (9) has a finite-horizon since there exists a maximal age for the agent, who cannot live beyond date T_m, such that $p_{T_m+1|T_m} = 0$.

Value of life. A very important concept in our paper is the value of a statistical life, or value of life, which is crucial for determining saving behaviors in presence of death risk. The value of

life denoted VSL_t at the age of date t can be expressed as the opposite of the marginal rate of substitution between the mortality rate and consumption at that age. Noting $q_{t+1|t} = 1 - p_{t+1|t}$ the mortality rate at the age of date t, we formally define the value of life as follows:

$$VSL_t = -\frac{\frac{\partial U_t^A}{\partial q_{t+1|t}}}{\frac{\partial U_t^A}{\partial c_t}} = \frac{\frac{\partial U_t^A}{\partial p_{t+1|t}}}{\frac{\partial U_t^A}{\partial c_t}}.$$

The value of life VSL_t is equal to the quantity of consumption an agent would be willing to give up for a marginal decrease in the hazard rate. Our definition of the value of life is standard and consistent with that of Johansson (2002) for example. Using equation (9), we derive the following expression for the value of a statistical life:

$$VSL_t = \frac{\beta}{1-\beta} \frac{E_t \left[\Phi \left(U_{t+1}^A \right) - \Phi \left((1-\beta)v(w_{t+1}) + \beta v(0) \right) \right]}{u'(c_t)\Phi' \left(\frac{1}{\beta} \left(U_t^A - (1-\beta)u(c_t) \right) \right)}. \tag{10}$$

3 Specifications of utility functions

We now specify the functional forms for felicity functions u and v and for the aggregator Φ.

3.1 Felicity function specification

We begin with specifying u and v. We assume that the agent has a constant intertemporal elasticity of substitution, which means that $-\frac{u'(c)}{c\,u''(c)}$ is constant. This implies that u is equal, up to an affine transformation, to:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma} & \text{if } \sigma \neq 1, \\ \ln(c) & \text{if } \sigma = 1. \end{cases} \tag{11}$$

where the parameter $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. The above specification is such that $u(1) = 0$. It embeds therefore a normalization assumption, which is without generality loss. The case $\sigma = 1$ is obtained by continuity from the general case.

The felicity derived from bequeathing the wealth w for a dead agent is assumed to have the following functional form:

$$v(w) = \begin{cases} -v_0 + \frac{\theta}{1-\sigma} \left[(\hat{w} + w)^{1-\sigma} - \hat{w}^{1-\sigma} \right] & \text{if } \sigma \neq 1, \\ -v_0 + \theta \ln \left(\frac{\hat{w}+w}{\hat{w}} \right) & \text{if } \sigma = 1. \end{cases} \tag{12}$$

where $v_0 \in \mathbb{R}$, $\theta \geq 0$ and σ is the inverse of the intertemporal elasticity of substitution used in the expression (11) of the felicity of u. As for u, the case $\sigma = 1$ in (12) is obtained by continuity from the general case.

We can distinguish two components in the specification of v in equation (12). The first one is the constant v_0, which corresponds to the difference in utility between being alive and consuming 1

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unit or being dead and bequeathing nothing. A higher (resp. lower) value of v_0 will be associated with a higher (resp. lower) valuation of being alive, compared to being dead. The value of v_0 thus strongly connects to the value of life. The second part, $\frac{1}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]$ measures the contribution of bequest to post-mortem felicity. This extra felicity derived from bequest is assumed to be continuous in zero, increasing in the amount of bequest and exhibiting bounded and decreasing marginal felicity. The rationale for this functional expression is the following one. Heirs may dispose of individual resources summarized by the quantity $\hat{w}$ and they enjoy bequest in addition to these resources $\hat{w}$. The felicity derived by heirs from bequest is proxied by the quantity $\frac{1}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]$.[3] The agent values the felicity of her heirs with the weight θ that can therefore be interpreted as a parameter for the altruism intensity. With $\hat{w} > 0$, bequests are a luxury good, as reported in the data (e.g., in Hurd and Smith, 2002). Moreover, the derivative $v'(0)$ is finite, so that agents bequeath only when their wealth is large enough. This functional form has been chosen for example in De Nardi (2004), De Nardi et al. (2010), Ameriks et al. (2011), and Lockwood (2012, 2014). It is sometimes assumed in the literature that $v_0 = \frac{1-\theta\hat{w}^{1-\sigma}}{1-\sigma}$ so that $v(w) = \frac{1}{1-\sigma}\left[\theta\left(\hat{w}+w\right)^{1-\sigma} - 1\right]$, which has some advantage in terms of tractability.[4] However, this constraint on v_0 implies to impose a nontrivial relationship between the utility of bequest and the value of life. In particular, if θ is set to zero (no altruism) and $\sigma > 1$, then the utility of being dead is always higher to that of being alive, implying a negative value of life. We will not make assumptions of these kinds as we want our model to match standard empirical estimates for the value of life.

3.2 Risk-sensitive preferences

Regarding Φ that determines risk preferences, we consider several functional forms. First, we consider risk-sensitive preferences:

$$\Phi(u) = \begin{cases} -\frac{1}{k}\left(\exp(-ku) - 1\right) & \text{if } k \neq 0, \\ u & \text{if } k = 0, \end{cases} \tag{13}$$

where k is a constant driving risk aversion. The case $k = 0$ corresponds to the usual additive model and is obtained by continuity of the general case. For an agent endowed with risk-sensitive preferences to be more risk averse than in the usual additive model, we need to assume that $k > 0$. Risk-sensitive preferences have been introduced in Hansen and Sargent (1995) and axiomatized in

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[3]The proxy is exact if (i) heirs have the same intertemporal elasticity of substitution as the donator and (ii) heirs fully annuitize their wealth.

[4]Additional tractability can then be obtained by setting $\hat{w} = 0$, in order to have homogeneous specifications, as in Inkmann, Lopes and Michaelides (2011) for example.

Strzalecki (**?**). As shown in Bommier, Kochov and LeGrand (2016), this is the only functional form Φ for which preferences represented by the utility function in recursion (7) are monotone. The monotonicity of preferences has to be understood as the monotonicity with respect to first-order stochastic dominance. Preference monotonicity means that if two uncertain consumption streams are available and that the first one is preferred to the other one in any possible state of the world, the former will always be preferred to the latter. This is distinct from and not implied by the fact that more certain consumption is preferred to less, which is in our setup equivalent to an increasing felicity function u. Moreover, as proved in Bommier and LeGrand (2014), risk-sensitive preferences are well-ordered with respect to risk aversion, both "in the large" (i.e., in terms of willingness-to-pay to eliminate all risks) but also "in the small" (i.e. in terms of willingness-to-pay for marginal risk reductions). This last aspect is important when addressing problems where complete risk elimination is not possible, or simply not optimal, as it is the case in the portfolio choice that we study.

