Social Security and the Interactions between Aggregate & Idiosyncratic Risk

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Motivation

- Question: Welfare effects of expanding PAYG system?
- Trade-off: Insurance vs. crowding out
- Social security as insurance against
  - Idiosyncratic risk (e. g., Imrohoroglu, et al. (1999))

    AND

  - Aggregate risk (e. g., Krueger & Kubler (2006))

Interactions

1. Life-cycle interaction (*LCI*)
   - Idiosyncratic wage and aggregate return shocks increase variance of savings
   - Variance of retirement consumption increases
   - Interaction term: *LCI*
   - *LCI* large, because long time horizon until retirement
2. Counter-cyclical variance of income risk (*CCV*)
   - Idiosyncratic risk higher in downturn than in boom
   - Mankiw(1986), Storesletten, et al. (2004)

Approach

- Two-generations model: Main mechanisms
- Quantitative overlapping generations model
- Calibration to the U. S.
- Experiment: Increase social security contributions from 0% to 2%
- Decomposition analysis: Quantify insurance against various sources and interactions
- Robustness and replication of previous literature

Main Results

- Analytically: life-cycle interaction *LCI*
- Positive welfare gains across all calibrations
- Interaction terms ($LCI + CCV$) account for 50-60%
- E. g. baseline calibration (most conservative):
  - Welfare gains in GE: $+1.4\%$
  - Benefits from insurance: $+3.8\%$
  - Losses from crowding out: -2.4%
  - Interactions account for 1/2 of benefits

Two-Generations Model: Households

- Households live 2 periods, consume only when old
- Lifetime utility:

$$U_{i,t} = \beta \frac{1}{1-\theta} \; c_{i,2,t+1}^{1-\theta}$$

- Budget constraint:

$$c_{i,2,t+1} = a'_{i,1,t}(1 + r_{t+1}) + b_{t+1}$$
$$a'_{i,1,t} = (1-\tau)\eta_{i,1,t} w_t$$

Two-Generations Model: Endowments

- Partial equilibrium factor prices:

$$1 + r_t = \varrho_t \bar{R}$$

$$w_t = \zeta_t \bar{w}_t = \zeta_t \bar{w}_{t-1}(1+g)$$

- PAYG social security:

$$b_t = \tau w_t$$

- Distribution: jointly log-normal, mean one, independent

Two-Generations Model: Main Result

Proposition

A marginal introduction of social security increases $E_{t-1}U_t$ if

$$(1+g)\cdot(1+V)^{\theta} > \bar{R},$$

where

$$V \equiv var(\eta_{i,1,t}\zeta_t\varrho_{t+1}) = \underbrace{\sigma_\eta^2}_{IR} + \underbrace{\sigma_\zeta^2 + \sigma_\varrho^2 + \sigma_\zeta^2\sigma_\varrho^2}_{AR} + \underbrace{\sigma_\eta^2\left(\sigma_\zeta^2 + \sigma_\varrho^2 + \sigma_\zeta^2\sigma_\varrho^2\right)}_{LCI=IR\cdot AR}.$$

Two-Generations Model: Welfare Decomposition

Definition

- Consumption equivalent variation, $g_c(\cdot)$:

$$\begin{aligned} g_c(IR) &= g_c(0) + dg_c(IR) \\ g_c(AR) &= g_c(0) + dg_c(AR) \\ g_c(AR, IR) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI) \end{aligned}$$

- First-order Taylor series approximation of $g_c(AR, IR)$ gives:

$$g_c(AR, IR) \approx \underbrace{\frac{1+g}{\bar{R}} - 1}_{g_c(0)} + \underbrace{\theta \frac{1+g}{\bar{R}} AR}_{dg_c(AR)} + \underbrace{\theta \frac{1+g}{\bar{R}} IR}_{dg_c(IR)} + \underbrace{\theta \frac{1+g}{\bar{R}} LCI}_{dg_c(LCI)}$$

Quantitative Model: Summary

1. Scale-up and extend simple model: ▸ Show details
   (a) 70 generations, 1-year periods
   (b) Population growth
   (c) Wage shocks ⇒ TFP shocks
   (d) Return shocks ⇒ depreciation shocks
   (e) (Auto-)correlation (TFP, depreciation) unrestricted
   (f) Idiosyncratic risk: autocorrelated, *CCV*
   (g) Deterministic age-income profile
   (h) Epstein-Zin preferences
2. Additional elements: ▸ Show details
   (a) Two assets: risk-free bond in addition to risky stock
   (b) Representative firm with capital structure
3. General equilibrium

