Pure Altruism and Time Inconsistency: An Axiomatic Foundation

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Motivation

- People often care about consequences of **present** decisions on **future** generations
  - parents' sacrifices for kids' education
  - bequests for descendants
  - protection of environment and natural resources
  - donations to medical research
  - balanced public finances (e.g., pension system) in long run
  - foundations of prosperous and sustainable economy
- Many models of **intergenerational altruism**
- Lack of solid foundations (exception: Koopmans' (1960) model and EDU)
  - which **assumptions** characterize those models?
  - which **properties of decisions** do they imply?

This Paper

- General axiomatic foundation of **direct pure** altruism towards future generations
  - **pure**: caring about descendants' overall **well-being** (including *their* altruism)
  - **direct**: caring about **all** descendants **directly**

- **Primitive**: observable preference of present generation ("generation 0") over infinite, deterministic consumption paths (Koopmans (1960))

- General representation

$$U(c_0, c_1, \ldots) = V(c_0, U(c_1, c_2, \ldots), U(c_2, c_3, \ldots), \ldots)$$

$U(c_t, c_{t+1}, \ldots)$ = well-being that **present generation ascribes** to generation t by "projecting" its preference onto generation t

- Koopmans' model: $U(c_0, c_1, \ldots) = V(c_0, U(c_1, c_2, \ldots))$

This Paper

- Direct pure altruism $\Rightarrow$ time **in**consistency in the form of **present bias**
- New tractable class of models based on **impartial + coherent** consideration of future generations

$$U(c_0, c_1, \ldots) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t G(U(c_t, c_{t+1}, \ldots))$$

where G = **pure-altruism utility** and $\alpha \in (0,1)$

- Implied properties
  - selfishness always dominates despite altruism
  - Bellman-like equation for dynamic-allocation problems
  - **discounting** of consumption utility u + **dependence on consumption** levels
  - G linear $\Leftrightarrow$ consumption **in**dependence + β-δ discounting
- Develops **method** to deal with well-being interdependences (widely applicable)
- **Welfare** with intergenerational altruism + existence of time-consistent planner

Related Literature

- *Intergenerational altruism + applications*: national savings (Ramsey ('28), Phelps-Pollack ('68)), growth (Bernheim-Ray ('89)), charitable giving (Andreoni ('89)), family economics (Bergstrom ('95)), public finance (Barro ('74)), environmental econ (Weitzman ('99), Dasgupta ('08), Schneider et al. ('12))
- *Representability of pure altruism in terms of u's* (Bergstrom ('99), Saez-Marti & Waibull ('05), Fels-Zeckhauser ('08))
  - u-representation $\rightarrow$ properties of consumption decisions
  - this paper: different, more general approach and answers
- *Axiomatizations of intertemporal preferences* (Koopmans ('60) ...)
  - β-δ model (Hayashi ('03), Olea-Strzalecki ('14), Echenique et al. ('14))
- *Sources of time inconsistency of preferences* (Strotz ('55), Akerlof ('91), Gul-Pesendorfer ('01), Halevy ('08), Saito ('11), Köszegi-Szeidl ('12))
- *Normative social choice* (Asheim ('10)): sensitivity to future generations' well-being, impartiality, coherence = normative appealing properties

Setup

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- Society = sequence of generations ("gens"): $t \in \{0, 1, 2, \ldots\}$
- Consumption of gen t: $c_t \in X$
- Consumption *streams/paths*: $C = X^{\mathbb{N}}$
- Consumption path from t onward: ${}_tc = (c_t, c_{t+1}, \ldots)$

Object of study: preference $\succ$ of present gen ("gen 0") over C

Interpretation: $\succ$ revealed by gen 0's choices with commitment

Classic primitive environment as in Koopmans (1960)

Setup

Standard axioms on $\succ$:
- Completeness
- Transitivity
- Continuity
- Constant-flaw dominance

$\Rightarrow$ continuous $U : C \to \mathbb{R}$ that represents $\succ$

Interpretation: $U(c) =$ total utility or **well-being** of gen 0 from path c

Axiom (Non-triviality)