3.3 Epstein-Zin preferences

We now present Epstein-Zin isoelastic preferences, which correspond to the following functional form for Φ:

$$\Phi(u) = \begin{cases} \frac{1}{1-\gamma}(1 + (1-\sigma)u)^{\frac{1-\gamma}{1-\sigma}} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\sigma}\ln(1 + (1-\sigma)u), & \text{if } \gamma = 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\gamma}e^{(1-\gamma)u} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma = 1, \\ u, & \text{if } \gamma = 1 \text{ and } \sigma = 1, \end{cases} \tag{14}$$

where $\gamma \in \mathbb{R}$ and $1 + (1-\sigma)u \geq 0$.[5] Remark that whenever $\gamma = \sigma$ (but possibly different from 1), we get $\Phi(u) = u$ and Epstein-Zin preferences are additive. It is also well-known from Tallarini (2000), and directly visible from the last two lines of (14), that when $\sigma = 1$ Epstein-Zin preferences coincide with risk-sensitive preferences. Thus, the cases where $\sigma = 1$ are already addressed with risk-sensitive preferences and do not need further consideration. We will therefore exclude them whenever we refer to Epstein-Zin preferences below. For $\sigma \neq 1$, the constraint $1 + (1-\sigma)u \geq 0$ is not trivial. It holds whenever the agent is alive, since we have $1 + (1-\sigma)u(c) = c^{1-\sigma}$, but imposes constraints on the felicity of bequest defined in equation (12). The constraint $1 + (1-\sigma)u \geq 0$ is

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[5]Epstein Zin preferences are often introduced with a different but equivalent normalization for the function u (e.g. using $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ instead $u(c) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma}$), and therefore different functions Φ (the constant 1 being no longer needed). Our (equivalent) approach has however the advantage to have the cases $\sigma = 1$ or $\gamma = 1$ directly obtained as limit cases of the others, while keeping v_0 or $\hat{w}$ independent of σ and γ. This normalization choice has no impact on our results.

equivalent to

$$\begin{cases} v_0 \leq \frac{1}{1-\sigma} & \text{if } \sigma < 1, \\ v_0 \geq \frac{1-\theta \hat{w}^{1-\sigma}}{1-\sigma} & \text{if } \sigma > 1. \end{cases}$$

Isoelastic Epstein-Zin preferences are very popular in macroeconomics and finance, one of their main advantage being that they usually provide homogeneous specification, which is key for tractability. Note that this is not the case in our setup, where Epstein-Zin preferences are not homothetic. The reason is not the normalization that we made in equation (14), but stems from our choice of imposing plausible value of life.

4 Calibration and computation

In this section, we first give an overview of our calibration strategy. We then discuss the resulting parametrization in detail. The last section discusses the major difficulties in solving the model computationally.

4.1 Calibration strategy

Our calibration shares many common aspects with the related literature but differs mainly in that we target the value of a statistical life explicitly.[6] Given a realistic value of life, the objective of the calibration exercise is to highlight the impact of risk aversion. To this aim, we consider three agents: one with standard additively separable preferences, one with Epstein-Zin preferences corresponding to the aggregator (14), and one with risk-sensitive preferences corresponding to the aggregator (13).[7] We will henceforth refer to the three agents as the additive, the Epstein-Zin and the risk-sensitive agent, respectively. Importantly, we calibrate only the additive agent to the data. The other two only differ in that they have a marginally larger risk aversion.

We now describe our strategy for calibrating the additive agent; the resulting parameter values are discussed in the following sections. We set the intertemporal elasticity of substitution, $\frac{1}{\sigma}$, to a standard value. We then jointly calibrate the discount factor, β, the bequest motive, θ, and the life-death utility gap, v_0, to match the following three targets. The first target is an estimate of the value of a statistical life at age 45, VSL_{45}, as defined in equation (10). Targeting this is central to our exercise. The second and third targets are average assets at age 45 and average bequests at age 85, respectively. All other parameters are set to values that are taken directly from available data

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[6]Note however that some aspects of the current calibration are still preliminary.

[7]Recall from Section 3 that the additively separable case is nested in Epstein-Zin preferences when $\gamma = \sigma$, and in risk-sensitive preferences when $k = 0$.

or related studies. Table 2 summarizes the parameter values, displaying endogenously calibrated values in italics.

After having calibrated the additive agent, we set the risk aversion of the Epstein-Zin agent to a slightly higher value $\gamma^{EZ} > \sigma$, while keeping all other parameters the same. This isolates how in a given economy risk aversion impacts the lifecycle savings choice and the value of a statistical life, among others. Similarly, we increase risk aversion k of the risk-sensitive agent by setting $k > 0$. More precisely, we calibrate k to produce the same average savings at age 45 as the Epstein-Zin agent, i.e., such that $E_0[s_{45}^{RS} + b_{45}^{RS}] = E_0[s_{45}^{EZ} + b_{45}^{EZ}]$. We do this, because we want the increases in risk-aversion to have a similar meaning when compared to the additive agent. Thus, we can compare on one hand the Epstein-Zin agent with the additive one, and on the other hand the risk-sensitive agent with the additive one. Note that the Epstein-Zin and risk-sensitive agents are not comparable with each other in terms of risk aversion.

4.2 Demographics

Agents start being economically active in the model at the working age of 21. They exogenously retire at the fixed age of 65, which corresponds to the statutory retirement age in the U.S. Mortality rates are taken from the Human Mortality Database for the U.S. for 2007. The maximum biological age is capped at 100, since mortality estimates become inaccurate after that.

4.3 Preferences

The intertemporal elasticity of substitution is set to 0.5, a common value in the literature, so that its inverse is $\sigma = 2$. For the Epstein-Zin agent we increase the risk aversion parameter moderately to $\gamma^{EZ} = 3$, since we do want not deviate too much from the additive agent. Last, for the risk-sensitive agent, we calibrate the risk aversion parameter to match the same average savings as the Epstein-Zin agent at the age of 45, which yields $k = 0.08$.

We then calibrate v_0 and β jointly so that the additive agent has a value of a statistical life at age 45 and assets at age 45 that match their empirical counterparts. For VSL we target US$ 6.5 million, which is in the middle of available estimates.[8] For average individual assets we target US$ 100 000, which is consistent with Census data. This yields $v_0 = 30.0$ and $\beta = 0.96$, which we keep constant for the three agents.

The strength of the bequest motive will be calibrated jointly with v_0 and β in order to match average bequests at age 85 but at the moment is set to a preliminary value of $\theta = 20$. The exogenous lifetime endowment of the offspring, $\hat{w}$, shifts bequest utility and is mostly needed for

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[8]Viscusi and Aldy (2003) provide a discussion of available estimates of the value of life.