Quantitative Model: Equilibrium and Solution

- Competitive recursive equilibrium: ▸ Show details

  competitive prices $\{r, r_f, w\}$, optimal household choices $\{c, a', \kappa\}$ and firm choices $\{K, L\}$, market clearing, soc. sec. budget balance $\{\tau, b\}$, law of motion

- Law of motion (Krusell & Smith (1997)):

  (i) capital stock, (ii) equity premium ▸ Show details

- Simulation periods > 80.000
- Endogenous grid method (Carroll (2006)) ▸ Show details
- Parallel on 16 cores, computation time 20 - 80 hrs

Quantitative Model: Baseline Calibration

Parameter	*Target (Source)*	*Value*
Working age, retirement age, maximum age		21, 65, 78
Age productivity	earnings profiles (PSID)	$\{\epsilon_j\}_1^J$
Population growth, n	U.S. Social Sec. Adm. (SSA)	0.011
Technol. growth, g	TFP growth (NIPA)	0.018
Capital share, α	wage share (NIPA)	0.32
Leverage ratio, d	U.S. capital structure (Croce (2010))	0.66
Autocorrelation of η	(Storesletten, et al. (2004))	0.952
CCV, $\sigma_{\nu,t}$	(Storesletten, et al. (2004))	$\{0.21, 0.13\}$
EIS, φ	exogenous (various)	0.5
CRRA, θ	exogenous in baseline	3.0
Discount factor, β	$K/Y = 2.65$ (NIPA)	0.986
Mean depreciation, $\bar{\delta}$	$E(r_f) = 2.3\%$ (Shiller)	0.10
Std. depreciation, σ_δ	$\sigma(\frac{C_{t+1}}{C_t}) = 0.03$ (NIPA)	0.08
Std. TFP shocks, σ_ζ	$\sigma(TFP) = 0.029$ (NIPA)	0.029
Prob($\zeta' = \zeta_i \| \zeta = \zeta_i$)	$autoc(TFP) = 0.88$ (NIPA)	0.941
Prob($\delta' = \delta_i \| \zeta' = \zeta_i$)	$cor(TFP, r) = 0.50$ (NIPA, Shiller)	0.885

Results: General Equilibrium

- Experiment: $\tau = 0\% \rightarrow \tau = 2\%$, unanticipated
- g_c: ex-ante expected CEV of a newborn

	GE
g_c	+1.38%
$\Delta K/K$	-10.42%
Δr	+0.88%
Δr_f	+0.89%
$\Delta w/w$	-3.47%

Results: Partial Equilibrium

- PE: "Small open economy"
- Prices same as in GE, determined by "world"
- No costs of crowding out, isolates benefits

	GE	PE	Crowd Out
g_c	+1.38%	+3.76%	-2.38%

Results: Decomposition Procedure

- Same PE experiment
- Sequentially "turn off" each risk
- Look at welfare change for each economy
- Recall decomposition of CEV:

$$
\begin{aligned}
g_c(AR, IR, CCV) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI) + dg_c(CCV) \\
g_c(AR, IR) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI) \\
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Results: Decomposition of Welfare Effects

Welfare effects in PE

g_c	$g_c(0)$	$dg_c(AR)$	$dg_c(IR)$	$dg_c(LCI)$	$dg_c(CCV)$
3.76% =	-0.62%	+1.86%	+0.66%	+1.06%	+0.80%

- Gains from "pure" AR + IR: $dg_c(AR) + dg_c(IR) = 2.52\%$
- Gains from interactions: $dg_c(LCI) + dg_c(CCV) = 1.86\%$
- $\frac{dg_c(LCI)+dg_c(CCV)}{g_c} = 0.50$
- $\frac{dg_c(LCI)}{dg_c(AR)} = 0.57$

Results: Overview of Calibration Strategies

1) IES=0.5
   i) Conservative baseline
   ii) Sharpe ratio
   iii) Equity premium
2) IES=1.5
   i) Conservative baseline
   ii) Sharpe ratio
   iii) Equity premium
3) Alternative calibrations
   i) Contribution rate $\tau = 0.12$
   ii) Mortality risk
   iii) Previous literature

