

EXPECTATIONS-BASED REFERENCE-DEPENDENT LIFE-CYCLE CONSUMPTION*

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Abstract

I incorporate expectations-based reference-dependent preferences into a dynamic stochastic model to explain three major life-cycle consumption facts; the intuitions behind these three implications constitute novel connections between recent advances in behavioral economics and prominent ideas in the macro consumption literature. First, expectations-based loss aversion rationalizes excess smoothness and sensitivity in consumption, the puzzling empirical observation of lagged consumption responses to income shocks. Intuitively, in the event of an adverse shock, the agent delays painful cuts in consumption to allow his reference point to decrease. Second, the preferences generate a hump-shaped consumption profile. Early in life, consumption is low due to a first-order precautionary-savings motive, but as uncertainty resolves, this motive is dominated by time-inconsistent overconsumption, forcing consumption to decline toward the end of life. Third, consumption drops at retirement. When uncertainty is absent, the agent does not overconsume because he dislikes the associated certain loss in future consumption. Additionally, I obtain several new predictions about consumption; compare the preferences with habit formation, hyperbolic discounting, and temptation disutility; and structurally estimate the preference parameters.

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I INTRODUCTION

Expectations-based reference-dependent preferences formalize the notion that changes in expectations about consumption generate instantaneous utility; moreover, losses in expectations about consumption hurt more than gains give pleasure. These preferences were developed by Koszegi and Rabin (2006, 2007, 2009) to discipline the insights of prospect theory¹ and have since been shown to explain behavioral evidence in a variety of domains. This paper incorporates these preferences into a fully dynamic and stochastic consumption-savings model to explain three major life-cycle consumption facts – excess smoothness and sensitivity in consumption, a hump-shaped consumption profile, and a drop in consumption at retirement. The intuitions behind these implications connect several prominent ideas. Expectations-based loss aversion makes consumption excessively smooth, as unexpected cuts in consumption today are more painful than expected reductions in the future. Moreover, expectations-based loss aversion introduces a first-order precautionary-savings motive because expected fluctuations in consumption are less painful higher on the utility curve; however, it also generates a time inconsistency because today the agent increases his consumption above expectations, whereas yesterday he also considered how this increase in consumption would have increased his expectations.

I first explain the preferences in greater detail. In each period, the agent’s instantaneous utility consists of the following components. “Consumption utility” is determined by his level of consumption and corresponds to the standard model of utility. “Contemporaneous and prospective gain-loss utility” is determined by his expectations about consumption relative to his reference point and corresponds to a prospect-theory model of utility. The agent’s reference point corresponds to his prior beliefs about both his present consumption and his entire stream of future consumption. The agent experiences “contemporaneous gain-loss utility” when he compares his actual present consumption with his probabilistic beliefs about present consumption, experiencing a sensation of gain or loss relative to each consumption outcome that he had previously expected.

¹Prospect theory (Kahneman and Tversky (1979)) states that people care about gains and losses relative to a reference point, whereby small losses hurt more than small gains give pleasure: people are loss averse.

Additionally, the agent experiences “prospective gain-loss utility” when he compares his updated beliefs about future consumption with his previous beliefs, experiencing gain-loss utility over what he has learned about future consumption. Thus, gain-loss utility can be interpreted as utility over good and bad news.²

I analyze an agent with such “news-utility” preferences in a partial-equilibrium life-cycle consumption model.³ The agent lives for a finite number of periods; at the beginning of each period, he observes the realization of a permanent and transitory income shock and then optimally decides how much to consume and save. I first assume that the agent’s consumption utility is an exponential function. This assumption produces a closed-form solution, which allows me to gain a precise understanding of the intuitions behind the life-cycle consumption implications. Because the assumption of exponential utility is often considered unrealistic, I show that all of the implications hold if I instead assume a power-utility function.

As the first key implication, the preferences generate excess smoothness and sensitivity in consumption, which refer to the inherently related empirical observations that consumption initially underresponds to income shocks and then adjusts with a delay.⁴ Such consumption responses are puzzling from the perspective of the standard model but are perfectly consistent with expectations-based loss aversion.⁵ A simplified intuition is that unexpected losses in consumption today are relatively more painful than expected losses in consumption tomorrow. Accordingly, the agent delays painful reductions in consumption below expectations today until tomorrow, by which point his expectations will have adjusted. This logic overcomes the fact that the agent experiences both

²Koszegi and Rabin (2009) develop the dynamic preferences from the static model of Koszegi and Rabin (2006, 2007) by introducing contemporaneous and prospective gain-loss utility in the instantaneous utility function. In so doing, the authors generalize the static “outcome-wise” gain-loss comparison to a “percentile-wise” comparison. I generalize the static comparison slightly differently and refer to this new comparison as the separated comparison, as the agent separates realized and non-realized uncertainty, only experiencing gain-loss utility over uncertainty that has been realized. For contemporaneous gain-loss utility, the two comparisons yield the same value. However, for prospective gain-loss utility, the comparisons yield small quantitative differences. Because the separated comparison is a linear operator and preserves an outcome-wise structure, the model is considerably more tractable. Moreover, because the psychological intuition of the separated comparison is also reasonable, I see this modification as a minor methodological contribution.

³Similar model environments are assumed by Carroll (2001) and Gourinchas and Parker (2002).

⁴See, e.g., Flavin (1985), Campbell and Deaton (1989), and Jappelli and Pistaferri (2010).

⁵Alternative explanations exist, which I review in the next section.

contemporaneous and prospective gain-loss utility. In the event of an adverse shock, contemporaneous losses loom larger than prospective losses because a share of the latter depends on future uncertainty. This share is unaffected by today's income shock because the agent's future reference point will have incorporated today's shock and conditional changes in consumption.

Beyond resolving these puzzles, the preferences are consistent with another stylized fact about lifetime consumption: a hump-shaped life-cycle consumption profile, which is characterized by increasing consumption at the beginning but decreasing consumption toward the end of life.⁶ This hump results from the net of two preference features, a first-order precautionary-savings motive and a beliefs-based time inconsistency. First, the preferences motivate precautionary savings because loss aversion increases the painfulness of anticipated fluctuations in consumption, which hurt, however, relatively less higher on the concave utility curve. Accordingly, this precautionary-savings motive depends on concavity and is a first-order⁷ consideration, as opposed to the precautionary-savings motive under standard preferences. Second, the preferences are subject to a beliefs-based time inconsistency. The agent behaves inconsistently, because he takes his expectations as given when increasing consumption today, but considers his expectations when increasing consumption tomorrow. However, once tomorrow rolls around, he will only consider the joy of increasing consumption above expectations. As a result, the agent overconsumes relative to his optimal pre-committed consumption path that maximizes his expected utility. Consequently, the precautionary-savings motive keeps consumption low at the beginning of life. However, the need for precautionary savings decreases when uncertainty resolves over time. Then, the beliefs-based time inconsistency causes overconsumption such that consumption is forced to decline by the end of life.

Finally, the preferences cause a drop in consumption at retirement.⁸ During retirement, income

⁶See, e.g., Fernandez-Villaverde and Krueger (2007) and Gourinchas and Parker (2002).

⁷The precautionary-savings motive does not go to zero when uncertainty becomes small. Alternatively, in a first-order approximation of savings, the effect of uncertainty depends on the second derivative of the utility function. In contrast, in a second-order approximation of savings in the standard model, the effect of uncertainty depends on the third derivative of the utility function. This result has been obtained by Koszegi and Rabin (2009) in a two-period, two-outcome model.

⁸The empirical evidence regarding the prevalence of a drop in consumption at retirement is mixed. A series of papers, e.g., Banks et al. (1998), Bernheim et al. (2001), and Battistin et al. (2009), find that consumption drops at retirement, and my data display such a drop. However, Aguiar and Hurst (2005) find that the drop is absent when properly controlling for health shocks, work-related expenses, and home production. In contrast, Schwerdt (2005)

uncertainty is absent, which eliminates both the precautionary-savings motive and the beliefs-based time inconsistency. The latter is eliminated because time-inconsistent overconsumption is associated with a certain loss in future consumption. This certain loss hurts the agent more than his overconsumption gives him pleasure; thus, he only overconsumes when allocating labor income, which he was not sure to receive, rather than trading off a certain loss in future consumption. Because the agent suddenly controls his time-inconsistent desire to overconsume, his consumption drops at retirement. Beyond these three implications, the preferences generate several new and testable predictions about consumption and savings.

This unified explanation for the three main life-cycle consumption facts, as surveyed by Atanasio and Weber (2010), validates the news-utility model in a domain that it was not designed to explain. Moreover, I analyze habit-formation, hyperbolic-discounting, temptation-disutility, and standard preferences to show that news utility is the only preference specification that provides such a unified explanation independent of institutional or environmental assumptions that are commonly made in the literature, e.g., hump-shaped income profiles, liquidity constraints, preference shifters, or non-separabilities of consumption and leisure. However, I believe that a more important contribution is that the explanations' intuitions connect several compelling concepts in both the macro consumption literature and micro evidence that the preferences were designed to reconcile.

More precisely, I consider it a key contribution that two widely analyzed macro consumption puzzles are explained by loss aversion. Loss aversion is an experimentally robust risk preference, which has been used to explain both behavioral phenomena, such as the endowment and disposition effects, and macro phenomena, such as the equity-premium puzzle and stock market non-participation.⁹ The intuition for these puzzles, that cutting consumption today is more painful than

finds a drop although he explicitly controls for home production and focuses on German retirees, who receive high state-provided pensions and have health insurance.

⁹The endowment effect asserts that the willingness to accept (WTA) compensation for a good is greater than the willingness to pay (WTP) for it once the good is in one's possession, as once one owns the item, foregoing it feels like a loss. The most famous study is Kahneman et al. (1990), in which students are given a mug and then offered the chance to sell it. The authors find that participants' WTA compensation for the mug is approximately twice as high as their WTP for it. More recently, Ericson and Fuster (2010) demonstrate that subjects' expectation to keep rather than possess a good accounts for the gap between WTA and WTP. The disposition effect (Odean (1998)) is an anomaly related to the tendency of investors to sell winners (stocks that have gone up in value) but keep losers (stocks that have gone down in value) to avoid the realization of losses.

cutting consumption tomorrow when the reference point has decreased, seems intuitively appealing and extends the importance of expectations-based endowment effects beyond mugs and pens. Moreover, the resolution of the excess-smoothness puzzle nicely parallels the equity-premium puzzle because loss aversion makes consumption excessively smooth relative to changes in permanent income.¹⁰ To explain the other life-cycle facts, the preferences intuitively unify precautionary savings, which have been studied extensively in the standard consumption literature, and a beliefs-based time inconsistency, which is reminiscent of hyperbolic discounting.