Table 2: Parameterization in baseline economy

Parameter	Value	Source, empirical counterpart, or target
Demographics		
Biological age at $t = 1$	21	age at labor force entry (college)
Model age at retirement, T_R	45	S.S.A. statutory retirement age of 65
Model age maximum, T_M	80	Biological maximum age of 100
Cond. mortality rates, $\{m_{t+1|t}\}$		Human Mortality Database, U.S. 2007
Preferences		
Inverse IES, σ	2.0	
Risk aversion, Epstein-Zin, γ^{EZ}	3.0	
Risk aversion, risk-sensitive, k	*0.08*	Assets of EZ at age 45
Life-death utility gap, v_0	*30.0*	$VSL_{45} = $ US\$ 6.5m (add. pref.)
Discount factor, β	*0.96*	$Assets_{45} = $ US\$ 100000 (add. pref.)
Strength of bequest motive, θ	20.0	Bequests at age 85 (add. pref.)
Exogenous offspring endowment, $\hat{w}$	1.5	
Endowments		
Average wage, y_0	0.3	
Pension, y_R	0.1	
Age productivity, $\{\mu_t\}$	cf. app.	Earnings profiles (PSID)
Labor income autocorrelation, ρ	0.95	Storesletten, et al. (2004)
Var. of persistent innovation, σ_π	0.3	Storesletten, et al. (2004)
Correlation with stock return, $\kappa_{R,\pi}$	0.15	Gomes and Michaelides (2005)
Var. of transitory innovation, σ_y	0.0	Preliminary
Asset Markets		
Gross risk-free return, R^f	1.01	Bond return (Shiller)
Equity premium, ν	0.02	Preliminary
Stock volatility, σ_R	0.18	Shiller data
Participation cost, F	0.2	Preliminary

Notes: Parts of this calibration are still preliminary. Values in italics have been calibrated to their respective targets.

making bequests a luxury good. We set it to $\hat{w} = 1.5$.

4.4 Endowments

The average wage in this economy is just a scaling factor and we set it to $y_0 = 0.3$ to keep the bounds of the state space small for computational purposes.[9] Pensions are set to 30 percent of the average wage, in line with the U.S. social security replacement rate. The deterministic age-productivity profile is taken from Harenberg and Ludwig (2015), who compute it from PSID data using the method of Huggett, Ventura and Yaron (2011). The values are displayed in the computational appendix.[10]

The values for the persistent income process are taken from Storesletten, Telmer and Yaron (2004). Using PSID data, they find an autocorrelation $\rho = 0.95$ and a variance of shocks of $\sigma_\pi = 0.3$. The correlation of the persistent income shocks with stock returns is set to the same value as in the baseline of Gomes and Michaelides (2005), $\kappa_{R,\pi} = 0.15$. In our preliminary calibration, the variance of the temporary income shocks, σ_y, is set to zero.[11]

4.5 Asset markets

The parameter values for asset markets are mostly preliminary. The gross risk-free rurn is set to the average bond return of the last 50 years in the data of Robert Shiller, $R^f = 1.01$ percent.[12] The equity premium takes a preliminary value of $\nu = 0.02$ percent (to be increased to 0.06 in the next version). Stock volatility is $\sigma_R = 0.18$, again as measured from Robert Shiller's data over the last 50 years. Participation cost is set to a preliminary value of $F = 0.2$ to get a reasonable stock market participation rate for the additive agent.

4.6 Computational solution

From a computational perspective, there are two difficulties when solving the model that are worth discussing. The first difficulty is that we want to numerically approximate the risks as precisely as possible, since the impact of the various risks is at the core of this paper. This is important for the autoregressive process driving the persistent income shock and for the correlation between income shocks and stock returns. The well-known discretization method of Tauchen (1986) and Tauchen and Hussey (1991) has been shown to be sensitive in both statistical and economic outcomes (e.g.,

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[9]Assets (or cash at hand) can become very large because we treat income shocks as unbounded. That is why a small y_0 is helpful for computational reasons.

[10]The computational appendix is available on request.

[11]This will be changed in further versions.

[12]Robert Shiller's data are freely available at `http://www.econ.yale.edu/~shiller/data.htm`.

Flodén 2008) and cannot handle cross-correlated processes (see Galindev and Lkhagvasuren 2010, for example). While several improvements have been proposed by Kopecky and Suen (2010) or Galindev and Lkhagvasuren (2010) for instance, it is not fully understood how sensitive to the discretization utility and choices are in a given model. Instead of relying on a finite-state approximation, we keep the continuous representation in equations (1) and (2) and treat π_t as an additional, continuous state variable. We use 24 gridpoints to approximate this continuous state and evaluate the expectations with Gauss-Hermite quadrature, for which convergence is well-known. To evaluate continuation utility at points off the grid, we use cubic two-dimensional B-splines. Details are provided in the computational appendix.

The second difficulty is that the model has a discrete choice—the stock market participation decision—which implies that the agent's problem is not (globally) differentiable in the continuous savings and portfolio choices.[13] As a consequence, we cannot rely on Euler equations and Newton-like nonlinear equation solvers. A brute-force maximization using discretization of the state space and the choices is also infeasible, because we have two continuous state variables, one binary state variable, 80 generations, along with two continuous and one discrete choice and want to calibrate the model to the data.[14] We solve this with a novel solution algorithm that is robust, fast, and generally applicable to finite-horizon problems. The main idea is to interpolate the expected continuation utility, $E_t U_{t+1}$, with a multi-dimensional cubic B-spline, because it can be proven that $E_t U_{t+1}$ is twice differentiable. The divide and conquer algorithm of Gordon and Qiu (2015) is then used to quickly find a bracket for a global maximum on a fine grid. Then, the maximum is computed with a high precision using a Newton-like maximizing routine, which can be defended with the result of Clausen and Strub (2012) on the local differentiability around an optimum.

On top of that, we speed up the algorithm by making use of the fact that, after minor transformations, the optimal stock choice can be represented and computed as a function of the optimal savings choice. Programmed in Fortran 2008, the code is parallelized and runs on 24 cores. Further details are provided in the computational appendix.

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[13]Even recent, more general envelope theorems are of only very limited use in a computational application. E.g., the very powerful result in Clausen and Strub (2012) is not directly applicable, because in a numerical solution we search for an optimal choice and need to evaluate continuation utility also at points that are not optimal and may therefore not be differentiable. The computational appendix provides more details.

[14]The two continuous states are cash-at-hand, x_t, and the stochastic state, π_t, the discrete states are the 80 generations and the stock market participation indicator. The continuous choices are bond and stock investments and the discrete choice is stock market participation.

5 Results

We now proceed to our results. We first describe the outcomes of the model, as calibrated in Section 4 and then provide some further explanation about the grounding of our findings.

5.1 Description

To present our results, we focus on the lifecycle profiles for agent choices. Each profile corresponds to the profile conditional on the agent surviving until the maximal age, averaged over all possible realizations for the income and investment risks. For agents that die before the maximal age, the savings-consumption profiles are simply truncated at the age of death. Lifecycle profiles –for savings for example– are computed as follows. For a given age, we compute the optimal saving response as a function of cash-at-hand as well as the distribution (conditional on surviving) of agents in terms of cash-at-hand.[15] From both optimal saving responses and conditional distribution, we directly get the average saving at each age.