Results: GE and PE Welfare across Calibrations

Consumption equivalent variation, g_c

	GE	PE	Crowd Out
$IES = 0.5$			
Baseline	+1.38%	+3.76%	-2.38%
Sharpe ratio	+1.54%	+4.56%	-3.02%
Equity premium	+1.46%	+4.19%	-2.73%
$IES = 1.5$			
Baseline	+1.78%	+2.53%	-0.75%
Sharpe ratio	+2.05%	+4.28%	-2.23%
Equity premium	+2.19%	+4.44%	-2.25%

Results: Decomposition of Welfare Effects

Welfare effects in PE

	$g_c(0)$	$dg_c(AR)$	$dg_c(IR)$	$dg_c(LCI)$	$dg_c(CCV)$
$IES = 0.5$					
Baseline	-0.62%	+1.86%	+0.66%	+1.06%	+0.80%
Sharpe ratio	-0.62%	+1.52%	+1.13%	+1.23%	+1.30%
Equity prem.	-0.62%	+1.43%	+0.98%	+0.99%	+1.41%
$IES = 1.5$					
Baseline	-0.62%	+1.28%	+0.60%	+0.81%	+0.45%
Sharpe ratio	-0.62%	+1.39%	+1.15%	+1.16%	+1.20%
Equity prem.	-0.62%	+1.44%	+0.95%	+1.15%	+1.52%

Results: Welfare Ratios across Calibrations

Welfare ratios

	$\frac{dg_c(LCI)}{dg_c(AR)}$	$\frac{dg_c(LCI)+dg_c(CCV)}{dg_c(AR)+dg_c(IR)}$	$\frac{dg_c(LCI)+dg_c(CCV)}{g_c}$
$IES = 0.5$			
Baseline	0.57	0.74	0.50
Sharpe ratio	0.81	0.95	0.55
Equity premium	0.69	1.00	0.57
$IES = 1.5$			
Baseline	0.63	0.67	0.50
Sharpe ratio	0.84	0.93	0.55
Equity premium	0.80	1.12	0.60

Results: Contribution Rate and Mortality Risk

- Contribution rate, $\tau = 0.12$
  - GE welfare: $+1.1\%$
  - Similar pattern, but smaller numbers
  - $\frac{dg_c(LCI)+dg_c(CCV)}{g_c} = 0.28$
- Mortality risk (*preliminary*)
  - Survival rates from HMD, same expected lifetime
  - Accidental bequests to newborn
  - Need CRRA < 1
  - GE welfare: $+7.3\%$

Results: Consistency with Previous Literature

- Calibration strategy
- Only idiosyncratic risk
  - GE welfare: -1.35%
- Only aggregate risk
  - Not yet computed

Conclusion

- Introduction of social security leads to robust welfare gains in GE
- Interaction terms account for at least 1/2 of the benefits
- Social security provides more insurance against aggregate risk than against idiosyncratic income risk
- The larger the social security system, the smaller the welfare gains
- Life-cycle interaction *LCI* exposed in theoretical model

Outlook: Directions for Future Research

- Companion paper: analytical GE extension ▸ Show details
- Endogenous labor
- Optimal size and/or structure of social security
- Government debt / buffer in pension system

Appendix overview I

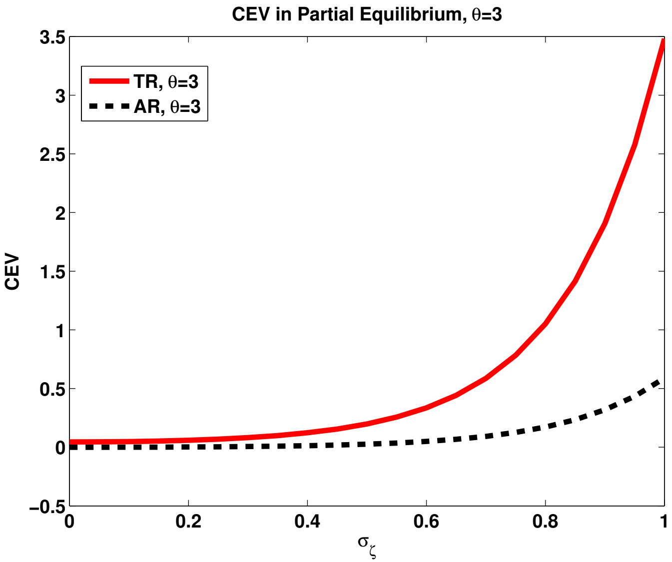
1. Related Literature
2. Two-Generations Model: GE extension
3. Quantitative Model: Market Structure
4. Quantitative Model: Demographics
5. Quantitative Model: Preferences
6. Quantitative Model: Endowments
7. Quantitative Model: Firms
8. Quantitative Model: Government and social security
9. Quantitative Model: Transformations and definitions
10. Stationary recursive competitive equilibrium
11. Quantitative Model: Household Problem