To quantitatively evaluate the model, I first calibrate it in line with the microeconomic literature; this exercise is straightforward, as all preference parameters have narrow ranges that are determined by existing behavioral evidence and reasonable introspection. I then show that this calibration simultaneously generates reasonable attitudes towards small and large wealth bets and that the model's quantitative predictions match the empirical evidence for the consumption puzzles.¹¹ Beyond calibrating the model, I structurally estimate the preference parameters and obtain estimates that match those found in the microeconomic literature.¹² All five preference parameters are identified because each parameter generates specific variation in consumption growth over the life-cycle; this is not the case in existing structural estimations.¹³

The paper is organized as follows. After a literature review, I explain the model environment, preferences, and equilibrium concept in Section III. Then, I derive the model's main predictions in closed form under the assumption of exponential utility in Section IV. After briefly outlining the power-utility model, I then calibrate both models to assess whether the quantitative predictions match the empirical evidence and structurally estimate the model's parameters in Section V. Section VI outlines several extensions and the model's welfare implications. Section VII concludes.

¹⁰Because consumption is too smooth relative to movements in asset prices, a high equity premium in the canonical asset-pricing economy requires unreasonably high second-order risk aversion.

¹¹I use NIPA consumption and income data following Ludvigson and Michaelides (2001).

¹²I follow the two-stage method-of-simulated-moments methodology of Gourinchas and Parker (2002) and use pseudo-panel data from the Consumer Expenditure Survey as provided by the NBER.

¹³For instance, Barseghyan et al. (2010) cannot separately identify the weight of gain-loss utility relative to consumption utility and the coefficient of loss aversion.

II LITERATURE REVIEW

I contribute to the life-cycle literature by exploring a new preference specification that has been used in a variety of contexts to explain experimental and other microeconomic evidence.¹⁴ The preferences' predictions regarding consumption and savings modify and extend the two-period, two-outcome model introduced by Koszegi and Rabin (2009). In particular, I generalize the implications for precautionary savings, overconsumption in deterministic settings, and the potential delay of reductions in consumption. Moreover, my paper relates to Bowman et al. (1999), in which loss aversion causes delayed adjustments to adverse income shocks. However, in both Koszegi and Rabin (2009) and Bowman et al. (1999), the agent delays losses only to remain at his deterministic reference point.¹⁵ In contrast, I consider a model in which the reference point is stochastic, as consumption is continuously distributed. The stochastic reference point induces delayed consumption adjustments to both good and bad income shocks.

A very incomplete list of papers in the life-cycle consumption literature with standard preferences is Deaton (1991), Carroll (1997), and Gourinchas and Parker (2002). As shown by Ludvigson and Michaelides (2001), among others, any time-separable utility function cannot generate excess smoothness in consumption. Borrowing constraints are a potential additional assumption; however, the agent expects these constraints and ensures that they are not binding for most income realizations. Angeletos et al. (2001) and Laibson et al. (2012) analyze hyperbolic-discounting

¹⁴Heidhues and Koszegi (2008, 2010), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing, which are tested by Karle et al. (2011), Herweg et al. (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design. An incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Abeler et al. (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2010) on the disposition effect, and Ericson and Fuster (2010) on the endowment effect. Barseghyan et al. (2010) structurally estimate a model of insurance-deductible choice. Suggestive evidence is provided by Crawford and Meng (2009) on labor supply, Pope and Schweitzer (2011) on golf players' performance, and Sydnor (2010) on deductible choice. Moreover, the numerous conflicting papers on the endowment effect can be reconciled with the notion of expectations determining the reference point. All of these papers consider the static preferences, but as the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension, the evidence is equally valid for the dynamic preferences. Moreover, the notion that agents are loss averse with respect to news about future consumption is indirectly supported by all experiments, which use monetary payoffs because these concern future consumption.

¹⁵An asymmetric response to income innovations would also be predicted by liquidity constraints, the empirical evidence for which is very mixed (see Jappelli and Pistaferri (2010) for a survey). A famous paper by Shea (1995) finds that consumption is more excessively sensitive to expected income declines than increases. The author notes that this finding is inconsistent with liquidity constraints or myopia but consistent with loss aversion.

preferences in a life-cycle context. The sophisticated hyperbolic-discounting agent restricts his consumption opportunities with illiquid savings but can borrow up to some constraint.¹⁶ To the extent that his borrowing constraint binds or his liquid asset holdings bunch at zero, his consumption is excessively sensitive. Moreover, the standard and hyperbolic agents' consumption profiles are hump shaped under the assumption of power utility, sufficient impatience, and a hump-shaped income profile.

By providing a purely preference-based explanation for the consumption puzzles, I resume a literature pioneered by Fuhrer (2000) and Michaelides (2002), who assume internal multiplicative habit formation. The basic concept of news utility appears similar to habit formation. However, the life-cycle implications are very different; most important, I confirm the conclusion of Michaelides (2002) that habit formation only generates excess smoothness at the cost of unreasonably high wealth accumulation. Furthermore, Chetty and Szeidl (2010) incorporate adjustment costs in consumption, and Reis (2006) assumes that agents face costs when processing information and thus optimally decide to update their consumption plans sporadically.

III THE LIFE-CYCLE CONSUMPTION MODEL

I first define a general life-cycle model environment to formally introduce the preferences and equilibrium concepts.

III.1 The model environment

The agent lives for a total of T discrete periods indexed by $t \in \{1, \dots, T\}$. At the beginning of each period, a vector $S_t \sim F_{S_t}$ realizes that consists of random shocks, which are independent of each other and over time. The realization of S_t is denoted s_t . The model's exogenous state variables are

¹⁶Demand for commitment is also generated by temptation-disutility preferences, as specified in Gul and Pesendorfer (2004) and analyzed by Buccioli (2012) in a life-cycle context.

represented by the vector Z_t , which evolves according to the following law of motion

$$Z_t = f^Z(Z_{t-1}, S_t). \quad (1)$$

After observing s_t and Z_t , the agent decides how much to consume C_t .¹⁷ The model's endogenous state variable is cash-on-hand X_{t+1} and is determined by the following budget constraint

$$X_{t+1} = f^X(X_t - C_t, Z_t, S_{t+1}). \quad (2)$$

All of the model's variables that are indexed by t realize in period t . Because the agent's preferences are defined over both outcomes and beliefs, I explicitly define his probabilistic "beliefs" about each of the model's period t variables from the perspective of any prior period as follows.

Definition 1. Let $I_t = \{X_t, Z_t, s_t\}$ denote the agent's information set in some period $t \leq t + \tau$; then, the agent's probabilistic beliefs about any model variable $V_{t+\tau}$ conditional on period t information is denoted by $F_{V_{t+\tau}}^t(v) = Pr(V_{t+\tau} < v | I_t)$, and $F_{V_{t+\tau}}^{t+\tau}$ is degenerate.

Throughout the paper, I assume rational expectations, i.e., the agent's beliefs about any of the model's variables equal the objective probabilities determined by the economic environment.

III.2 Expectations-based reference-dependent preferences

Having outlined the model environment, I now introduce the agent's preferences. To facilitate the exposition, I first explain the static model of expectations-based reference dependence, as specified in Koszegi and Rabin (2006, 2007), and then introduce the dynamic model of Koszegi and Rabin (2009).

¹⁷Throughout most of the paper, I consider a standard life-cycle environment in which the agent's stochastic labor income is $Y_t = f^Y(P_{t-1}, S_t^p, S_t^T)$, which depends on his permanent income P_{t-1} , a permanent shock $S_t^p \sim F_{S_t^p}$, and a transitory shock $S_t^T \sim F_{S_t^T}$. He decides how much to consume C_t and how much to save in a risk-free asset that pays a net return r such that his cash-on-hand X_{t+1} is determined by $X_{t+1} = (X_t - C_t)(1 + r) + Y_{t+1}$.

The static preferences. The agent’s utility function consists of two components. First, he experiences “consumption utility” $u(c)$, which corresponds to the standard model of utility and is solely determined by consumption c . Second, he experiences “gain-loss utility” $\mu(u(c) - u(r))$. The gain-loss utility function $\mu(\cdot)$ corresponds to the prospect-theory model of utility determined by consumption c relative to the reference point r . $\mu(\cdot)$ is piecewise linear with slope η and a coefficient of loss aversion λ , i.e., $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta \lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the gain-loss utility component relative to the consumption utility component and $\lambda > 1$ implies that losses are weighed more heavily than gains, i.e., the agent is loss averse. Koszegi and Rabin (2006, 2007) allow for stochastic consumption, distributed according to $F_c(c)$, and a stochastic reference point, distributed according to $F_r(r)$. Then, the agent experiences gain-loss utility by evaluating each possible outcome relative to all other possible outcomes

$$\int_{-\infty}^{\infty} (\eta \int_{-\infty}^c (u(c) - u(r)) dF_r(r) + \eta \lambda \int_c^{\infty} (u(c) - u(r)) dF_r(r)) dF_c(c). \quad (3)$$

A, the authors make the central assumption that the distribution of the reference point F_r equals the agent’s fully probabilistic rational beliefs about consumption c .

The dynamic preferences. In the dynamic model of Koszegi and Rabin (2009), the utility function consists of consumption utility, “contemporaneous” gain-loss utility about current consumption, and “prospective” gain-loss utility about the entire stream of future consumption. Thus, total instantaneous utility in period t is given by

$$U_t = u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{\infty} \beta^{\tau} n(F_{C_{t+\tau}}^{t,t-1}). \quad (4)$$

The first term in equation (4), $u(C_t)$, corresponds to consumption utility in period t . Before turning to the subsequent terms in equation (4), which consider consumption and beliefs, I define an “admissible consumption function”. This function allows me to explicitly describe the probabilistic structure of the agent’s beliefs about any of the model’s variables at any future date. Because the

agent fully updates his beliefs in each period and because the shocks are independent over time, I consider a stationary function that depends only on this period's cash-on-hand X_t , the vector of exogenous state variables Z_t , the realization of the vector of shocks s_t , and calendar time t .