Table 3: Selected Lifecycle Statistics

	Additive	Epstein-Zin	Risk-sensitive
Assets, age 45, in US $1 000	*100.0*	58.5	*58.5*
VSL, age 45, in US $1 000 000	*6.50*	12.8	15.3

Notes: Values in italics are calibrated as described in section 4.1.

Table 3 reports average life-cycle assets and value of life at the age of 45 for the additive, risk-sensitive and Epstein-Zin agents. Asset holdings of the additive agent match the empirical counterpart for individual savings at age 45 of US $100 000, because that is a calibration target. Asset holdings for the other two agents are lower, amounting to US $58 500. They are the same for both agents because we calibrated the parameters for the risk-sensitive agent to match the asset holdings of the Epstein-Zin agent at age 45, as is explained in Section 4.1. The value of a statistical life at age 45 is US $6.5 millions for the additive agent, which was a calibration target. For the other two agents, the value of life at age 45 is higher, because we increase risk aversion while holding all other parameters—in particular v_0—constant. Thus, the agent is more averse to the risk of dying and values mortality risk reduction more. The values in Table 3 were part of the calibration strategy and represent a particular point of the full lifecycle profile, which we turn to next.

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[15]Since the shocks are continuous, we get a continuous distribution over cash-at-hand. We approximate this distribution with a piecewise linear function over 3600 points in the cash at hand grid. For details, see the computational appendix.

We now plot the lifecycle profile for savings in Figure 1. The three profiles, corresponding to the additive, risk-sensitive and Epstein-Zin agents exhibit a similar pattern. Agents build up savings during their working age, until they reach the exogenous retirement age of 65. During retirement, agents gradually decumulate their savings. The second element of this graph is that the savings of the additive agent are much larger that those of both the Epstein-Zin and risk-sensitive agents. Recall that the Epstein-Zin and risk-sensitive agents are both more risk averse than the additive one. The theoretical impact of risk aversion on savings is not clear cut when the three risks are simultaneously present,[16] but the main result of our quantitative exercise is unambiguous. An increase in risk aversion diminishes savings when investment, income and mortality risks are all present and properly calibrated. Our quantitative result means that the effect of the mortality risk dominates the effect of investment and income risks. This notably implies that it is crucial to properly take into account the value of life in the modeling and in the calibration. To conclude the discussion of saving profiles in Figure 1, note that the impact of a change in risk aversion (despite of a similar calibration) is not strictly identical for risk-sensitive and Epstein-Zin agents, except at age 45 (which is the age used for the value of life calibration). In particular, savings decrease more for the risk-sensitive agent at earlier age (before 45), but the pattern is reversed for later ages (after 45).

We relatedly plot the lifecycle consumption profiles in Figure 1. They are consistent with lifecycle saving profiles. Note that consumption profiles for the three agents are hump-shaped. Risk averse agents consume more at earlier ages (between ages 30 and 60) than the additive agent. The opposite holds at older ages, greater than 60. A greater risk aversion tends therefore to increase consumption at earlier age and to decrease it a later age. Again, this is consistent with the fact that the impact of mortality risk dominates the impact of other risks. A more risk averse agent will be more prone to consume early and to save less in order to reduce the risk of dying while having large savings (even though they can be bequeathed).

We now plot the lifecycle participation rate as a function of age in Figure 3. First, since the participation cost is paid once in a life, the participation rate is an increasing function of age. Moreover, the other feature is that the stock market participation rate decreases with risk aversion for both the Epstein-Zin and the risk-sensitive agents. More risk averse agents choose to participate less in risky markets and therefore to have a smaller exposure to the investment risk. This is also consistent with the fact that more risk averse agents save less and are therefore less prone to pay a cost for participating to stock markets.

This pattern for participation rate is confirmed by the lifecycle profile of stock portfolio shares

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[16]See Table 1 and the related discussion in the introduction and in Section 5.2.1 for the impact of risk aversion on savings when only one the risk is present.
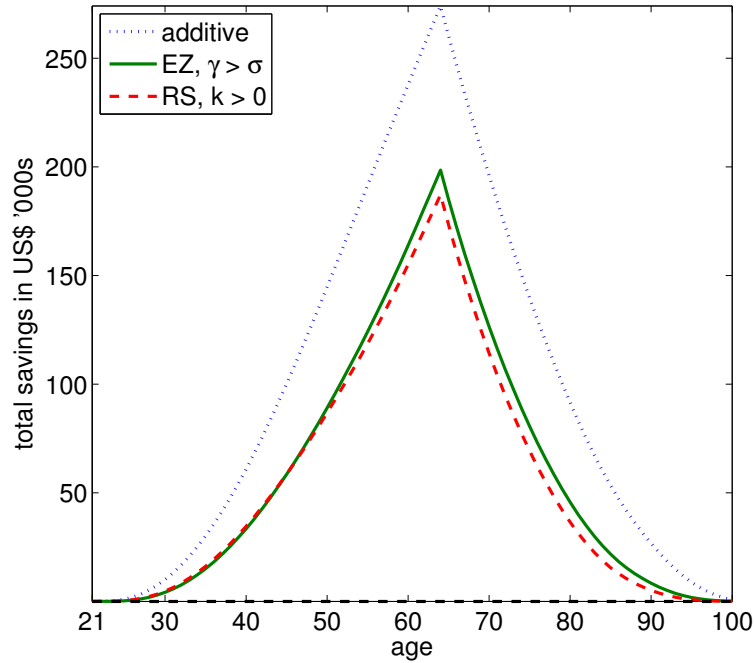
Figure 1: Lifecycle saving profiles

plotted in Figure 4. Conditional on the participation to stock markets, more risk averse agents hold a smaller share of their savings in stocks, except at later ages (above 85), where the wealth of agents becomes very low. More risk averse agent opt for less risky portfolios and a smaller exposure to the investment risk.

To summarize our findings in this quantitative exercise, we can state that in our environment in which agents simultaneously face investment, income, and mortality risks, more risk averse agents save less, choose to participate in the stock market less frequently and when participating, hold a smaller share of risky assets in their portfolios.