There exist $x, x', \hat{x} \in X$ *and* $c, c', \hat{c} \in C$ *s. t.* $(x, \hat{c}) \succ (x', \hat{c})$ *and* $(\hat{x}, c) \succ (\hat{x}, c')$

$\to$ altruism: gen 0 cares about consumption of **some** future gen

Intergenerational Pure Altruism

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Pure (non-paternalistic) altruism $\stackrel{def}{=}$ gen 0 cares about future gens' *well-being*,
not consumption per se

If gen 0's $\succ$ exhibits this, then $\succ$ reveals gen 0's **perception** of future gens' well-being

How does this perception work?

This paper's **view**: gen 0 "projects" its $\succ$ onto future gens

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Pure-Altruism Representation

Minimal property: given c_0, if gen 0 **thinks** all future gens will be indifferent between c and c', then gen 0 is indifferent

Axiom

If ${}_tc \sim {}_tc'$ for ***all*** *$t > 0$, then $(c_0, {}_1c) \sim (c_0, {}_1c')$*

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Theorem (Pure-Altruism Representation)

Previous axioms hold iff there exists function V *such that*

$$U(c) = V(c_0, U({}_1c), U({}_2c), \ldots),$$

where V *is nonconstant in* c_0 *and* ***some*** $U({}_tc)$

Includes EDU: $U(c) = u(c_0) + \delta u(c_1) + \delta^2 u(c_2) + \ldots = u(c_0) + \delta U({}_1c)$

Terminology:
- $U(c) = V(c_0, U({}_1c), U({}_2c), \ldots) \leftrightarrow$ **direct** pure altruism
- $U(c) = V(c_0, U({}_1c)) \leftrightarrow$ **indirect** pure altruism

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Time (In)consistency

Suppose **all** gens have **same** preference $\succ$ and are purely altruistic

Definition (Time Consistency of Sequence of $\succ$)

If consumption path starting at t is preferable according to $\succ^t$, then it remains preferable, **from t onward**, according to $\succ^{t-1}$

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Remark: time consistency $\Rightarrow$ **indirect** pure altruism, i.e., $U(c) = V(c_0, U({}_1c))$

Lesson: pure altruism *beyond* immediate descendant **causes** time inconsistency

Example: grandma and son disagree on best consumption allocation
because they internalize **his** daughter's well-being differently

Which **form** of time inconsistency?

Time (Inconsistency): Present Bias

Definition ("Present Bias")

Let x be "better than" y. If $(z_0, \ldots, z_t, x, \hat{x}, c') \sim (z_0, \ldots, z_t, y, \hat{y}, c')$ for $t \geq 0$, then $(x, \hat{x}, c') \succ (y, \hat{y}, c')$

Preference $\succ$ exhibits more patience in long than in short run

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Proposition

If $U(c) = V(c_0, U({}_1c), \ldots)$ represents $\succ$ & V strictly increasing in all $U({}_tc)$, then $\succ$ exhibits present bias

Intuition:
- take grandma's viewpoint
- shift consumption from son to granddaughter
- **his** well-being $\downarrow$ for lower consumption and $\uparrow$ for her higher well-being
- grandma cares directly about granddaughter's well-being

$\Rightarrow$ grandma thinks son should shift more consumption to granddaughter than if grandma were in son's position

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Time-separable, Stationary Preferences

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Back to single preference $\succ$ of gen 0

Goal: sharper predictions + tractability + normative appeal

Main properties: *Intergenerational Separability* + *Altruism Stationarity*

Intergenerational Separability

Intuition:
- how gen 0 enjoys its consumption is independent of future gen's well-being
- how gen 0 evaluates gen t's well-being is independent of gen $\hat{t}$'s well-being (impartiality/fairness)

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Axiom (Intergenerational Separability)

Let Π consist of all unions of subsets of $\{\{1\},\{2\},\{3,4,\ldots\}\}$. Fix any $\pi \in \Pi$. If c, $\hat{c}$, c', $\hat{c}' \in C$ satisfy

(i) ${}_tc \sim {}_t\hat{c}$ and ${}_tc' \sim {}_t\hat{c}'$ for all $t \in \pi$,

(ii) ${}_tc \sim {}_tc'$ and ${}_t\hat{c} \sim {}_t\hat{c}'$ for all $t \in \mathbb{N} \setminus \pi$,

(iii) either $c_0 = c_0'$ and $\hat{c}_0 = \hat{c}_0'$, or $c_0 = \hat{c}_0$ and $c_0' = \hat{c}_0'$,
then $c \succ c'$ if and only if $\hat{c} \succ \hat{c}'$.