Definition 2. The consumption function in any period t is admissible if it can be written as a function $C_t = g_t(X_t, Z_t, s_t)$ that is strictly increasing in the realization of each shock $\frac{\partial g_t(X_t, Z_t, s_t)}{\partial s_t} > 0$. Repeated substitution of the law of motion, equation (1), and the budget constraint, equation (2), allows me to rewrite $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, S_{t+\tau}) = h_{t+\tau}^t(X_t, Z_t, s_t, S_{t+1}, \dots, S_{t+\tau})$.

I now return to the two remaining terms in equation (4). The first term, $n(C_t, F_{C_t}^{t-1})$, corresponds to gain-loss utility over contemporaneous consumption; here, the agent compares his present consumption C_t with his beliefs $F_{C_t}^{t-1}$. According to Definition 1, the agent's beliefs $F_{C_t}^{t-1}$ correspond to the conditional distribution of consumption in period t given the information available in period $t-1$. The agent experiences gain-loss utility over “news” about contemporaneous consumption as follows

$$n(C_t, F_{C_t}^{t-1}) = \eta \int_{-\infty}^{C_t} (u(C_t) - u(c)) dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t) - u(c)) dF_{C_t}^{t-1}(c) \quad (5)$$

where C_t and $F_{C_t}^{t-1}(c)$ are explicitly described via the admissible consumption function, i.e., $C_t = g_t(X_t, Z_t, s_t) = h_t^{t-1}(X_{t-1}, Z_{t-1}, s_{t-1}, s_t)$ and $F_{C_t}^{t-1}(c) = Pr(h_t^{t-1}(X_{t-1}, Z_{t-1}, s_{t-1}, S_t) < c)$.

The third term in equation (4), $\gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{t+\tau}}^{t,t-1})$, corresponds to gain-loss utility, experienced in period t , over the entire stream of future consumption. Prospective gain-loss utility about period $t + \tau$ consumption depends on $F_{C_{t+\tau}}^{t-1}$, the agent's beliefs he entered the period with, and on $F_{C_{t+\tau}}^t$, the agent's updated beliefs about consumption in period $t + \tau$. Again the probabilistic structure of these beliefs can be explicitly described via the admissible consumption function, i.e., $h_{t+\tau}^t(X_t, Z_t, s_t, S_{t+1}, \dots, S_{t+\tau})$. Importantly, the prior and updated beliefs about $C_{t+\tau}$, $F_{C_{t+\tau}}^{t-1}$ and $F_{C_{t+\tau}}^t$, are not independent distribution functions because future shocks $S_{t+1}, \dots, S_{t+\tau}$ are contained in both. Thus, there exists a joint distribution, which I denote by $F_{C_{t+\tau}}^{t,t-1} \neq F_{C_{t+\tau}}^t F_{C_{t+\tau}}^{t-1}$.¹⁸ Because

¹⁸I calculate prospective gain-loss utility $n(F_{C_{t+\tau}}^{t,t-1})$ by generalizing the “outcome-wise” comparison, specified in

the agent compares his newly formed beliefs with his prior beliefs, he experiences gain-loss utility over “news” about future consumption as follows

$$\mathbf{n}(F_{C_{t+\tau}}^{t,t-1}) = \int_{-\infty}^{\infty} (\eta \int_{-\infty}^c (u(c) - u(r)) + \eta \lambda \int_c^{\infty} (u(c) - u(r))) dF_{C_{t+\tau}}^{t,t-1}(c, r) \quad (7)$$

with $F_{C_{t+\tau}}^{t,t-1}(c, r)$ given by $F_{C_{t+\tau}}^{t,t-1}(c, r) = Pr(h_{t+\tau}^t(X_{t-1}, Z_{t-1}, s_{t-1}, s_t, S_{t+1}, \dots, S_{t+\tau}) < c, h_{t+\tau}^{t-1}(X_{t-1}, Z_{t-1}, s_{t-1}, S_t, S_{t+1}, \dots, S_{t+\tau}) < r)$.

The agent exponentially discounts prospective gain-loss utility by $\beta \in [0, 1]$. Moreover, he discounts prospective gain-loss utility relative to contemporaneous gain-loss utility by a factor $\gamma \in [0, 1]$. Thus, he puts the weight $\gamma\beta^\tau < 1$ on prospective gain-loss utility regarding consumption in period $t + \tau$. Because both contemporaneous and prospective gain-loss utility are experienced over news, the preferences can be referred to as “news utility”.

III.3 The model solution

The news-utility agent’s lifetime utility in each period $t = \{1, \dots, T\}$ is

$$u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau \mathbf{n}(F_{C_{t+\tau}}^{t,t-1}) + E_t[\sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau}], \quad (8)$$

Koszegi and Rabin (2006, 2007) and reported in equation (15), to account for the potential dependence of F_r and F_c , i.e.,

$$\mathbf{n}(F_{c,r}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{c,r}(c, r). \quad (6)$$

If F_r and F_c are independent, equation (6) reduces to equation (15). However, if F_r and F_c are non-independent, equation (6) and equation (15) yield different values. Suppose that F_r and F_c are perfectly correlated, as though no update in information occurs. Equation (15) would yield a negative value because the agent experiences gain-loss disutility over his previously expected uncertainty, which is unrealistic. In contrast, equation (6) would yield zero because the agent considers the dependence of prior and updated beliefs, which captures future uncertainty, thereby separating uncertainty that has been realized from uncertainty that has not been realized. Thus, I call this gain-loss formulation the separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison to a “percentile-wise” ordered comparison. The separated and ordered comparisons are equivalent for contemporaneous gain-loss utility. However, for prospective gain-loss utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous gain-loss utility.

where $\beta \in [0, 1)$, $u(\cdot)$ is a HARA¹⁹ utility function, $\eta \in [0, \infty)$, $\lambda \in [1, \infty)$, and $\gamma \in [0, 1]$. I also consider hyperbolic-discounting or $\beta\delta$ -preferences, as developed by Laibson (1997); the $\beta\delta$ -agent's lifetime utility is given by $u(C_t^b) + bE_t[\sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^b)]$ where $b \in [0, 1]$ is the hyperbolic-discount factor. Needless to say, standard preferences, as analyzed by Carroll (2001), Gourinchas and Parker (2002), or Deaton (1991), are a special case of the above models for either $\eta = 0$ or $b = 1$. I now define two equilibrium concepts: the monotone-personal equilibrium and monotone-pre-committed equilibrium.

The monotone-personal equilibrium. I define the model's "monotone-personal" equilibrium in the spirit of the preferred-personal equilibrium solution concept, as defined by Koszegi and Rabin (2009), but within the outlined environment and admissible consumption function as follows.

Definition 3. The family of admissible consumption functions $C_t = g_t(X_t, Z_t, s_t)$ is a monotone-personal equilibrium for the news-utility agent if, in any contingency, $C_t = g_t(X_t, Z_t, s_t)$ maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau})$. In each period t , the agent takes his beliefs about consumption $\{F_{C_{t+\tau}}^{t-1}\}_{\tau=0}^{T-t}$ as given in the maximization problem.

The monotone-personal equilibrium can be obtained by simple backward induction; thus, it is time consistent in the sense that beliefs map into correct behavior and vice versa. In other words, I derive the equilibrium consumption function under the premise that the agent enters period t , takes his beliefs as given, optimizes over consumption, and rationally expects to behave in this manner in the future. If I obtain a consumption function by backward induction that is admissible, then the monotone-personal equilibrium corresponds to the preferred-personal equilibrium as defined by Koszegi and Rabin (2009). For the hyperbolic-discounting agent, the monotone-personal equilibrium corresponds to the solution of Laibson (1997).

¹⁹A utility function $u(c)$ exhibits hyperbolic absolute risk aversion (HARA) if the risk tolerance $-\frac{u''(c)}{u'(c)}$ is a linear function of c .

The monotone-pre-committed equilibrium. The monotone-personal equilibrium maximizes the agent’s utility in each period t when he takes his beliefs as given. However, if the agent could pre-commit to his consumption in each possible contingency, he would choose a different consumption path. I define this path as the model’s “monotone-pre-committed” equilibrium in the spirit of the choice-acclimating equilibrium concept, as defined by Koszegi and Rabin (2007), but within the outlined environment and admissible consumption function as follows.

Definition 4. The family of admissible consumption functions, $C_t = g_t(X_t, Z_t, s_t)$ for each period t , is a monotone-pre-committed equilibrium for the news-utility agent, if, in any contingency, $C_t = g_t(X_t, Z_t, s_t)$ maximizes (8) subject to (2) and (1) under the assumption that all future consumption corresponds to $C_{t+\tau} = g_{t+\tau}(X_{t+\tau}, Z_{t+\tau}, s_{t+\tau})$. In each period t , the agent’s maximization problem determines both his beliefs $\{F_{C_{t+\tau}}^{t-1}\}_{\tau=0}^{T-t}$ and consumption $\{C_{t+\tau}\}_{\tau=0}^{T-t}$.

I derive the equilibrium consumption function under the premise that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency and thus jointly optimizes over consumption and beliefs. This equilibrium is not time consistent because the agent would deviate if he were to take his beliefs as given and optimize over consumption alone.

Equilibrium existence and uniqueness. I demonstrate the existence and uniqueness of the monotone-personal and monotone-pre-committed equilibria for special environments, such as exponential utility and permanent and transitory normal shocks, and under certain parameter conditions. In particular, F_{S_t} must be sufficiently dispersed such that the equilibrium consumption functions fall into the class of admissible consumption functions. For the monotone-pre-committed equilibrium, an additional parameter constraint $\eta(\lambda - 1) < 1$ is required to ensure global concavity of the agent’s maximization problem. For other environments, such as power utility and permanent and transitory log-normal shocks, simulations using numerical backward induction suggest that the monotone-personal and monotone-pre-committed equilibria exist and are unique for most reasonable calibrations.²⁰

²⁰Carroll (2011) and Harris and Laibson (2002) demonstrate the existence and uniqueness of equilibria for the standard and sophisticated hyperbolic-discounting agent in similar environments. In these models, the equilibrium

IV THEORETICAL PREDICTIONS ABOUT CONSUMPTION

In the following, I explain the closed-form solution of the exponential-utility model in detail to illustrate the model's predictions formally and intuitively. After briefly outlining the model's monotone-personal equilibrium in Proposition 1, I flesh out the second-to-last period's decision problem to explain the model's theoretical predictions. Proposition 2 and Corollary 1 formalize excess smoothness and sensitivity in consumption. Lemma 1 discusses how the precautionary-savings motive competes with the prospective gain-loss discount factor; the net of these forces leads to a hump-shaped consumption profile, which is formalized in Proposition 3. Proposition 4 determines consumption during retirement, and Proposition 5 characterizes when consumption drops at retirement. After these main predictions, I discuss several more subtle consumption implications and new comparative statics. Finally, Proposition 6 characterizes the implications of the monotone-pre-committed equilibrium.