Appendix overview II

Related Literature

- Quantitative OLG (e.g. Auerbach and Kotlikoff (1987))
- Idiosyncratic risk (e. g. Conesa and Krueger (1999), Imrohoroğlu, Imrohoroğlu, and Joines (1995),Fehr, Habermann, and Kindermann (2008))
- Aggregate risk (e. g. Krueger and Kubler (2006), Bohn (1998))
- Portfolio choice, reasonable equity premium (e.g. Gomes and Michaelides (2008))
- Counter-cyclical variance of income risk / CCV (e.g. Storesletten, Telmer, and Yaron (2007), Constantinides and Duffie (1996), Mankiw (1986)

Two-Generations Model: Welfare Illustration



Two-Generations Model: GE Extension

- General equilibrium (work in progress)
  - Production economy (Cobb-Douglas)
  - Savings in first period
  - Additional assumptions: log utility, 100% depreciation
- Two additional channels:
  - Precautionary savings
  - Crowding out
- Impact of (interaction of) risks on these two channels

Go to appendix

Two-Generations Model: GE Extension

- General equilibrium extension:
  - Savings in first period
  - Idiosyncratic risk in second period (subperiod structure)
- Two additional effects:

Mechanism	Welfare Effect	Interaction
Precautionary savings	positive	positive
Crowding-out	negative	negative

▸ Go back

Two-Generations Model in GE: Households

- Utility:

$$E_t U_t = u(c_{i,1,t}) + \beta E_t \left[u(c_{i,2,t+1}) \right]$$

- Budget constraints:

$$c_{i,1,t} + s_{i,1,t} = (1 - \tau) w_t$$

$$c_{i,2,t+1} = s_{i,1,t}(1 + r_{t+1}) + \lambda \eta_{i,2,t+1} w_{t+1} (1 - \tau) +$$

$$+ (1 - \lambda) b_{t+1}$$

Two-Generations Model in GE: Government

- Budget constraint:

$$b_t(1-\lambda)N_{2,t} = \tau w_t\,(1+\lambda)\,N_{1,t}, \text{ because } N_{2,t} = N_{1,t},$$

- Therefore:

$$b_t = \tau w_t \frac{1+\lambda}{1-\lambda}.$$

Two-Generations Model in GE: Firms

- Profits:

$$\Pi = \zeta_t F(K_t, \Upsilon_t L_t) - (\bar{\delta} + r_t)\varrho_t^{-1} K_t - w_t L_t$$

- Production function:

$$F(K_t, \Upsilon_t L_t) = K_t^{\alpha} \left(\Upsilon_t L_t\right)^{1-\alpha},$$

- First-order conditions:

$$1 + r_t = \alpha k_t^{\alpha-1} \zeta_t \varrho_t = \bar{R}_t \zeta_t \varrho_t$$

$$w_t = (1-\alpha)\Upsilon_t k_t^{\alpha} \zeta_t = \bar{w}_t \zeta_t.$$

Two-Generations Model in GE: More Assumptions

1. Log utility: $u(c) = \ln(c)$
2. 100% depreciation: $\bar{\delta} = 1$

Two-Generations Model in GE: Equilibrium

Proposition

Equilibrium dynamics in the economy are given by

$$k_{t+1} = \frac{1}{(1+g)(1+\lambda)} \chi (1-\tau)(1-\alpha)\zeta_t k_t^\alpha$$

where the savings rate χ is given by

$$\chi \equiv \frac{\beta \bar{E}}{1+\beta \bar{E}} = \frac{1}{1+\left(\beta \bar{E}\right)^{-1}}$$

and

$$\bar{E} \equiv E_t \left[\frac{1}{1 + \frac{1-\alpha}{\alpha(1+\lambda)\varrho_{t+1}} \left(\lambda \eta_{i,2,t+1} + \tau \left(1 + \lambda(1-\eta_{i,2,t+1})\right)\right)} \right]$$

Two-Generations Model in GE: MSE

Definition

Mean shock equilibrium (MSE): $\zeta_t = E\zeta_t = 1$, $\varrho_t = E\varrho_t = 1 \ \forall t$.