5.2 Result discussion

Our discussion is threefold. First, we provide further intuitions about the fact that relationship between risk aversion and savings is ambiguous in presence of mortality, income, and investment risks. Second, we make a couple of back of the envelope computations to illustrate the orders of magnitude of mortality and income risks. Finally, we further investigate the importance of mortality risk and compute how much of the total variance in ex post utilities is explained by mortality. These two last computations show that mortality risk is the main risk from a lifecycle perspective. It is therefore not surprising that the effect of mortality risk dominates the ones of income and investment risks regarding the relationship between risk aversion and savings.
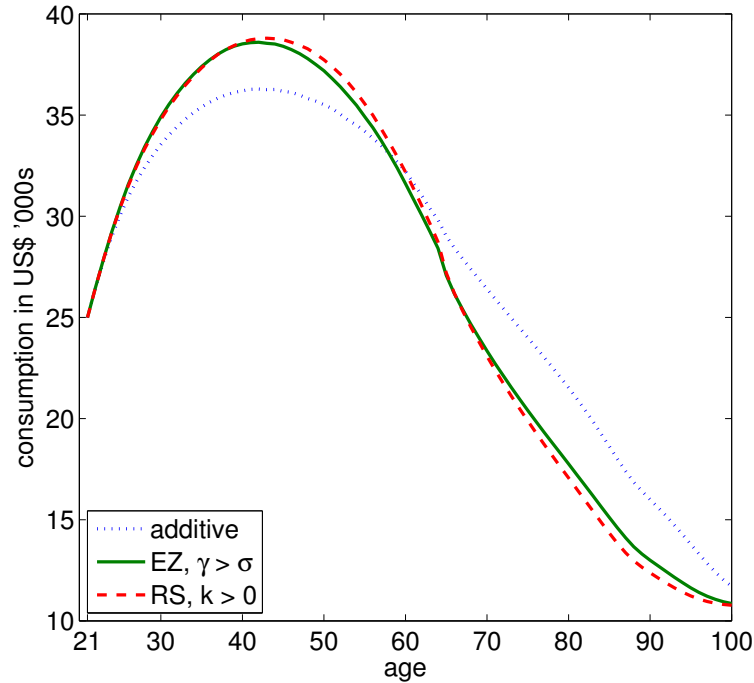
19

Figure 2: Lifecycle consumption profiles

5.2.1 When can the relationship between risk aversion and savings be ambiguous?

To understand that the impact of risk aversion on savings is not clear-cut when the three risks (death, income and mortality) are present, let us start with focusing on situations where only one risk is present. To make the intuition more transparent, let us restrict to a two-state two-period economy. For each of the two states, there is an optimal state-specific saving in the first period, which corresponds to the optimal saving for a given state realization in absence of risk. The optimal saving in presence of risk can then be seen as a linear combination of these two state-specific optimal savings, where the weights depend on risk aversion. The more risk averse the agent, the greater the weight on the saving of the bad state. This reflects the intuitive fact that a risk averse agent cares a lot about bad state realizations.

We can now discuss the impact of risk aversion on savings in presence of one of the three aforementioned risks. In presence of income risk only, the bad state is a low income realization in the next period. The state-specific saving in the bad state is larger than the one in the good state. Therefore, in presence of income risk, a more risk averse agent will save more. When mortality is the only risk, the bad state is to die early and to leave savings, which are not consumed. In that case, a more risk averse agent will save less to avoid holding savings while dead. Finally, when the risk is an investment risk (through an uncertain financial rate of return), the state specific response mixes
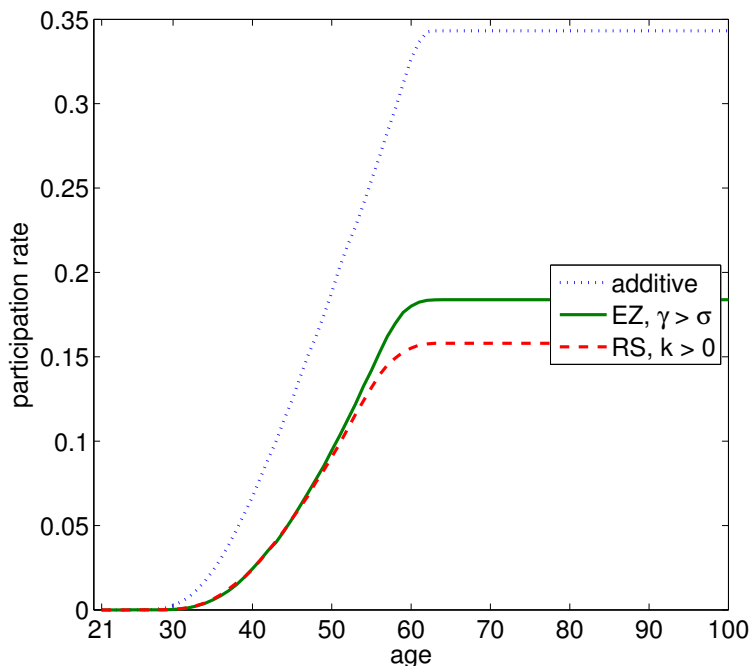
20

Figure 3: Lifecycle stock market participation

income and substitution effects and the best response depends on the intertemporal elasticity of substitution. When the intertemporal elasticity of substitution is greater (resp. lower) than one, the substitution (resp. income) effect dominates and bad-state-specific savings are lower (resp. higher) than the good state one. A more risk averse agent will thus save less (resp. more). These effects in presence of a single risk have been formally stated in BCL and summarized in Table 1.

In consequence, the direction of the impact of risk aversion on savings depends on the risk which is at stake, and when the three risks are present, this impact is possibly ambiguous and will depend on the respective magnitude of individual effects. We explore magnitudes in the two following subsections.

5.2.2 How bad is the mortality risk compared to the income risk?

We make a couple of back of the envelope computations to assess a first order estimate for the magnitudes of mortality and income risks.

First regarding mortality risk, we obtain from our database that the life expectancy at age 20 amounts to 58.5 years (i.e., expected death at age 78.5) with a standard error of 14.5 years. This means that if we approximate the mortality risk by its standard error, the mortality risk amounts to 14.5 years. To convert this risk in monetary terms, note that we consider a value of life at age 45 equal to $ 6.5 millions, while the life expectancy at that age is 35 years. The monetary value
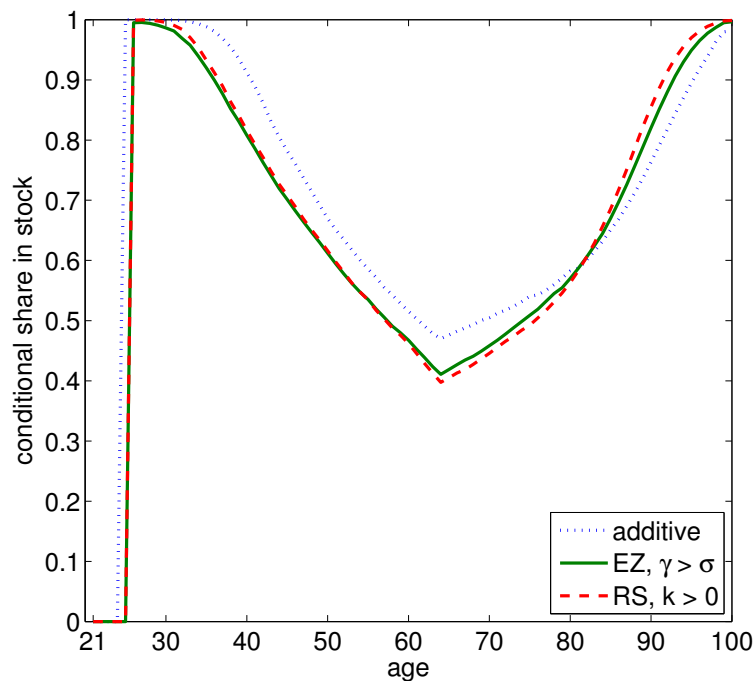
Figure 4: Share of wealth invested in stocks (conditional on participating)

of one year of being alive is therefore approximately $ 186000. We therefore deduce that as a first approximation, the mortality risk at age 20 amounts to $ 2.7 millions in monetary terms.