I begin by briefly explaining the model's environment and stating the equilibrium consumption function to convey a general impression of the model's solution. The agent's utility function is exponential $u(C) = -\frac{1}{\theta}e^{-\theta C}$, where $\theta \in (0, \infty)$. His additive income process $Y_t = P_{t-1} + s_t^P + s_t^T$ is characterized by a permanent $S_t^P \sim N(\mu_{P_t}, \sigma_{P_t}^2)$ and transitory $S_t^T \sim N(\mu_{T_t}, \sigma_{T_t}^2)$ normal shocks, and his permanent income is $P_t = P_{t-1} + s_{t+1}^P$. His end-of-period asset holdings are denoted $A_t = X_t - C_t$ and his budget constraint is given by

$$X_{t+1} = (X_t - C_t)(1 + r) + Y_{t+1} \Rightarrow A_{t+1} = A_t R + Y_{t+1} - C_{t+1}. \quad (9)$$

In Appendix B.2.1, I show that the agent's optimal consumption function is

$$C_t = (1 - a(T - t))(1 + r)A_{t-1} + P_{t-1} + s_t^P + (1 - a(T - t))s_t^T - a(T - t)\Lambda_t. \quad (10)$$

consumption functions fall in the class of admissible consumption functions. For the standard agent, the monotone-personal equilibrium corresponds to the pre-committed equilibrium. For the sophisticated hyperbolic-discounting agent, the monotone-personal equilibrium does not correspond to the monotone-pre-committed equilibrium, which instead corresponds to the standard agent's equilibrium.

His consumption depends on his assets, income, horizon, and interest rate; the latter two are captured in the annuitization factor $a(T-t) = \frac{\sum_{j=0}^{T-t-1} (1+r)^j}{\sum_{j=0}^{T-t} (1+r)^j}$. Moreover,

$$\Lambda_t = \frac{1}{\theta} \log \left(\frac{1-a(T-t)}{a(T-t)} \frac{\psi_t + \gamma Q_t (\eta F(s_t^P + (1-a(T-t))s_t^T) + \eta \lambda (1 - F(s_t^P + (1-a(T-t))s_t^T)))}{1 + \eta F(s_t^P + (1-a(T-t))s_t^T) + \eta \lambda (1 - F(s_t^P + (1-a(T-t))s_t^T))} \right), \quad (11)$$

where $F(\cdot) = F_{s_t^P + (1-a(T-t))s_t^T}(\cdot)$ and ψ_t and Q_t are constant. Thus, Λ_t varies with the shock realizations but is independent of permanent income or assets.

Proposition 1. *There exists a unique monotone-personal equilibrium in the finite-horizon exponential-utility model if $\sqrt{\sigma_{P_t}^2 + (1-a(i))^2 \sigma_{T_t}^2} \geq \sigma_i^*$ for all $t \in \{1, \dots, T\}$.*

The standard and hyperbolic-discounting agents' monotone-personal equilibria have the same structure except that Λ_t^s and Λ_t^b only vary with the agent's horizon. This proof and proofs of the following propositions can be found in Appendix B.4. All of the following propositions are derived within this model environment and hold in any monotone-personal equilibrium if one exists.

IV.1 Excess smoothness and sensitivity in consumption

Excess smoothness and sensitivity in consumption are two robust empirical observations, which emerged from tests of the permanent income hypothesis. The permanent income hypothesis postulates that the marginal propensity to consume out of permanent income shocks is one and that future consumption growth is not predictable using past variables. However, numerous studies find that the marginal propensity to consume is less than one because consumption underresponds to permanent income shocks; thus, consumption is excessively smooth according to Deaton (1986). Moreover, numerous studies find that past changes in income have predictive power for future consumption growth because consumption adjusts with a delay; thus, consumption is excessively sensitive according to Flavin (1985). Campbell and Deaton (1989) explain how these observations are intrinsically related; consumption underresponds to permanent income shocks and thus adjusts with a delay. In this spirit, I define excess smoothness and sensitivity for the exponential-utility

model as follows.²¹

Definition 5. Consumption is excessively smooth if $\frac{\partial C_t}{\partial s_t^P} < 1$ everywhere and excessively sensitive if $\frac{\partial \Delta C_{t+1}}{\partial s_t^P} > 0$ everywhere.

This definition has an empirical analogue: an ordinary least squares (OLS) regression of period $t + 1$ consumption growth on the realization of the permanent shock in periods $t + 1$ and t ; for the two OLS coefficients β_1 and β_2 , the above definition implies that consumption is excessively smooth if $\beta_1 = \frac{\partial C_t}{\partial s_t^P} |_{s_t^P = \mu_{P_t}} < 1$ and excessively sensitive if $\beta_2 = \frac{\partial \Delta C_{t+1}}{\partial s_t^P} |_{s_t^P = \mu_{P_t}} > 0$.

Proposition 2. *The news-utility agent's consumption is excessively smooth and sensitive.*

I briefly present a simplified intuition for this result to then explain the agent's first-order condition in greater detail and provide the full intuition. The agent's marginal gain-loss utility today is more sensitive to his savings than his marginal gain-loss utility tomorrow, as his reference point today is invariable while his reference point tomorrow will have adjusted to his savings plan today. As a result, in the event of an adverse shock, the agent prefers to delay the reduction in consumption until his reference point has decreased. Additionally, in the event of a good shock, the agent prefers to delay the increase in consumption until his reference point has increased.

To explain this result in greater detail, I flesh out the agent's decision-making problem in the second-to-last period assuming that transitory shocks are absent, $A_{T-2} = P_{T-2} = 0$, and the permanent income shock is independent and identically distributed (i.i.d.) normal $S_{T-1}^P, S_T^P \sim F_P = N(\mu_P, \sigma_P)$. In period $T - 1$, the agent chooses how much to consume C_{T-1} and save $s_{T-1}^P - C_{T-1}$. His optimal consumption growth is given by

$$\Delta C_T = s_T^P + \frac{1}{\theta} \log\left((1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} (\eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P))}\right). \quad (12)$$

I explain each component of the fraction in equation (12) in detail. The denominator is marginal consumption and contemporaneous gain-loss utility in period $T - 1$; the latter consists of two terms. First, the agent compares his actual consumption to all consumption outcomes that would have

²¹This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

been less favorable and experiences a gain weighted by η , i.e., $\eta \int_{-\infty}^{C_{T-1}} (u(C_{T-1}) - u(c)) F_{C_T}^{T-1}(c)$. Second, the agent compares his actual consumption to all outcomes that would have been more favorable and experiences a loss weighted by $\eta\lambda$, i.e., $\eta\lambda \int_{C_{T-1}}^{\infty} (u(C_{T-1}) - u(c)) F_{C_T}^{T-2}(c)$. Because the agent takes his beliefs as given in the monotone-personal equilibrium, his marginal consumption and marginal contemporaneous gain-loss utility equals

$$u'(C_{T-1}) + u'(C_{T-1})(\eta F_{C_{T-1}}^{T-2}(C_{T-1}) + \eta\lambda(1 - F_{C_{T-1}}^{T-2}(C_{T-1}))). \quad (13)$$

This expression can be simplified by replacing $F_{C_{T-1}}^{T-2}(C_{T-1})$ with $F_P(s_{T-1}^P)$ because any admissible consumption function is increasing in the shock realization.

The second term of the numerator in equation (12) is marginal prospective gain-loss utility over future consumption $C_T = (s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P + S_T^P$. I denote the expected marginal utility of the last period's income shock $Q_{T-1} = \beta E_{T-1}[u'(S_T^P)]$.²² As the agent's admissible future consumption is increasing in the shock realization and he takes his beliefs as given, his marginal prospective gain-loss utility corresponds to the same weighted sum of $F_P(s_{T-1}^P)$

$$(1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P)\gamma Q_{T-1}(\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))). \quad (14)$$

The first term of the numerator in equation (12) is marginal future consumption and gain-loss utility. I denote the expected marginal consumption and gain-loss utility of the last period's income shock ψ_{T-1} , which equals Q_{T-1} plus $\beta E_{T-1}[\eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(s)) dF_P(s)]$. Consequently, marginal expected consumption and gain-loss utility are given by $(1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P)\psi_{T-1}$.

The fraction in equation (12) is increasing in s_{T-1}^P for any γ iff $\psi_{T-1} > Q_{T-1}$. The difference between ψ_{T-1} and Q_{T-1} corresponds to expected marginal gain-loss utility that is constant because the future reference point adjusts to today's savings plan. Thus, a positive share of tomorrow's marginal utility is inelastic to today's savings, which implies that tomorrow's marginal utility is less

²²Exponential utility implies that $u'(*+\cdot) = u'(*)u'(\cdot)$ and thus works well with additive risk.

sensitive to changes in savings than today's marginal utility. Today's marginal contemporaneous and prospective gain-loss utility is relatively high or low in the event of an adverse or positive shock. In contrast, expected marginal gain-loss utility is constant because tomorrow's reference point will have adjusted to today's plan. Thus, the agent will consume relatively more in the event of an adverse shock and relatively less in the event of a positive shock. According to Definition 5, consumption is excessively smooth $\frac{\partial C_{T-1}}{\partial s_{T-1}^P} < 1$ and excessively sensitive $\frac{\partial \Delta C_T}{\partial s_{T-1}^P} > 0$.

In contrast, the standard agent's consumption growth is $\Delta C_T^s = s_T^P + \frac{1}{\theta} \log((1+r)Q_{T-1})$, and the hyperbolic-discounting agent's consumption growth is $\Delta C_T^b = s_T^P + \frac{1}{\theta} \log((1+r)bQ_{T-1})$. Thus, the consumption of these agents is neither excessively smooth nor excessively sensitive.