Like Koopmans' (1960) separability, but applied to c_0, U_1, U_2, and $(U_3, U_4, \ldots)$ rather than to c_0, c_1, and $(c_2, c_3, \ldots)$

Altruism Stationarity

Focuses on altruistic component of gen 0's preference

Idea: if gen 0 cares **directly** about gens beyond gen 1 **in coherent way**, then it should be possible to "remove" gen 1 & preserve how gen 0 ranks others' well-being

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Axiom (Altruism Stationarity)

If $c, c' \in C$ *satisfy* $c_0 = c'_0$ *and* ${}_1c \sim {}_1c'$, *then*

$$c \succsim c' \Leftrightarrow (c_0, {}_2c) \succsim (c'_0, {}_2c').$$

Intuition:
- grandma thinks **son** is overall indifferent between $(c_1, {}_2c)$ and $(c'_1, {}_2c')$
- well-being of **his** daughter, granddaughter, etc. $\Rightarrow$ grandma prefers c to c'
- if **son** dies, grandma continues to prefer ${}_2c$ to ${}_2c'$ for remaining descendants

Comparison with Koopmans' Stationarity

Axiom (Altruism Stationarity)

If $c, c' \in C$ *satisfy* $c_0 = c'_0$ *and* ${}_1c \sim {}_1c'$, *then*

$$(c_0, {}_1c) \succsim (c'_0, {}_1c') \Leftrightarrow (c_0, {}_2c) \succsim (c'_0, {}_2c')$$

vs.

Axiom (Koopmans' Stationarity)

If $c, c' \in C$ *satisfy* $c_0 = c'_0$, *then*

$$(c_0, {}_1c) \succsim (c'_0, {}_1c') \Leftrightarrow {}_1c \succsim {}_1c'$$

- gen 1's **perceived** well-being dominates altruistic component $\rightarrow$ why should it?
- implies **indirect** pure altruism: $U(c) = V(c_0, U({}_1c))$
- **different** from time consistency (it involves only one preference)

Monotonicity

(i) Grandma happier if she thinks son is happier, fixing well-being of other descendants

(ii) If grandma prefers initial part up to T of c to same part of c' *for any* T, then she prefers c to c' overall

Axiom (Monotonicity)

(i) If $c_0 = c'_0$, ${}_1c \succ {}_1c'$, *and* ${}_tc \sim {}_tc'$ *for all* $t > 1$, *then* $c \succ c'$

(ii) If for every T *and* $c'' \in C$ *we have* $(c_0, c_1, ..., c_T, c'') \succsim (c'_0, c'_1, ..., c'_T, c'')$, *then* $c \succsim c'$

Note: EDU satisfies all our axioms, **except** altruism stationarity

Additive Pure-Altruism Representation

Theorem (Additive Pure-Altruism Representation (G-representation))

Previous axioms hold iff U may be chosen so that

$$U(c) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t G(U({}_t c))$$

with u, G nonconstant & continuous, $\alpha \in (0,1)$, G strictly increasing & bounded

- uses known results in Debreu (1960) and Koopmans (1960)
- **complication**: streams of future gens' well-being $\neq$ Cartersian-product space (interdependences through altruism)
- approach may be useful for other forms of interdependences across agents

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with u, G nonconstant & continuous, $\alpha \in (0,1)$, G strictly increasing & bounded

Proposition (Characterization)

- *Given representation (1), U "continuous in tail" of c and for every v, v' in range of U,*

$$|G(v) - G(v')| < \frac{1-\alpha}{\alpha}|v - v'|$$

- *If G strictly increasing, bounded, $\frac{1-\alpha}{\alpha}$-Lipschitz, then (1) has unique, "tail continuous" solution U*