Let us now consider the income risk from a lifecycle perspective. To measure this risk, we compute the distribution of future lifetime income seen from age 20. The lifetime income is simply the sum of all per period incomes discounted at the riskless interest rate to age 20. Since there is no income risk after age 65, we discard incomes after that age. Due to the discounting (and without mentioning shock persistence), a bad income shock in earlier ages will have a much stronger impact on the lifetime income risk than the same bad shock at an older age. Using our calibration, we obtain an average lifetime income of $ 1.1 million with a standard deviation of $ 0.8 million.

Using these two proxies, we deduce that the mortality risk is approximately three times as "large" as the income risk. Moreover, as will be illustrated in the Section 5.2.3, the impact of the dispersion in lifetime incomes on lifetime utilities is dampened by the concavity of instantaneous utilities, while the impact of mortality risk on lifetime utilities is almost linear. In consequence, the mortality risk is not only greater than the income risk, but we also expect the mortality risk to have an even greater impact on the lifetime risk (measured as the dispersion of lifetime utilities).

5.2.3 How much of the lifetime risk is explained by mortality risk?

We finally assess the share of the total lifetime risk that can be explained by the mortality risk. We measure the lifetime risk as the risk associated to the distribution of (ex post) lifetime utilities. Any agent in our model enters the economy (alive) at age 20, experiences a sequence of income and investment shocks while alive, and finally dies at a given age. During her lifetime, the agent has made a sequence of consumption and saving decisions. It is therefore possible to compute the lifetime utility associated to this agent, once her life is over. Experiencing bad income shocks will negatively affect consumptions and thus lifetime utilities. Dying early, by shortening the number of consumption periods, will also have a negative effect on lifetime utilities.

To measure how much of the lifetime risk is explained by mortality, we simply regress lifetime utilities on dummy variables capturing the different possible lifetimes. The coefficient of determination (i.e., the R^2) of this regression indicates how much of the variance in lifetime utilities is explained by the variance in lifetimes. In other words, this tells us the share of the lifetime risk, which is explained by the mortality risk.

Formally, for any agent i in our population sample I, we denote V^i her lifetime utility and T_i her lifetime. Consistently with our model notations, the lifetime is normalized to vary between 1 and T_m. The regression can be expressed as follows

$$\forall i \in I, \ V_i = \sum_{t=1}^{T_m} \alpha_t 1_{t=T_i} + \sigma_\nu \nu_i, \tag{15}$$

where $\{\nu_i\}_{i \in I} \sim \mathcal{N}(0,1)$ are IID standard normal variables.

[To be completed]

6 Relation to previous literature

Our results indicate that savings decrease with risk aversion. This contrasts with the predictions of Gomes and Michaelides (2005, 2008), and those of HPSA when the elasticity of substitution is smaller than one, where they find the opposite result. The divergence between these studies and ours comes from how mortality is take into account. The simplest way to get the intuition is probably to refer to Section 5.2.1 of the current paper or to Section 4.3 of BCL who refer the role of risk aversion in a simple two period model, where mortality is the only risk at play. It is explained that if (i) living two periods is better than living one period and (ii) an agent –with no second period labor income– saves more when anticipating to live for two periods than when anticipating to live only for one period, then risk aversion decreases savings. Intuitively, greater risk aversion leads to take saving decisions which are closer to those that would be optimal in the

bad state of nature (i.e., living for one period here). This means saving less when assumptions (i) and (ii) are fulfilled. Assumption (i) means a positive value of life, while Assumption (ii) means that survival and consumption are complementary: In other words, it is assumed that saving for a non-existing future is of very little interest. Although BCL do not discuss it, their conclusion would be reversed if Assumption (i) or (ii) would be reversed, that is if assuming a negative value of life, or assuming that consumption and survival are substitute.

Gomes and Michaelides (2008) maintain the assumption that survival and consumption are complementary, but assume a negative value of life. When neglecting the impact of next period survival on the current consumption, their recursive utility representation can be expressed as follows:

$$V_t = \left((1-\beta)c_t^{1-\frac{1}{\varepsilon}} + \beta E_t \left(p_t V_{t+1}^{1-\rho} \right)^{\frac{1-\frac{1}{\varepsilon}}{1-\rho}} \right)^{\frac{1}{1-\frac{1}{\varepsilon}}}, \tag{16}$$

where V_t is the utility conditional of being alive at time t, c_t is consumption in period t, and p_t is the probability of being alive in period $t+1$. The parameter ε is the elasticity of substitution and ρ the coefficient of relative risk aversion. Direct calculations provide:

$$\frac{\partial V_t}{\partial p_t} = \frac{\beta}{1-\rho} E_t \left(V_{t+1}^{1-\rho} \right) E_t \left(p_t V_{t+1}^{1-\rho} \right)^{\frac{\rho-\frac{1}{\varepsilon}}{1-\rho}} V_t^{\frac{1}{\varepsilon}}.$$

Since Gomes and Michaelides (2008) assume that ρ is larger than 1, it follows that $\frac{\partial V_t}{\partial p_t} < 0$ indicating a negative value of life. Utility is then maximal when death occurs with probability one.

Gomes and Michaelides (2005) further include bequest in representation (16) and consider the following expression:[17]

$$V_t = \left((1-\beta p_t)c_t^{1-\frac{1}{\varepsilon}} + \beta E_t \left(p_t V_{t+1}^{1-\rho} + (1-p_t)b\frac{(X_{t+1}/b)^{1-\rho}}{1-\rho} \right)^{\frac{1-\frac{1}{\varepsilon}}{1-\rho}} \right)^{\frac{1}{1-\frac{1}{\varepsilon}}}, \tag{17}$$

where X_{t+1} is the amount of bequests left in case of death and b is a parameter driving the strength of the bequest motive. Utility represented by recursion (17) exhibits two unconventional features. First, utility derived from the date-t consumption depends on survival probability at date $t+1$. Second, the utility decreases with the amount of bequest: $\frac{\partial V_t}{\partial X_{t+1}} < 0$: the agent prefers to die while bequeathing nothing to her heirs (since X_{t+1} is constrained to be non-negative).[18]

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[17]The actual representation used in Gomes and Michaelides (2005) also features a dependence of the date-t consumption utility in the date-$t+1$ survival probability (i.e., $(1-\beta p_t)c_t^{1-\frac{1}{\varepsilon}}$ instead of $(1-\beta)c_t^{1-\frac{1}{\varepsilon}}$).

[18]Note that this is not true at the maximal age T_m. Indeed, they use the terminal terminal condition $V_{T_m} = b\frac{(X_{T_m+1}/b)^{1-\rho}}{1-\rho}$, where bequest has a positive impact on welfare ($\frac{\partial V_{T_m}}{\partial X_{T_m+1}} > 0$).