IV.2 The hump-shaped consumption profile

Fernandez-Villaverde and Krueger (2007), among others, show that lifetime consumption profiles are hump shaped, even when controlling for cohort, family size, number of earners, and time effects.²³ In the following, I demonstrate that the preferences generate a hump-shaped consumption profile as a result of the net of two competing features – an additional first-order precautionary-savings motive and the agent's discount factor on prospective gain-loss utility γ .

Precautionary savings and prospective news discounting. Income uncertainty has a first-order effect on savings in the news-utility model. This “first-order precautionary-savings motive” is added to the precautionary savings motive of the standard agent, which is a second-order motive.²⁴ This result is highlighted by Koszegi and Rabin (2009) in a two-period, two-outcome model.²⁵

Definition 6. There exists a first-order precautionary-savings motive iff $\frac{\partial (s_{T-1}^P - C_{T-1})}{\partial \sigma_P} \Big|_{\sigma_P=0} > 0$.

However, the agent wishes to increase his consumption and decrease his savings if he dis-

²³Moreover, Fernandez-Villaverde and Krueger (2007) find suggestive evidence that non-separability between consumption and leisure, which was promoted by Attanasio (1999) and previous papers, cannot explain more than 20% of the hump in consumption.

²⁴Refer to Gollier (2001).

²⁵This result and those following can be generalized to any HARA utility function, arbitrary horizons, and labor income uncertainty.

counts prospective gain-loss utility relative to contemporaneous gain-loss utility, i.e., $\gamma < 1$. This discounting is reminiscent of $\beta\delta$ -preferences. The following lemma formalizes these two opposing forces.

Lemma 1.

1. *Precautionary savings: News utility introduces a first-order precautionary-savings motive.*
2. *Implications for consumption growth: There exists a $\bar{\gamma}^s < 1$, implicitly determined by $\Delta C_T = \Delta C_T^s$, such that, iff $\bar{\gamma}^s < \gamma$, the news-utility agent's consumption growth in period T is higher than the standard agent's consumption growth for any realization of s_{T-1}^P and s_T^P , and $\frac{\partial \bar{\gamma}^s}{\partial \sigma_p} < 0$.*

The intuition for the first part of the lemma is as follows. The agent anticipates being exposed to gain-loss fluctuations in period T , which are painful in expectation because losses hurt more than gains give pleasure. Additionally, the painfulness of these fluctuations is proportional to marginal consumption utility, which is lower higher on the utility curve. Thus, the agent has an additional incentive to increase savings. The intuition for the second part of the lemma is straightforward. If $\gamma < 1$, the agent is more concerned about contemporaneous than prospective gain-loss utility; thus, he wishes to increase his consumption and decrease his savings. Consequently, the presence of news utility might increase or decrease consumption relative to the standard model depending on the net of two parameters $\sigma_p > 0$ and $\gamma < 1$.

In the following, I develop a more formal intuition for the standard and additional precautionary-savings motive and demonstrate that the assumption $\psi_{T-1} > Q_{T-1}$, which I made previously, always holds. As shown above, the marginal value of savings is $(1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P)\psi_{T-1}$, where ψ_{T-1} equals the shock's expected marginal consumption plus expected marginal gain-loss utility

$$\beta E_{T-1}[u'(S_T^P)] + \beta E_{T-1}[\eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(s)) dF_P(s)]. \quad (15)$$

The integral in equation (15) reflects the expected marginal utility of all gains and losses, which partly cancel, such that only the overweighted component of the losses remains, i.e., $\eta(\lambda -$

1)(·). The key point is that this integral is always positive if $u'' < 0$ and thus captures the additional precautionary-savings motive, implies that $\psi_{T-1} > Q_{T-1}$, and is increasing in η , λ , and σ_P . Because $\frac{\partial(s_{T-1}^P - C_{T-1})}{\partial\sigma_P}|_{\sigma_P=0} > 0$, this motive is first order, as the news-utility agent is first-order risk averse. In contrast, the standard precautionary-savings motive is captured by $Q_{T-1} = \beta E_{T-1}[u'(S_T^P)]$, which is larger than $\beta u'(E_{T-1}[S_T])$ if $u''' > 0$, according to Jensen's inequality. This standard precautionary-savings motive is second order, i.e., $\frac{\partial(s_{T-1}^P - C_{T-1})}{\partial\sigma_P}|_{\sigma_P=0} = 0$, as the standard agent is second-order risk averse.²⁶

The resulting hump-shaped consumption profile. The two competing news-utility features – the additional precautionary-savings motive and $\gamma < 1$ – make it likely that the life-cycle consumption profile is hump shaped.

Definition 7. I say that the agent's consumption profile is hump shaped if consumption is increasing at the beginning of his life $\Delta C_1 > 0$ and decreasing $\Delta C_T < 0$ at the end of his life.

Proposition 3. Suppose $\sigma_{P_t} = \sigma_P$ for all t and T is large; then, there exists a σ_P in $[\underline{\sigma}_P, \overline{\sigma}_P]$ such that, if $\gamma < 1$, $\log((1+r)\beta) \in [-\Delta, \Delta]$, and Δ is small, the news-utility agent's lifetime consumption path is hump shaped.

The basic intuition is illustrated in Lemma 1. The relative strengths of the additional precautionary-savings motive and $\gamma < 1$ determine whether the presence of gain-loss utility increases or decreases the news-utility agent's consumption relative to the standard model. When the agent's horizon increases, the precautionary-savings motive accumulates because uncertainty accumulates. Accordingly, at the beginning of life, the presence of gain-loss utility is likely to reduce consumption and increase consumption growth unless γ is small. Toward the end of life, however, the additional precautionary-savings motive is relatively small, and $\gamma < 1$ is likely to decrease consumption growth. More formally, the two conditions $\Delta C_{t+1} \leq 0$ reduce to $\Lambda_t \leq 0$ as $T - t$ becomes large or

²⁶As shown by Benartzi and Thaler (1995) and Barberis et al. (2001), first-order risk aversion resolves the equity premium puzzle, which highlights that agents must have implausibly high second-order risk aversion to reconcile the historical equity premium because aggregate consumption is smooth compared with asset prices. The excess-smoothness puzzle highlights that aggregate consumption is too smooth compared to labor income, and again, first-order instead of second-order risk aversion is a necessary ingredient for resolving the puzzle.

$T - t$ becomes small. The sign of Λ_t is determined by the relative values of $\frac{\Psi_t}{Q_t} > 1$ and $\gamma < 1$. As $T - t$ increases, $\frac{\Psi_t}{Q_t}$ increases such that $\gamma < 1$ loses relative importance and Λ_t is more likely to be positive. In contrast, $\frac{\Psi_{T-1}}{Q_{T-1}}$ is small such that $\gamma < 1$ is likely to cause Λ_{T-1} to be negative.

IV.3 News-utility consumption during and at retirement

IV.3.1 News-utility consumption during retirement

I now add a retirement period at the end of life. I assume that in periods $t \in \{T - R, T\}$, the agent earns his permanent income without uncertainty. I first formalize a general prediction of the news-utility agent's consumption during retirement, in which I generalize a result obtained by Koszegi and Rabin (2009) in a two-period model.²⁷

Proposition 4. *If uncertainty is absent, both the monotone-personal equilibrium and monotone-pre-committed equilibrium of the news-utility agent correspond to the standard agent's equilibria iff $\gamma \geq \frac{1}{\lambda}$. Iff $\gamma < \frac{1}{\lambda}$ then the monotone-pre-committed equilibrium of the news-utility agent corresponds to the standard agent's equilibrium and the monotone-personal equilibrium of the news-utility agent corresponds to a $\beta\delta$ -agent's monotone-personal equilibrium with the hyperbolic-discount factor given by $b = \frac{1+\gamma\eta\lambda}{1+\eta}$.*

The news-utility agent is likely to follow the standard agent's path if uncertainty is absent. The basic intuition is that the agent associates a certain loss in future consumption, which is very painful, with an increase in present consumption. Thus, unless the agent discounts prospective gain-loss utility significantly, he follows the utility-maximizing standard agent's path. More formally, suppose that the agent allocates his deterministic cash-on-hand between consumption today C_{T-1} and tomorrow C_T . Under rational expectations, he cannot fool himself; hence, he cannot experience actual gain-loss utility in equilibrium in a deterministic model. Accordingly, his expected-utility maximization problem corresponds to the standard agent's maximization problem, and his monotone-pre-committed equilibrium thus corresponds to the standard agent's problem

²⁷This result can be generalized to a HARA utility function.

determined by $u'(C_{T-1}) = \beta(1+r)u'(C_T)$. Suppose that the agent's beliefs about consumption in both periods correspond to this pre-committed equilibrium path. Taking his beliefs as given, the agent will deviate if the gain from consuming more today exceeds the discounted loss from consuming less tomorrow, i.e.,

$$u'(C_{T-1})(1+\eta) > \beta(1+r)u'(C_T)(1+\gamma\eta\lambda).$$

Thus, he follows the standard agent's path iff $\gamma \geq \frac{1}{\lambda}$ because the pain of the certain loss in future consumption is greater than the pleasure gained from present consumption. However, if $\gamma < \frac{1}{\lambda}$, the agent chooses a consumption path that just meets the consistency constraint and behaves as a $\beta\delta$ - or hyperbolic-discounting agent with hyperbolic discount factor $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

IV.3.2 News-utility consumption at retirement

During retirement, the implications of the agent's prospective gain-loss discount factor γ are simple: it needs to be sufficiently low to overcome the certain loss in future consumption. I now examine the pre-retirement period to derive two additional implications of $\gamma < 1$. The first concerns a drop in consumption at retirement, and the second shows how excess sensitivity in consumption arises in the absence of future uncertainty.

The drop in consumption at retirement. The empirical evidence on the prevalence of a drop in consumption at retirement is mixed. While a series of papers (see Attanasio and Weber (2010) for a survey) have found that consumption drops at retirement, Aguiar and Hurst (2005) cannot confirm this finding when controlling for the sudden reduction of work-related expenses, the substitution of home production for market-purchased goods and services, and health shocks. In my data, I find such a drop in consumption at retirement even for non-work-related expenditures. Moreover, I consider the evidence provided by Schwerdt (2005) compelling because the author explicitly controls for home production and focuses on German retirees, who receive large state-provided pensions, which require little self organization, and for whom health is a complement to consumption thanks

to proper insurance coverage. I first define a drop in consumption as follows.