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Working with G-representation: "cake-eating" problem

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Gen 0 commits to allocation $(c_0, c_1, \ldots) \in \mathbb{R}_+^{\mathbb{N}}$ to maximize

$$U(c_0, c_1, \ldots) = u(c_0) + \sum_{t>0} \alpha^t G(U({}_t c)) \qquad \text{subject to} \qquad \sum_{t\geq 0} c_t \leq b$$

Letting $C(b) \subset C$ denote set of all feasible streams, value function given by

$$U^*(b) = \sup_{c_0 \leq b} \{u(c_0) + \alpha A(b - c_0)\}$$

where

$$A(b') = \sup_{c' \in C(b')} \sum_{t\geq 0} \alpha^t G(U({}_t c'))$$

Sufficient to solve for A...

Working with G-representation: "cake-eating" problem

For every $b \geq 0$, $A(b)$ satisfies

$$A(b) = \sup_{c_0 \leq b} \left\{ \sup_{c' \in C(b-c_0)} \left\{ G\left(u(c_0) + \alpha \sum_{t \geq 0} \alpha^t G(U({}_t c')) \right) + \alpha \sum_{t \geq 0} \alpha^t G(U({}_t c')) \right\} \right\}$$

which yields the following Bellman-like equation for A:

$$A(b) = \sup_{c_0 \leq b} \{ G(u(c_0) + \alpha A(b - c_0)) + \alpha A(b - c_0) \}$$

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Analysis of equilibrium without commitment is harder, but feasible too (Ray ('87), Bernheim & Ray ('89), Harris & Laibson ('01))

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Let c, c' be identical except that $c_0 = c'_t = x$ and $c_t = c'_0 = y$ with $u(x) > u(y)$.
Then $\succ$ exhibits selfishness if $c \succ c'$

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Corollary

G-representation $\Rightarrow$ $\succ$ *exhibits selfishness*

With **finite** horizon, possible to choose $\alpha < 1$ and G so that gen 0 **willing to sacrifice** own consumption for benefit of descendants *close in lineage* (interior optimum)

EDU with $\delta > 1 \Rightarrow$ sacrifice for benefit of *last* generation (corner optimum)

Intergenerational Discounting

Intergenerational Discounting

Corollary ("u-representation")

G-representation $\Rightarrow$ there exists $\overline{U}$ such that $U(c) = \overline{U}(u(c_0), u(c_1), \ldots)$ for all $c \in C$

Marginal rate at which gen 0 substitutes **consumption utility** between itself and gen t

$$d(t,c) = \frac{\partial \overline{U}(u_0, u_1, \ldots)/\partial u_t}{\partial \overline{U}(u_0, u_1, \ldots)/\partial u_0}$$

EDU model: $d(t,c) = \delta^t$

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Proposition (Intergenerational discount function)

G-representation + differentiability of G $\Rightarrow$

$$d(t,c) = \alpha^t G'(U({}_tc)) \left[1 + \sum_{\tau=1}^{t-1} G'(U({}_{t-\tau}c)) \prod_{s=1}^{\tau-1} (1 + G'(U({}_{t-s}c))) \right]$$

Intergenerational Discounting

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depends on **intermediate** consumption: gen 0 thinks intermediate gens are also altruistic

depends on consumption **after** gen t: gen 0 anticipates gen t's altruism

Corollary

Suppose c, c' satisfy $u(c_t) \geq u(c'_t)$ for all $t > 0$. Then, $d(t,c) \leq (\geq)\ d(t,c')$ for all $t > 0$ if and only if G' is decreasing (increasing)

Suppose G' is decreasing:

- gen 0 learns future living standards won't improve as expected $\Rightarrow$ **more** willing to sacrifice own satisfaction to improve that of future gens
- gen 0 prefers *well-being* smoothing across gens

Intergenerational Discounting

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β-δ Discounting ("Imperfect" Altruism)