We can then compute the following derivative expression:

$$V_t^{-\frac{1}{\varepsilon}} \frac{\partial V_t}{\partial p_t} = -\frac{\beta}{1 - \frac{1}{\varepsilon}} c_t^{1 - \frac{1}{\varepsilon}} \tag{18}$$

$$+ \frac{\beta}{1 - \rho} E_t \left(V_{t+1}^{1-\rho} - b\frac{(X_{t+1}/b)^{1-\rho}}{1 - \rho} \right) E_t \left(p_t V_{t+1}^{1-\rho} + (1 - p_t)b\frac{(X_{t+1}/b)^{1-\rho}}{1 - \rho} \right)^{\frac{\rho - \frac{1}{\varepsilon}}{1 - \rho}}$$

The sign of the derivative (18) is ambiguous if $\rho > 1$ and $\varepsilon < 1$, as is the case in Gomes and Michaelides (2005)'s calibration. Indeed, the first line has a positive sign (due to the impact of next period survival probability on the utility of current consumption), while the second line has an unambiguous negative sign (since bequests are not valued by the agent, they do not contribute to make life more enjoyable). Overall, there is no guarantee that the sign of the derivative (18) is positive and that the value of life is positive in Gomes and Michaelides (2005).[19]

In cases where the value of life is negative, the intuition of BCL is reversed. A bad realization is a long life, while a good one is an early death. Greater risk aversion suggests to transfer welfare from good to bad realizations, or from early death to long life, which means increasing savings. This negative value of life explains why our results differ from those of Gomes and Michaelides (2005, 2008).

HPSA, who are interested in endogenous health investment, takes great care of using a model that assumes a positive value of life. When the elasticity of substitution is smaller than one, as in their fitted specification, their model assumes however that consumption and survival are substitute, rather than complementary. To clarify this, one may rewrite the model of HPSA while focusing on mortality risk and leaving health and financial risks aside. The HPSA utility function is then given by:

$$U = \left(\int_0^\infty e^{-\delta t} s(t)^{\frac{1 - \frac{1}{\varepsilon}}{1 - \lambda}} (c(t) - a)^{1 - \frac{1}{\varepsilon}} dt \right)^{\frac{1}{1 - \frac{1}{\varepsilon}}} \tag{19}$$

where a is a minimal subsistence level, δ the exogenous rate of time preference, $s(t)$ the probably of being alive age t, $c(t)$ consumption at age t, ε the elasticity of substitution, and $\lambda \in [0,1)$ mortality risk aversion (denoted by λ_{m_0} in their paper). When $\varepsilon < 1$, such specification may cause convergence problems if mortality rates goes above some level.[20] Let us assume that this

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[19]It might be the case that the equation (2) of Gomes and Michaelides (2005), reproduced above in (17), is in fact just a "typoized" version of the specification:

$$V_t = \left((1 - \beta)c_t^{1 - \frac{1}{\varepsilon}} + \beta E_t \left(p_t V_{t+1}^{1-\rho} + (1 - p_t)b(X_{t+1}/b)^{1-\rho} \right)^{\frac{1 - \frac{1}{\varepsilon}}{1 - \rho}} \right)^{\frac{1}{1 - \frac{1}{\varepsilon}}}$$

which was used later on in Inkmann et al. (2011). But, then, with $\rho > 1$, a positive value of life is only obtained if $b \geq \left(\frac{E_t[V_{t+1}^{1-\rho}]}{E_t[X_{t+1}^{1-\rho}]} \right)^{1/\rho}$ which is most likely not the case with the values that they consider for b. Gomes and Michaelides (2005) consider for example $b = 0$ as a plausible value, in which case the inequality is for sure not fulfilled.

[20]This does not occur in HPSA which is a perpetual youth model à la Blanchard (1985). However convergence issues would systematically occur in realistic life-cycle models where mortality tends to be large at old age.

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technical point can be solved by truncating the integral after some finite time T and assuming that survival never reaches zero before T. The utility function (19) assumes that the marginal utility of consumption is positive and that survival increases the value function. The value of life is thus always positive, as is emphasized in HPSA. The model is well-behaved in that respect. Let us however look at the marginal rate of substitution between consumption in period t and consumption in period 0:

$$\frac{\frac{\partial U}{\partial c(t)}}{\frac{\partial U}{\partial c(0)}} = e^{-\delta t} s(t)^{\frac{1-\frac{1}{\varepsilon}}{1-\lambda}}$$

If $\varepsilon < 1$, this marginal rate of substitution is decreasing with $s(t)$. In other words, the less likely is survival in period t, the more valuable it is to make provision for consumption in that period. According to this model, the rate of time discounting at time t is in fact $\delta + \frac{1-\frac{1}{\varepsilon}}{1-\lambda}\mu(t)$ where $\mu(t) = -\frac{s'(t)}{s(t)} > 0$ is the mortality rate at age t. Thus if $\varepsilon < 1$, mortality reduces impatience. This is reflected in HPSA analysis, where it is found that when $\varepsilon < 1$, the propensity to consume is decreasing with mortality (see their Theorem 1 and their discussion p. 678). The effect is limited in HPSA, since their calibrated model, which does not account for age effects, considers relatively low mortality rates. However, if applying the same model to realistic demographic data (as suggested in Appendix B of HPSA), the fact that $\mu(t)$ gets large at old age would imply (when $\varepsilon < 1$) that mortality would make people becoming extremely patient at the end of their life cycle. In fact, agents with utility (19) would keep most of their wealth for consumption at very old ages (e.g., ages greater than 110), precisely because these are ages they will most likely never reach. This is of course implausible, but a logical consequence of the assumption of substitutability between consumption and survival which is embedded in equation (19). This eventually explains the divergence between the results of HPSA and ours.

7   Conclusion

We have studied the role of risk aversion in a model that accounts for labor income, investment and mortality risks. The first message that one can take from our paper is that, under reasonable calibration, mortality is found to be the main source of risk in life. This resonates with basic intuition, death being generally considered to be more dramatic than bankruptcy or job loss. Discussing the role of risk aversion requires therefore to carefully account for mortality risk, with appropriate assumptions on the complementary between survival and consumption and reasonable (positive) levels for the value of life. Once this is done, economic analysis provides a simple message: the main impact of risk aversion is to decrease life-cycle savings. The basic intuition is the following: saving, which involves keeping resources for an uncertain future, is a risk taking

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behavior. As a risk taking behavior, it is found less appealing by more risk averse agents.

In addition to mortality risk, our analysis accounts for labor income and asset return uncertainty. Labor income risk typically leads the agents to save more, an effect that is amplified by risk aversion. This effect is however secondary compared that driven by mortality risk and only marginally reduces the impact of risk aversion on savings previously mentioned. Similarly, asset return uncertainty may reduce or amplify the effect of risk aversion on savings (depending on whether the elasticity of substitution is larger or smaller than one) but, again, this is of secondary importance. The main finding that risk aversion decreases savings is therefore quite robust.