Definition 8. There occurs a drop in consumption at retirement if consumption growth at retirement ΔC_{T-R} is negative and smaller than consumption growth after retirement ΔC_{T-R+1} .

As an example, if $\gamma \geq \frac{1}{\lambda}$, the news-utility agent's post-retirement consumption growth equals that of the standard agent's, i.e., $\frac{1}{\theta} \log(\beta(1+r)) \approx 0$, whereas consumption growth at retirement is $\frac{1}{\theta} \log(\beta(1+r)) + \frac{1}{\theta} g^s$ with $g^s \in \{\log(\frac{1+\gamma\eta\lambda}{1+\eta\lambda}), \log(\frac{1+\gamma\eta}{1+\eta})\} < 0$ for the news-utility agent and remains zero for the standard agent.²⁸

Proposition 5. *If $\gamma < 1$, $\log((1+r)\beta) \in [-\Delta, \Delta]$, and Δ is small, the news-utility agent's monotone-personal consumption path is characterized by a drop at retirement.*

After the beginning of retirement the agent is less inclined to overconsume than before. The basic intuition for overconsumption in the pre-retirement period is that the agent allocates house money, i.e., labor income that he was not certain that he would receive, and thus prefers to surprise himself with additional consumption today iff $\gamma < 1$. During retirement, the agent associates a certain loss in future consumption with a surprise in present consumption. In contrast, in the pre-retirement period, the agent finds the loss in future consumption merely as painful as a slightly less favorable realization of his labor income, i.e., the agent trades off being somewhere in the gain domain today versus being somewhere in the gain domain tomorrow instead of a sure gain today with a sure loss tomorrow. The agent's first-order condition in period $T-1$ absent uncertainty in period T is given by

$$u'(C_{T-1}) = \beta(1+r)u'(C_T) \frac{1 + \gamma(\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))}. \quad (16)$$

In equation (16), it can immediately be seen that iff $\gamma = 1$, contemporaneous and prospective marginal gain-loss utility cancel. However, iff $\gamma < 1$, the agent reduces the weight on future utility relative to present utility by a factor between $\frac{1+\gamma\eta\lambda}{1+\eta\lambda}$ and $\frac{1+\gamma\eta}{1+\eta} < 1$. During retirement, the

²⁸This and the following results can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

news-utility agent follows the standard agent's consumption path if γ is sufficiently high and a $\beta\delta$ -agent's consumption path with discount factor $b = \frac{1+\gamma\eta\lambda}{1+\eta}$ otherwise. Because $\frac{1+\gamma\eta}{1+\eta} < \min\{\frac{1+\gamma\eta\lambda}{1+\eta}, 1\}$ iff $\gamma < 1$, the agent's factor that reduces the weight on future utility is necessarily lower in the pre-retirement period than after retirement, which implies that a drop in consumption occurs at retirement.²⁹ The other agents' consumption paths do not exhibit a drop in consumption at retirement.

I believe that the drop in consumption is an interesting prediction because it is driven by the reduction in income uncertainty rather than a drop in income at retirement. Nevertheless, this prediction depends entirely on the assumption of no uncertainty during retirement; needless to say, this assumption is rather unrealistic.³⁰ Furthermore, if I were to observe a consumption path that is much flatter during retirement than before retirement and interpret this finding from the perspective of the standard model, I may conclude that the agent does not decumulate assets sufficiently rapidly after retirement compared to his pre-retirement asset decumulation. Thus, the model is able to explain another puzzle observed by Bucciol (2012) and Disney (1996), namely, the lack of asset decumulation during retirement.

Excess sensitivity in the pre-retirement period. In the following, I outline an additional result regarding excess sensitivity in the pre-retirement period.³¹ This prediction is related to Proposition 7 in Koszegi and Rabin (2009), in which the authors find that if $\frac{1}{\lambda} < \gamma < 1$, then the news-utility agent might entirely consume small gains but entirely delay small losses when he is surprised by

²⁹What happens if uncertainty in the pre-retirement period becomes small? The model's result depends on the support of uncertainty. First, suppose the agent expects a continuous shock, the variance of which becomes small. So long as a monotone-personal equilibrium exists, there occurs a drop at retirement. However, if the variance of the shock becomes very small, the agent will follow a flat consumption path at some point. Nevertheless, the agent will not be able to follow his deterministic consumption path, but reduces the weight on future marginal value by a factor in the range of $\{\frac{1+\gamma\eta}{1+\eta}, \frac{1+\gamma\eta\lambda}{1+\eta}\}$. Thus, if $\frac{1}{\lambda} \leq \gamma \leq 1$, there occurs a drop for good realizations, and if $\gamma < \frac{1}{\lambda}$, there occurs a drop for all realizations. Second, suppose the agent expects a shock with some probability. If the probability of a shock occurring becomes small, the agent's consumption in the pre-retirement period approaches his deterministic consumption path; this eliminates the drop because the agent's first-order condition is no longer subject to a change in the weighting of future versus present marginal value.

³⁰The assumption of no uncertainty during retirement is made in all standard life-cycle consumption model, as they abstract from portfolio choice. Moreover, because mortality risk does not affect my result, the drop in consumption at retirement is a necessary artifact of news-utility preferences in the standard life-cycle environment.

³¹This result can be generalized to a HARA utility function, arbitrary horizons, and arbitrary income uncertainty.

them.³²

Corollary 1. *Iff $\gamma < 1$, the news-utility agent's monotone-personal equilibrium consumption is excessively smooth and sensitive in the pre-retirement period.*

The basic intuition is that the agent can effectively reduce his sense of loss by delaying the cut in consumption. Iff $\gamma < 1$, the agent cares more about contemporaneous than prospective gain-loss utility and thus overconsumes in the presence of uncertainty, as explained above. Moreover, he overconsumes even more when experiencing a relatively bad realization because losses are overweighted. Because the agent overconsumes relatively more in the event of a bad shock and relatively less in the event of a good shock, he delays his adjustment to consumption. Mathematically, the agent behaves like a $\beta\delta$ -agent, weighting future consumption by a factor of $b \in \{\frac{1+\gamma\eta\lambda}{1+\eta}, \frac{1+\gamma\eta}{1+\eta}\}$. Thus, the agent's weight on future consumption is particularly low when the income realization is relatively bad, i.e., $F_P(s_{T-1}^P) \approx 0$. In turn, variation in $F_P(s_{T-1}^P)$ leads to variation in $\Delta C_T = \frac{1}{\theta} \log(\beta(1+r)) + \frac{1}{\theta} \log(\frac{1+\gamma(\eta F_P(s_{T-1}^P) + \eta\lambda(1-F_P(s_{T-1}^P)))}{1+\eta F_P(s_{T-1}^P) + \eta\lambda(1-F_P(s_{T-1}^P))})$ and consumption is excessively smooth and sensitive because an increase in the permanent shock increases the fraction determining ΔC_T . Moreover, for any given η and λ consumption is more excessively smooth and sensitive if γ is low.

IV.4 New predictions about news-utility consumption

In the following, I highlight several additional news-utility predictions for life-cycle consumption that are new and testable comparative statics. I first explain the agent's consumption function, equation (10), in detail to highlight some subtle predictions about how the marginal propensity to consume varies with the realization of the permanent and transitory shocks and the agent's

³²In this example, the agent's consumption is excessively smooth and sensitive for surprise losses, but the opposite is true for surprise gains. In the same setup, the agent's consumption would also be excessively smooth and sensitive, according to Definition 5, for gains if they are sufficiently large and thus not entirely consumed or if they are expected. The agent might entirely consume an unexpected gain because it brings about a change in the weighting of future versus present marginal value in the agent's first-order condition. More formally, absent uncertainty, the agent follows the standard agent's path, as $\frac{1}{\lambda} < \gamma$, whereas in the event of a surprise gain, he puts a weight of $\frac{1+\gamma\eta\lambda}{1+\eta} < 1$ on future consumption. Thus, if the gain is small, the change in the weight the agent places on future marginal consumption utility induces the agent to consume the entire gain.

horizon. To illustrate the implications of the additive consumption function, I assume that $T - t$ is large such that $a(T - t) \approx \frac{1}{1+r}$. Then, in each period t , the agent consumes the interest payments of his last period's asset holdings rA_{t-1} , his entire permanent income $P_{t-1} + s_t^P$, and the per-period value of his temporary shock $\frac{r}{1+r}s_t^T$. Λ_t captures the agent's patience compared to the market, his precautionary savings, and his marginal gain-loss utility. In the event of a negative shock, Λ_t is low and the agent consumes more out of his end-of-period asset holdings and thus spreads the consumption adjustment to his entire future. Λ_t varies more with the permanent shock than with the transitory shock because the agent is only consuming the per-period value of the transitory shock, such that $F_{C_t}^{t-1}(C_t)$ varies little with it. This observation constitutes the first novel prediction of the news-utility model: consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. In an environment with transitory shocks alone, however, news-utility consumption is excessively sensitive with respect to transitory shocks.

A second prediction is that the degree of excess smoothness and sensitivity is decreasing in income uncertainty σ_P . If σ_P is small, the agent's beliefs change more rapidly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around μ_P .³³ A third prediction is that any bell-shaped shock distribution induces bounded variation in Λ_t and thus the agent's excess sensitivity. If the agent is affected by a tail realization, the actual value of the low-probability shock matters less because neighboring states have very low probability; thus, the variation in Λ_t is bounded. A fourth prediction is that consumption is more excessively

³³I return to the two-period, one-shock model to discuss an interesting prediction of flat consumption. Suppose that the absolute level of the shock increases; then, holding C_{T-1} constant, the marginal value of savings declines and the agent's first-order condition implies that consumption should increase. However, $F_P(s_{T-1}^P)$ also increases, and marginal gain-loss utility is lower, such that the agent's optimal consumption should decrease. Suppose that s_{T-1}^P increases marginally but $F_P(s_{T-1}^P)$ increases sharply, which could occur when F_P is a narrow, bell-shaped distribution. In this case, the lower marginal gain-loss utility that decreases consumption dominates such that the first-order condition predicts decreasing consumption over some range in the neighborhood of the expected value μ_P where F_P increases most sharply. However, a decreasing consumption function cannot be an equilibrium because the agent would experience, unnecessarily, gain-loss utility over the decreasing part of consumption, which is negative on average. Instead the agent would choose a flat consumption function in this region. In that case, he does not respond to shocks at all, i.e., his consumption is perfectly excessively smooth and sensitive, which resembles liquidity constraints or adjustment costs to consumption. As explained in detail by Heidhues and Koszegi (2008), flat consumption implies that beliefs about consumption are also flat, which generates non-differentiabilities in the agent's maximization problem, renders the first-order condition inapplicable, and complicates the equilibrium-finding process. Although this aspect of the model is undoubtedly interesting, I avoid this problem by restricting myself to monotone consumption functions.

smooth and sensitive when the agent's horizon increases because the marginal propensity to consume out of permanent shocks declines when the precautionary-savings motive accumulates.