Equilibrium dynamics:

$$k_{t+1,ms} = \frac{1}{(1+g)(1+\lambda)}\chi(1-\tau)(1-\alpha)k_{t,ms}^{\alpha}$$

Definition

Stationary MSE (=stochastic steady state): all variables grow at constant rates: $k_{t,ms} = k_{ms}$ for all t.

$$k_{ms} = \left(\frac{1}{(1+g)(1+\lambda)}\chi(1-\tau)(1-\alpha)\right)^{\frac{1}{1-\alpha}}.$$

Two-Generations Model in GE: Welfare

Proposition

Marginal introduction of social security increases ex-ante expected utility in the long-run MSE iff

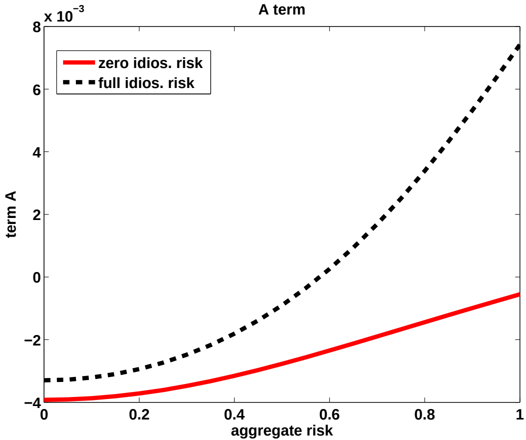
$$A + B + C > 0$$

$$A \equiv \beta E_{t-1}\left[\frac{\frac{(1-\alpha)}{\alpha}\frac{1}{\zeta_t\varrho_{t+1}} - \frac{(1-\alpha)\lambda}{\alpha(1+\lambda)}\frac{\eta_{i,2,t+1}}{\zeta_t\varrho_{t+1}} - 1}{1 + \frac{(1-\alpha)\lambda}{\alpha(1+\lambda)}\frac{\eta_{i,2,t+1}}{\zeta_t\varrho_{t+1}}}\right] - 1 \gtrless 0$$

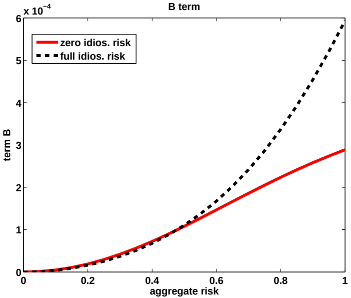
$$B \equiv \beta \underbrace{\epsilon_{\chi,\tau}}_{<0} \underbrace{\left(\bar{\bar{\bar{E}}} - \bar{E}\right)}_{\leq 0} \geq 0$$

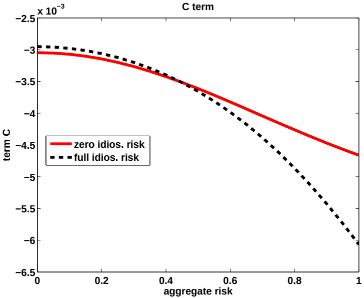
$$C \equiv -\underbrace{\left(\alpha(1+\beta) - \beta(1-\alpha)\bar{\bar{\bar{E}}}\right)}_{>0} \frac{1}{1-\alpha} \underbrace{(1-\epsilon_{\chi,\tau})}_{>0} < 0$$

Two-Generations Model in GE: Term A



Two-Generations Model in GE: Terms *B&C*





Quantitative Model: Market Structure

- Discrete time $t = 0, \ldots, \infty$
- Aggregate shock z_t: Markov chain with $\pi(z_{t+1} \mid z_t)$
- Event tree $z^t = (z_0, z_1, ..., z_t)$
- Incomplete markets
  - Bond: one-period risk-free at known interest rate r^f_{t+1}
  - Stock: risky return r_{t+1}
- Natural borrowing limit

Quantitative Model: Demographics

- J overlapping generations, indexed by $j = 1, \ldots, J$
- Retirement age jr
- Survival probabilities s_{j+1}
- Accidental bequests are burned
- Population grows at rate n
- Continuum of agents in each generation
- Intragenerational heterogeneity denoted by i

Quantitative Model: Preferences

- Epstein-Zin preferences:

$$U_{i,j,t} = \left[c_{i,j,t}^{\frac{1-\theta}{\gamma}} + \beta s_{j+1} \left(\mathbb{E}\left[U_{i,j+1,t+1}^{1-\theta} \right] \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{1-\theta}}$$

- θ: Coefficient of relative risk-aversion
- φ: Elasticity of intertemporal substitution
- $\gamma = \frac{1-\theta}{1-\frac{1}{\varphi}}$