Risk aversion does not only impact the level of saving, but also portfolio choice. Our results indicate that more risk averse agents participate less to the stock market, which reflects an indirect wealth effect: more risk averse accumulate less savings and are therefore less likely to pay the fixed participation costs that our model assumes. Last, for those who are active on the stock market, risk aversion leads them to choose less risky portfolio, in line with standard results in finance.

One of the implications of our study, is that relatively low level of savings could be explained by risk aversion. While the economic literature abounds of works arguing that observed saving behaviors have to reflect strongly myopic preferences or some form of irrationality, our analysis suggests on the contrary that saving little could just be a rational decision for risk averse agents who are well aware that life duration is uncertain. Of course, low saving levels typically result in having a majority of (surviving) elderly declaring that they failed to save enough. But this is not evidence of under-savings. If they could be questioned, those who died before retirement would surely answer that they actually saved too much.

Another message brought by our analysis is that mortality risk being the most significant risk in life, the impact of changes in mortality risks have to be considered with great attention. In particular, one should use well-behaved models that properly account for risk aversion. Preliminary exploration can be found in Bommier (**?**), but this is the whole field of economics of aging that deserve to be revisited with non-additive models. That will unavoidably brings new technical challenges, as those we addressed for our numerical exercise, but this is a cost to pay to get a better modeling of the link between economics and demography.

References

Ameriks, J., Caplin, A., Laufer, S. and Van Nieuwerburgh, S.: 2011, The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Bequest and Precautionary Motives, *Journal of Finance* **66**(2), 519–561.

Blanchard, O. J.: 1985, Debt, Deficits and Finite Horizons, *Journal of Political Economy* **93**(2), 223–247.

Bommier, A.: 2006, Uncertain Lifetime and Intertemporal Choice: Risk Aversion as a Rationale for Time Discounting, *International Economic Review* **47**(4), 1223–1246.

Bommier, A.: 2013, Life Cycle Preferences Revisited, *Journal of European Economic Association* **11**(6), 1290–1319.

Bommier, A., Chassagnon, A. and Le Grand, F.: 2012, Comparative Risk Aversion: A Formal Approach with Applications to Saving Behaviors, *Journal of Economic Theory* **147**(4), 1614–1641.

Bommier, A., Kochov, A. and LeGrand, F.: 2016, On Monotone Recursive Preferences, *mimeo*.

Bommier, A. and LeGrand, F.: 2014, A Robust Approach to Risk Aversion, *Working paper*, ETH Zurich.

Clausen, A. and Strub, C.: 2012, Envelope Theorems for Non-Smooth and Non-Concave Optimization, *Working paper*, University of St. Gallen.

De Nardi, M.: 2004, Wealth Inequality and Intergenerational Links, *Review of Economic Studies* **71**(3), 743–768.

De Nardi, M., French, E. and Jones, J. B.: 2010, Why Do the Elderly Save? The Role of Medical Expenses, *Journal of Political Economy* **118**(1), 39–75.

Drouhin, N.: 2015, A Rank-Dependent Utility Model of Uncertain Lifetime, *Journal of Economic Dynamics and Conctrol* **53**, 208–224.

Epstein, L. G. and Zin, S. E.: 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica* **57**(4), 937–969.

Flodén, M.: 2008, A Note on the Accuracy of Markov-Chain Approximations to Highly Persistent AR(1) Processes, *Economics Letters* **99**(3), 516–520.

Galindev, R. and Lkhagvasuren, D.: 2010, Discretization of Highly Persistent Correlated AR(1) Shocks, *Journal of Economic Dynamics and Control* **34**(7), 1260–1276.

Gomes, F. J. and Michaelides, A.: 2008, Asset Pricing with Limited Risk Sharing and Heterogeneous Agents, *The Review of Financial Studies* **21**(1), 415–448.

Gomes, F. and Michaelides, A.: 2005, Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence, *The Journal of Finance* **60**(2), 869–904.

Gordon, G. and Qiu, S.: 2015, A Divide and Conquer Algorithm for Exploiting Policy Function Monotonicity, *CAEPR Working Paper 2015-002*, Center for Applied Economics and Policy Research, Indiana University.

Hansen, L. P. and Sargent, T. J.: 1995, Discounted Linear Exponential Quadratic Gaussian Control, *IEEE Transactions on Automatic Control* **40**(5), 968–971.

Harenberg, D. and Ludwig, A.: 2015, Social Security in an Analytically Tractable Overlapping Generations Model with Aggregate and Idiosyncratic Risk, *CER-ETH Working Paper 14/204*, ETH Zurich.

Huggett, M., Ventura, G. and Yaron, A.: 2011, Sources of Lifetime Inequality, *American Economic Review* **101**(7), 2923–2954.

Hugonnier, J., Pelgrin, F. and St-Amour, P.: 2012, Health and (Other) Asset Holdings, *The Review of Economic Studies* **80**(2), 663–710.

Hurd, M. D. and Smith, J. P.: 2002, Expected Bequests and their Distribution, *NBER Working Paper 9142*, National Bureau of Economic Research.

Inkmann, J., Lopes, P. and Michaelides, A.: 2011, How Deep Is the Annuity Market Participation Puzzle?, *Review of Financial Studies* **24**(1), 279–319.

Johansson, P.-O.: 2002, On the Definition and Age-Dependency of the Value of a Statistical Life, *Journal of Risk and Uncertainty* **25**(3), 251–263.

Kihlstrom, R. E. and Mirman, L. J.: 1974, Risk Aversion with many Commodities, *Journal of Economic Theory* **8**(3), 361–388.

Kopecky, K. A. and Suen, R. M.: 2010, Finite State Markov-Chain Approximations to Highly Persistent Processes, *Review of Economic Dynamics* **13**(3), 701–714.

Kreps, D. M. and Porteus, E. L.: 1978, Temporal Resolution of Uncertainty and Dynamic Choice Theory, *Econometrica* **46**(1), 185–200.

Lockwood, L. M.: 2012, Bequest Motives and the Annuity Puzzle, *Review of Economic Dynamics* **15**(2), 226–243.

Lockwood, L. M.: 2014, Bequest Motives and the Choice to Self-Insure Late-Life Risks, *NBER Working Paper 20745*, National Bureau of Economic Research.

Storesletten, K., Telmer, C. I. and Yaron, A.: 2004, Consumption and Risk sharing over the Life Cycle, *Journal of Monetary Economics* **51**(3), 609–633.

Tallarini, T. D. J.: 2000, Risk-Sensitive Real Business Cycles, *Journal of Monetary Economics* **45**(3), 507–532.

Tauchen, G.: 1986, Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions, *Economics Letters* **20**(2), 177–181.

Tauchen, G. and Hussey, R.: 1991, Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models, *Econometrica* **59**(2), 371–396.

Viscusi, W. K. and Aldy, J. E.: 2003, The Value of a Statistical Life: A Critical Review of Market Estimates Throughout the World, *The Journal of Risk and Uncertainty* **27**(1), 5–76.

Wakai, K.: 2007, Risk Non-Separability without Force of Habit, *Working paper*, The State University of New York at Buffalo.