IV.5 Comparison to the agent's pre-committed equilibrium

Now, I briefly explain the consumption implications of the monotone-pre-committed equilibrium that maximizes expected utility by jointly optimizing over consumption and beliefs. Without an appropriate commitment device, however, the pre-committed equilibrium is not credible because the agent overconsumes once he wakes up and takes his beliefs as given. I call this phenomenon beliefs-based present bias because the agent prefers to enjoy the pleasant surprise of increasing consumption above expectations today instead of increasing both his consumption and expectations tomorrow.³⁴ Empirically, there is abundant laboratory and field evidence for time-inconsistent overconsumption, preference reversals, and demand for commitment devices.³⁵ Theoretically, the hyperbolic-discounting model of Laibson et al. (2012) is very successful in explaining life-cycle consumption. In the next proposition, I formalize how the consumption implications differ in the monotone-pre-committed equilibrium if one exists. Then, I explain beliefs-based present bias in detail and show how it differs from hyperbolic discounting.

Proposition 6. *Comparison to the monotone-pre-committed equilibrium.*

1. *If $\sigma_{P_t} > 0$ for any t , then the monotone-pre-committed consumption path does not correspond to the monotone-personal equilibrium consumption path.*
2. *The news-utility agent's monotone-pre-committed consumption is excessively smooth and sensitive.*
3. *News-utility preferences introduce a first-order precautionary-savings motive in the monotone-pre-committed equilibrium, monotone-pre-committed consumption is lower $C_{T-1}^c < C_{T-1}$, and the gap increases in the event of good income realizations $\frac{\partial(C_{T-1}^c - C_{T-1})}{\partial s_{T-1}^p} > 0$.*

³⁴Koszegi and Rabin (2009) argue in Proposition 6 that the agent overconsumes relative to the optimal pre-committed path in the presence of uncertainty.

³⁵See, e.g., DellaVigna (2009), Frederick et al. (2002), or Angeletos et al. (2001) for a survey of the theory and empirical evidence.

5. *The news-utility agent's monotone-pre-committed consumption path is not characterized by a drop at retirement.*

Suppose that the agent can pre-commit to an optimal, history-dependent consumption path for each possible future contingency. Then, the agent's marginal gain-loss utility is no longer solely composed of the sensation of increasing consumption in that contingency; additionally, the agent considers that he will experience fewer sensations of gains and more feelings of loss in all other contingencies. Thus, marginal gain-loss utility has a second component, $-u'(C_{T-1})(\eta(1 - F_{C_t}^{t-1}(C_t)) + \eta\lambda F_{C_t}^{t-1}(C_t))$, which is negative such that the pre-committed agent consumes less. Moreover, this negative component dominates if the realization is above the median, i.e., $F_{C_t}^{t-1}(C_t) > 0.5$. Thus, in the event of good income realizations, pre-committed marginal gain-loss utility is negative. In contrast, non-pre-committed marginal gain-loss utility is always positive because the agent enjoys the sensation of increasing consumption in any contingency. Therefore, the degree of present bias is reference dependent and less strong in the event of bad income realizations, when increasing consumption is the optimal response even on the pre-committed path. Moreover, this additional variation in marginal gain-loss utility implies that pre-committed consumption is more excessively smooth and sensitive.

Beliefs-based present bias is both conceptually different from $\beta\delta$ -preferences and observationally distinguishable. In particular, there are three main differences between $\beta\delta$ -preferences and beliefs-based present bias. First, news utility introduces an additional precautionary-savings motive that is absent in the $\beta\delta$ -model. Second, the news-utility agent does not have a universal desire to pre-commit himself to the standard agent's consumption path, in contrast to $\beta\delta$ -preferences. Finally, news utility predicts that the agent's degree of present bias is reference dependent and lower in bad times; hence, he behaves better in bad times.

V QUANTITATIVE PREDICTIONS ABOUT CONSUMPTION

In the following, I assess whether the model's quantitative predictions match the empirical evidence. Because it is commonly argued that exponential utility is unrealistic, I also present the numerical implications of a power-utility model, i.e., $u(C) = \frac{C^{1-\theta}}{1-\theta}$, to demonstrate that all of the predictions hold in model environments that are commonly assumed in the life-cycle consumption literature.³⁶ In Section V.1, I calibrate the two model's parameters in accordance with the microeconomic literature and explain each choice in detail. In Section V.3, I employ my own data to structurally estimate the power-utility model's parameters and show that the estimates are in accordance with the existing micro evidence.

V.1 Calibration

The income processes and model environment. For the exponential-utility model, I retain the normal income process outlined in Section IV assuming that permanent and transitory shocks are i.i.d., i.e., $Y_t = P_{t-1} + s_t^P + s_t^T \sim N(P_{t-1} + \mu_P + \mu_T, \sigma_P^2 + \sigma_T^2)$. For the power-utility model, I follow Carroll (1997) and Gourinchas and Parker (2002), who specify income Y_t to be log-normal and characterized by a deterministic permanent income growth G_t , permanent shocks, and transitory shocks, which allow for a low probability of unemployment or illness

$$Y_t = P_t N_t^T = P_{t-1} G_t N_t^P N_t^T$$

$$N_t^T = \left\{ \begin{array}{ll} e^{s_t^T} & \text{with probability } 1 - p \text{ and } s_t^T \sim N(\mu_T, \sigma_T^2) \\ 0 & \text{with probability } p \end{array} \right\} N_t^P = e^{s_t^P} s_t^P \sim N(\mu_P, \sigma_P^2).$$

The life-cycle literature suggests fairly tight ranges for the parameters of the log-normal income process, which are approximately $\mu_T = \mu_P = 0$, $\sigma_T = \sigma_P = 0.1$, and $p \approx 0$. I choose the parameters for the exponential-utility model to roughly generate the volatility of the log-normal income pro-

³⁶The power-utility model cannot be solved analytically, but it can be solved by numerical backward induction, as shown by Gourinchas and Parker (2002) or Carroll (2001), among others. The numerical solution is illustrated in greater depth in Appendix B.5.4.

cess, i.e., $\mu_T = \mu_P = 0$ and $\sigma_T = \sigma_P = 0.06$ because the log-normal variance is $e^{0.5\sigma^2}(1 - e^{0.5\sigma^2})$. G_t typically implies a hump-shaped income profile. Nevertheless, I initially assume that $G_t = 1$ for all t to highlight the model's predictions in an environment that does not simply generate a hump-shaped consumption profile via a hump-shaped income profile. Additionally, I choose the agent's horizon T , his retirement period R , his initial wealth A_0 and P_0 , and the interest rate r in accordance with the life-cycle literature.

The preference parameters. I refer to the literature for the choice of the standard preference parameters $\beta \approx 1$ and $\theta \in [0.5, 4]$ but discuss the news-utility parameter values, i.e., η , λ , and γ , in greater detail. In particular, I demonstrate which choices are consistent with existing micro evidence on risk and time preferences. In Table IV in Appendix A, I illustrate the risk preferences over gambles with various stakes of the news-utility, standard, and habit-formation agents. In particular, I calculate the required gain G for a range of losses L to make each agent indifferent between accepting or rejecting a 50-50 win G or lose L gamble at a wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and Szeidl (2007).³⁷

First, I want to match risk attitudes towards bets regarding immediate consumption, which are determined solely by η and λ because it can be reasonably assumed that utility over immediate consumption is linear. Thus, $\eta = 1$ and $\lambda \approx 2.5$ are suggested by the laboratory evidence on loss aversion over immediate consumption, i.e., the endowment effect literature.³⁸ Of course, when I assume linear utility over immediate consumption, the standard and habit-formation agents are

³⁷In a canonical asset-pricing model, Pagel (2012) demonstrates that news-utility preferences constitute an additional step towards resolving the equity-premium puzzle, as they match the historical level and the variation of the equity premium while simultaneously implying plausible attitudes towards small and large wealth bets.

³⁸For illustration, I borrow a concrete example from Kahneman et al. (1990), in which the authors distribute a good (mugs or pens) to half of their subjects and ask those who received the good about their willingness to accept (WTA) and those who did not receive it about their willingness to pay (WTP) if they traded the good. The median WTA is \$5.25, whereas the median WTP is \$2.75. Accordingly, I infer $(1 + \eta)u(\text{mug}) = (1 + \eta\lambda)2.25$ and $(1 + \eta\lambda)u(\text{mug}) = (1 + \eta)5.25$, which implies that $\lambda \approx 3$ when $\eta \approx 1$. I obtain a similar result for the pen experiment. Unfortunately, thus far, I can only jointly identify η and λ . If the news-utility agent were only to exhibit gain-loss utility, I would obtain $\eta\lambda 2.25 \approx 5.25$ and $\eta 2.25 \approx 2.25$, i.e., $\lambda \approx 2.3$ and $\eta \approx 1$ both identified. Alternatively, if I assume that the market price for mugs (or pens), which is \$6 in the experiment (or \$3.75), equals $(1 + \eta)u(\text{mug})$ (or $(1 + \eta)u(\text{pen})$), then I can estimate $\eta = 0.74$ and $\lambda = 2.03$ for the mug experiment and $\eta = 1.09$ and $\lambda = 2.1$ for the pen experiment. These latter assumptions are reasonable given the induced-market experiments of Kahneman et al. (1990). $\eta = 1$ and $\lambda \approx 2.5$ thus appear to be reasonable choices that are typically used in the literature concerning the static preferences.