Quantitative Model: Endowments

- Dynamic budget constraint:

$$a'_{i,j,t} + c_{i,j,t} = a_{i,j,t}(1 + r_t^f + \kappa_{i,j-1,t-1}(r_t - r_t^f)) + y_{i,j,t}$$

$$\text{with} \quad a'_{i,J,t} \geq 0$$

- Income:

$$y_{i,j,t} = \begin{cases} (1-\tau)\,\eta_{i,j,t}\,w_t\,\epsilon_j & \text{for } j < j_{ret} \\ b_t & \text{for } j \geq j_{ret} \end{cases}$$

- Idiosyncratic stochastic component:

$$\ln \eta_{i,j,t} = \rho \ln \eta_{i,j-1,t-1} + \nu_{i,t}, \qquad \sigma_\nu^2(contr) > \sigma_\nu^2(expans)$$

Quantitative Model: Firms

- Neoclassical production:

$$Y_t = F(\zeta_t, K_t, L_t) = \zeta_t K_t^{\alpha} (\Upsilon_t L_t)^{1-\alpha}$$

- Wage rate:

$$w_t = \zeta_t (1-\alpha) k_t^{\alpha} (1+g) \Upsilon_{t-1}$$

- Net return on capital:

$$r_t^k = \zeta_t \alpha k_t^{\alpha-1} - \delta_t$$

- Leveraged stock return:

$$r_t = r_t^k (1+d) - d r_t^f$$

Quantitative Model: Government and social security

- Government collects accidental bequests and burns them
- PAYG budget constraint:

$$\tau_t w_t L_t = Ret_t \int P_{i,j,t} d\Phi$$

- For today:
  - fixed contribution rate: $\tau_t = \tau$
  - lump-sum benefits: $P_{i,j,t} = P_t$
- Experiment: single, unanticipated increase in τ

Quantitative Model: Transformations and definitions

- Rewrite in terms of cash at hand, x

$$x = a(1 + r^f + \kappa(r - r^f)) + y$$

- Denote measure over agents by $\Phi_t(j, x, \eta)$
- State space for each agent: $\mathcal{S} = (j, x, \eta, z, \Phi)$

Stationary recursive competitive equilibrium

- Price functions $\{r(\Phi, z), r^f(\Phi, z), w(\Phi, z)\}$
- Policy functions $c(\mathcal{S})$, $a'(\mathcal{S})$, $\kappa(\mathcal{S})$ that maximize the household's utility for given $\{r, r^f, w, \tau, b\}$
- Firm choice k that maximizes profits for given $\{r, r^f, w\}$
- Govt policies $\tau(\Phi, z), b(\Phi, z)$ implying budget balance
- Market clearing, in particular:

$$k'(\Phi', z') = \int a'(\mathcal{S})\, d\Phi(j, x, \eta)$$

$$B'(\Phi', z') = \int (1 - \kappa(\mathcal{S}))\, a'(\mathcal{S})\, d\Phi(j, x, \eta)$$

- A law of motion $\Phi' = H(\Phi, z, z')$ consistent with policies

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Quantitative Model: Household problem

- Euler equations

$$c^{\frac{1-\theta-\gamma}{\gamma}} - \tilde{\beta} \left(\mathbb{E} \left[u(j+1,\cdot)^{1-\theta} \right] \right)^{\frac{1-\gamma}{\gamma}} \dots$$
$$\cdot\, \mathbb{E} \left[u(j+1,\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}} (c')^{\frac{1-\theta-\gamma}{\gamma}} \tilde{R}' \right] = 0$$
$$\mathbb{E} \left[u(j+1,\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}} (c')^{\frac{1-\theta-\gamma}{\gamma}} \left(r' - r^{f'} \right) \right] = 0$$

- Endogenous grid method (Carroll 2006) applied to portfolio choice
  - Avoid collinear problem of jointly finding $\{a', \kappa\}$
  - Reduce 2-dimensional optimization to 2 sequential steps: first solve for κ, then for c

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Quantitative Model: Laws of motion

- Problem: agents need measure Φ to forecast prices
- Krusell and Smith (1998): approximate $\Phi' = H(\Phi, z, z')$ by low-dimensional object
- Our approximation is

$$(k', \mu) = \hat{H}(k, k^2, z)$$

where $\mu = \mathbb{E}r' - r^{f\prime}$, the expected equity premium

- To find $\hat{H}$, need to simulate and update until convergence
- Fit: $R^2 = 0.9999$