Linear $G'(U) = \gamma \Rightarrow d(t, c)$ independent of c

$$\Rightarrow d(t, c) = \beta\delta^t \text{ with } \beta = \frac{\gamma}{1+\gamma} \text{ and } \delta = (1+\gamma)\alpha < 1$$

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Axiom (Consumption Independence)

(i) $(c_0, c_1, {}_2c) \succ (c_0', c_1', {}_2c)$ *if and only if* $(c_0, c_1, {}_2c') \succ (c_0', c_1', {}_2c')$;
(ii) $(c_0, c_1, {}_2c) \succ (c_0', c_1, {}_2c')$ *if and only if* $(c_0, c_1', {}_2c) \succ (c_0', c_1', {}_2c')$.

i) $MRS(\text{grandma}, \text{son})$ independent of consumption of son's descendants
ii) $MRS(\text{grandma}, \text{descendants})$ independent of son's consumption

Theorem (Linear Pure-Altruism Representation)

Previous axioms hold if and only if there exists $\gamma \in (0, (1-\alpha)/\alpha)$ *such that*

$$U(c) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t \gamma U({}_tc)$$

$\gamma \approx$ how vivid well-being of **any** future gen is for gen 0

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$$U(c) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t \gamma U({}_t c) \qquad \Leftrightarrow \qquad U(c) = u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t)$$

where $\delta = (1+\gamma)\alpha < 1$ and $\beta = \gamma/(1+\gamma) < 1$

- Axiomatization of Phelps and Pollack's (1968) "imperfect" altruism:
  - gen 0 cares about its consumption and future gens' *well-being*
  - gen 0 takes into account future gens' altruism

    $\rightarrow$ *they* should be more generous towards their descendants (present-bias)

    $\rightarrow$ gen 0 treats **all** future gens' u in a uniformly different way ($\beta < 1$)
  - gen 0 treats future generations with impartiality and coherence
- New axiomatization of Laibson's ('97) quasi-hyperbolic discounting model

β-δ Discounting ("Imperfect" Altruism)

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  - gen 0 cares about its consumption and future gens' *well-being*
  - gen 0 takes into account future gens' altruism

    $\rightarrow$ *they* should be more generous towards their descendants (present-bias)

    $\rightarrow$ gen 0 treats **all** future gens' u in a uniformly different way ($\beta < 1$)
  - gen 0 treats future generations with impartiality and coherence
- New axiomatization of Laibson's ('97) quasi-hyperbolic discounting model

Welfare with Intergenerational Altruism

Welfare with Intergenerational Altruism

Direct pure altruism $\Rightarrow$ time-inconsistent preferences

- Justification for paternalism?
  No: time inconsistency not "irrationality" but logical consequence of richer altruism
- Time consistency vs. other normatively appealing properties?
  Our axioms isolate and highlight
  - gen 0 sensitive to well-being of gens **beyond** its immediate descendant
  - intergenerational separability (fairness)
  - altruism stationarity (coherence)
- Democratic governments may respond only to preference of gen 0
  $\rightarrow$ welfare properties of governments' decisions? shortcomings?

Welfare with Intergenerational Altruism

- EDU → **usual** welfare criterion: $U(c) = \sum_{t=0}^{\infty} \delta^t u(c_t)$ (gen 0's pref)

- This "libertarian" criterion may be **more** appropriate with direct pure altruism (despite time inconsistency): takes account of well-being of **all** future generations

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Summary

- Study of how direct pure altruism **shapes** each generation's preference

- **Axiomatic foundation** based on properties of revealed preference of present generation over infinite consumption paths

- Direct pure altruism **naturally causes** time inconsistency in the form of present bias

- New class of models founded on **impartial and coherent treatment** of **all** future generations' well-being (tractability and also normative appeal)

- New **characterization** of β-δ discounting (Phelps-Pollack ('68) and Laibson ('97))

- Rigorous treatment of delicate issue of how to conduct **welfare analysis** when generations' preferences are **time inconsistent**

- Possible **single-agent** interpretation: gen $t =$ **self** t (Strotz ('55), Frederick ('02))

Thank you!