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Quantitative Model: Mean shock equilibrium

- Auxiliary general equilibrium
- Degenerate laws of motion: $(k', \mu') = k, \mu$
- Solve household problem for all z
- Instead of simulating, set $z = \bar{z}$, with $\zeta(\bar{z}) = \mathbb{E}\zeta$ and $\delta(\bar{z}) = \mathbb{E}\delta$
- Find fixed point: (k', μ') is generated by $a'(\mathcal{S}, k, \mu)$ and $\kappa'(\mathcal{S}, k, \mu)$
- Can use $k_{ms}, \mu_{ms}, \Phi_{ms}$ as initial guesses for KS method

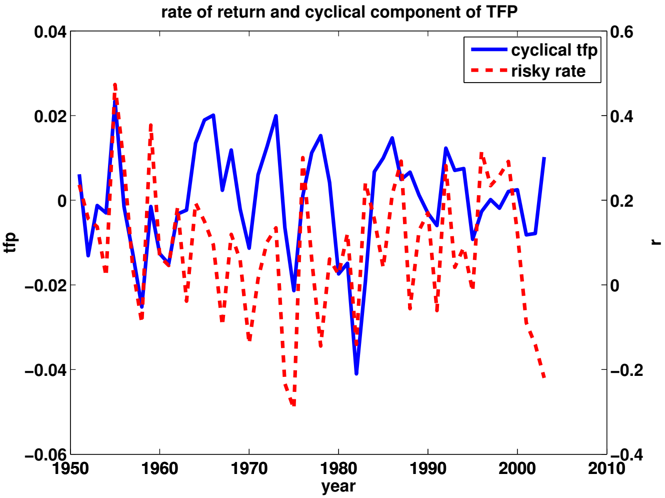
Quantitative Model: Transition Matrix $\pi(z'|z)$

- $\pi^{\zeta} = \pi(\zeta' = 1 - \bar{\zeta} \mid \zeta = 1 - \bar{\zeta})$
- $\pi^{\delta} = \pi(\delta' = \delta_0 + \bar{\delta} \mid \zeta' = 1 - \bar{\zeta}) = \pi(\delta' = \delta_0 - \bar{\delta} \mid \zeta' = 1 + \bar{\zeta})$
- both symmetric

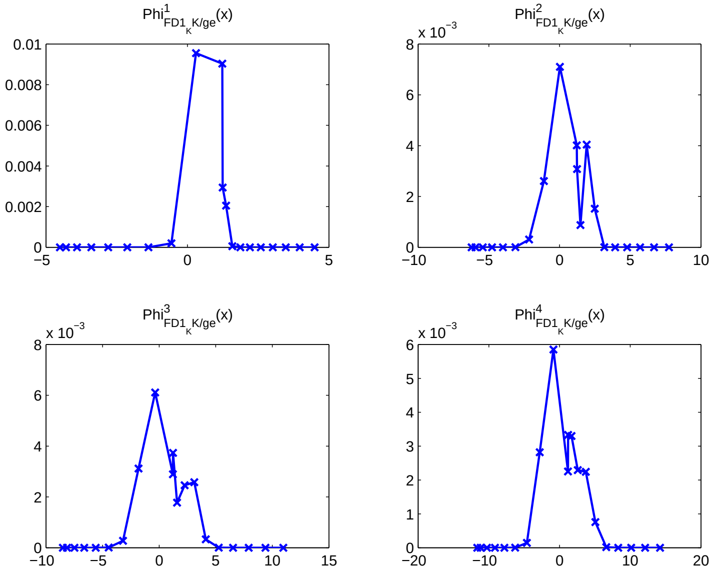
$$\pi^{z} = \begin{bmatrix} \pi^{\zeta}\cdot\pi^{\delta} & \pi^{\zeta}\cdot(1-\pi^{\delta}) & (1-\pi^{\zeta})\cdot(1-\pi^{\delta}) & (1-\pi^{\zeta})\cdot\pi^{\delta} \\ \pi^{\zeta}\cdot\pi^{\delta} & \pi^{\zeta}\cdot(1-\pi^{\delta}) & (1-\pi^{\zeta})\cdot(1-\pi^{\delta}) & (1-\pi^{\zeta})\cdot\pi^{\delta} \\ (1-\pi^{\zeta})\cdot\pi^{\delta} & (1-\pi^{\zeta})\cdot(1-\pi^{\delta}) & \pi^{\zeta}\cdot(1-\pi^{\delta}) & \pi^{\zeta}\cdot\pi^{\delta} \\ (1-\pi^{\zeta})\cdot\pi^{\delta} & (1-\pi^{\zeta})\cdot(1-\pi^{\delta}) & \pi^{\zeta}\cdot(1-\pi^{\delta}) & \pi^{\zeta}\cdot\pi^{\delta} \end{bmatrix}$$

- STY: $\pi^{\delta} = 1$
- GM: $\pi^{\delta} = 0.5$
- Our paper: $\pi^{\delta} = 0.7$

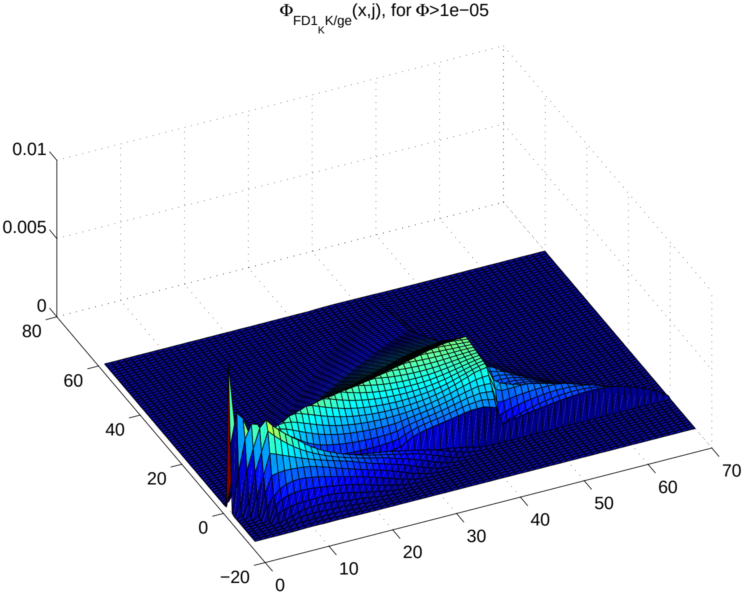
Quantitative Model: Correlation of TFP and Returns



Results: Distribution over Age and Cash-at-Hand



Results: Distribution over Age and Cash-at-Hand



Results: Economy without Aggregate Risk

- Only one asset
- Empirical average asset return (Siegel (2002)): 4.2%
- Model average asset returns

Equity premium calibration

Median portfolio return	3.07%
E(mpk)	4.70%
Capital-structure weighted average of $E(r)$ and $E(r_f)$	5.24%

- Comparable and consistent results

Show table

Results: Different average returns

Equity premium calibration with mortality

	$Median(pfr) = 3.07\%$	$E(mpk) = 4.70\%$	$mpk\left(E(r), E(r_f)\right) = 5.24\%$
$g(0, IR)$	1.360%	-0.099%	-0.438%
$g(0, 0)$	-0.102%	-0.912%	-1.101%
$dg(AR)$	1.066%	1.876%	2.066%
$dg(IR)$	1.461%	0.813%	0.664%
$dg(LCI)$	0.949%	1.598%	1.747%
$\mathbf{dg(LCI)/dg(AR)}$	**0.891**	**0.851**	**0.846**
$dg(AR) + dg(IR)$	2.528%	2.690%	2.730%
$dg(LCI) + dg(CCV)$	2.608%	3.256%	3.406%
$\mathbf{\frac{dg(LCI)+dg(CCV)}{g_c}}$	**0.294**	**0.367**	**0.384**

- $mpk\left(E(r), E(r_f)\right) = \frac{E(r)+\overline{d}\cdot E(r_f)}{1+\overline{d}}$

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Results: NC Calibration

	PC	NC
Target		
Corr. TFP, R, $corr(\zeta_t, R_t)$	0.50	-0.08
Main parameter		
Cond. prob. depr. shocks, π^δ	0.86	0.435
Adjustments		
Discount factor, β	0.97	0.96
Relative risk aversion, θ	8	12
Small adjustments in $\bar{\delta}, \sigma_\delta$		
Endogenous moments		
Corr. w, R, $corr(w_t, R_t)$	0.306	-0.33

Results: PC vs NC welfare

Welfare gains

	PC	NC
	3.52%	0.51%
$g_c(0,0)$	-2.00%	-2.00%
$dg_c(AR)$	3.26%	2.18%
$dg_c(IR)$	1.00%	1.04%
$dg_c(LCI)$	1.66%	0.14%
$dg_c(CCV)$	1.77%	0.47%

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