TABLE I:
CALIBRATION OF THE EXPONENTIAL- AND POWER-UTILITY MODELS

Exponential-utility model															
μ_P	σ_P	μ_T	σ_T	β	r	θ	η	λ	γ	A_0	P_0				
0	5%	0	7%	0.978	2%	2	1	2	0.75	0	0.1				
Power-utility model															
μ_P	σ_P	μ_T	σ_T	G_t	p	β	r	θ	η	λ	γ	h	b	τ	$\frac{A_0}{P_0}$
0	0.1	$\frac{-p}{1-p}$	0.1	1	0.01	0.97	1%	2	1	2	0.8	0.45	0.8	0.1	0.3

risk neutral. In contrast in Table IV, it can be seen that the news-utility agent’s contemporaneous gain-loss utility generates reasonable attitudes towards small and large gambles over immediate consumption. Second, I elicit the agents’ risk attitudes by assuming that each of them is presented the gamble after all consumption in that period has taken place. The news-utility agent will only experience prospective gain-loss utility over the gamble’s outcome that is determined by γ . Empirical estimates for the quasi-hyperbolic parameter β in the $\beta\delta$ –model typically range between 0.7 and 0.8 (e.g., Laibson et al. (2012)). Thus, the experimental and field evidence on peoples’ attitudes towards intertemporal consumption trade-offs dictates a choice of $b = \gamma \approx 0.75$ when $\beta \approx 1$. In Table IV, it can be seen that the news-utility agent’s risk attitudes take reasonable values for small, medium, and large stakes. The habit-formation agent is risk neutral for small and medium stakes and somewhat more risk averse for large stakes than the standard agent, who only exhibits reasonable risk attitudes for very large stakes. For the habit-formation agent, I roughly follow Michaelides (2002) and choose $h = 0.45$ to match the excess-sensitivity evidence. The tempted agent’s additional preference parameter $\tau = \frac{\lambda^{td}}{1+\lambda^{td}} = 0.1$ is chosen according to the estimates of Bucciol (2012). The calibrations of the exponential-utility and power-utility models can be found in Table I.

V.2 The model’s quantitative predictions

Excess smoothness and excess sensitivity in consumption. To illustrate the quantitative implications of excess sensitivity in the exponential-utility model, I run the data-free linear regression

TABLE II:
EXCESS-SMOOTHNESS AND SENSITIVITY REGRESSION RESULTS

Model	news-utility		habit		standard		hyperbolic		tempted	
	β_1	β_2	β_1^h	β_2^h	β_1^s	β_2^s	β_1^b	β_2^b	β_1^{td}	β_2^{td}
coefficient	0.67	0.27	0.69	0.38	0.93	0.01	0.94	0.01	1.01	-0.01
t-statistic	86.7	34.2	141	76.3	187	1.32	205	1.33	135	-1.76
e-s ratio	0.74		0.80		0.95		0.96		1.04	

Average results of 200 regressions with $N = 200$ simulated data points each at normalized wealth level $\frac{A_0}{P_0} = 1$ and date $t = 50$. The regression results are similar for different wealth levels and time horizons and nearly identical when I aggregate consumption to then run the regression instead of averaging the estimates.

of consumption growth on income

$$\Delta C_{t+1} = \alpha + \beta_1 \Delta Y_{t+1} + \beta_2 \Delta Y_t + \varepsilon_{t+1}.$$

For the news-utility model, I obtain $\beta_1 \approx 0.22$ and $\beta_2 \approx 0.18$ and a marginal propensity to consume out of permanent shocks of approximately 71%, whereas the marginal propensity is one for the standard model.³⁹ To illustrate the exponential-utility model, Figure I displays the consumption functions for realizations within two standard deviations of each shock, while the other is held constant. The flatter part of the news-utility consumption function generates excess smoothness and sensitivity.⁴⁰

For the power-utility model, I follow Campbell and Deaton (1989) and run the data-free regression

$$\Delta \log(C_{t+1}) = \alpha + \beta_1 \Delta \log(Y_{t+1}) + \beta_2 \Delta \log(Y_t) + \varepsilon_{t+1}.$$

The results are displayed in Table II. In aggregate data, regressing consumption growth on lagged

³⁹Running the regression

$$\Delta C_{t+1} = \alpha + \beta_1 (s_{t+1}^P - \mu_P) + \beta_2 \Delta (s_t^P - \mu_P) + \varepsilon_{t+1}$$

yields $\beta_1^s \approx 1$ and $\beta_2^s \approx 0$ in the standard model and $\beta_1 \approx 0.71$ and $\beta_2 \approx 0.31$ in the news-utility model. The transitory shock introduces a spurious positive correlation between ΔC_{t+1} and ΔY_t because $\Delta Y_{t+1} = s_{t+1}^P + s_{t+1}^T - s_t^T$ and $\Delta Y_t = s_t^P + s_t^T - s_{t-1}^T$.

⁴⁰All consumption adjustment takes place after a single period because the agent's preferences are characterized by full belief updating. However, the variation in Λ_t increases end-of-period asset holdings and is thereby spread out over the entire future. Empirically, Fuhrer (2000) and Reis (2006) find that consumption peaks one year after the shock and that the consumption response dies out briefly after this delayed response.

labor income growth, I obtain an OLS estimate for β_2 of approximately 0.23 and an excess-smoothness ratio, i.e., $\frac{\sigma(\Delta \log(C_t))}{\sigma(\Delta \log(Y_t))}$ as defined in Deaton (1986), of approximately 0.68.⁴¹ In the model, I obtain $\beta_2 \approx 0.27$ and $\frac{\sigma(\Delta \log(C_{t+1}))}{\sigma_p} \approx 0.74$, whereas in the standard model, I obtain $\beta_2^s \approx 0.01$ and $\frac{\sigma(\Delta \log(C_{t+1}^s))}{\sigma_p} \approx 0.95$. In addition to the standard and hyperbolic agents, I display results for internal, multiplicative habit-formation preferences, as assumed in Michaelides (2002), and temptation-disutility preferences, as developed by Gul and Pesendorfer (2004), following the specification of Bucciol (2012). The utility specifications can be found in Appendix B.1. Unsurprisingly, temptation disutility does not generate excess smoothness and sensitivity, while habit formation does. However, habit formation appears to generate too little excess smoothness and too much excess sensitivity and has unrealistic implications for the life-cycle consumption profile, which I examine next.

The hump-shaped consumption profile. For the exponential-utility model, Figure I displays the news-utility and standard agent's lifetime consumption profiles. The figure displays the average consumption profile of 300 identical agents who encounter different realizations of s_t^P and s_t^T and the consumption profile if $s_t^P = 0$ and $s_t^T = 0$ for all t . As can be observed from the figure, the news-utility agent's consumption profile is hump shaped. In contrast, the standard agent's profile is V-shaped, which demonstrates that exponential utility and a random-walk income process do not promote the desired hump.

For the power-utility model, Figure II contrasts the five agents' consumption paths with the average CEX consumption and income data, which I explain in Section V.3. The habit-formation agent's consumption profile is shown only in part because he engages in extremely high wealth accumulation due to his high effective risk aversion, even if I choose a lower value for h than the one that fits the excess-sensitivity evidence.⁴² Hyperbolic-discounting preferences tilt the consumption profile upward at the beginning and downward at the end of life. Temptation disutility causes severe overconsumption at the beginning of life, which then dies out when alternative consump-

⁴¹I follow Ludvigson and Michaelides (2001) and use NIPA deflated total, nondurable, or services consumption and total disposable labor income for the years 1947 to 2011.

⁴²This result confirms a finding by Michaelides (2002).

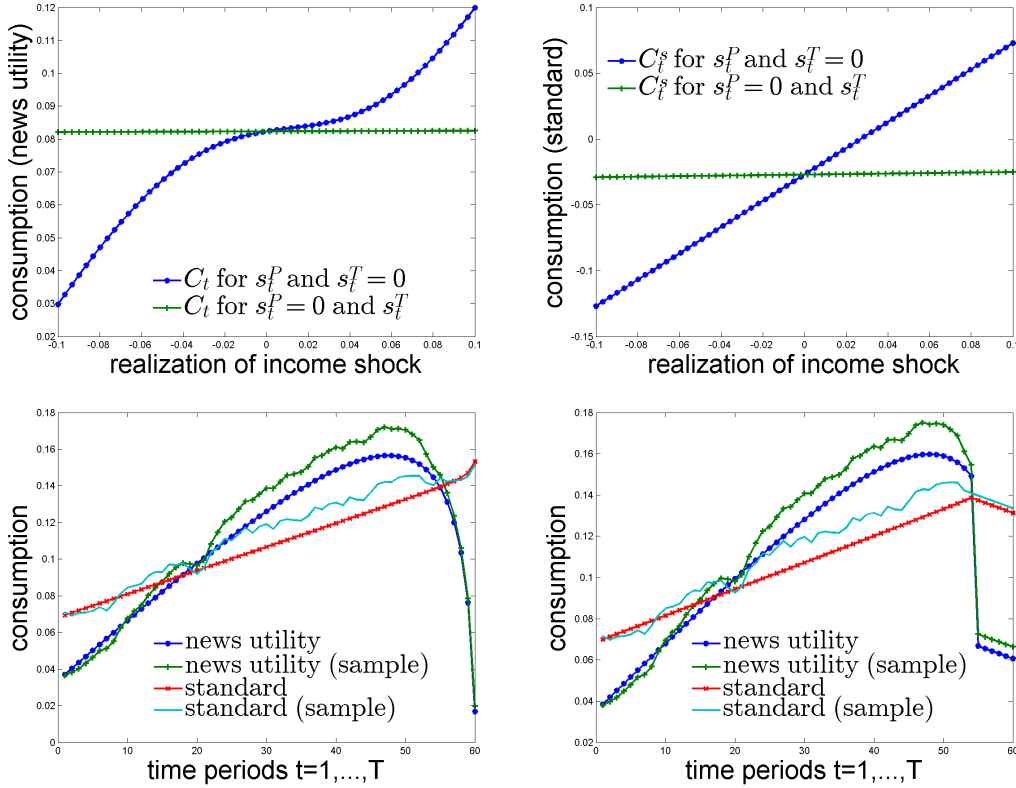


FIGURE I:
EXPONENTIAL-UTILITY CONSUMPTION FUNCTIONS AND LIFE-CYCLE PROFILES

tion opportunities diminish. All of the preference specifications except habit formation generate a hump-shaped consumption profile. The consumption path of all agents is increasing at the beginning of life because power utility renders them unwilling to borrow; however, all agents are sufficiently impatient such that consumption eventually decreases.⁴³ However, I argue that the news-utility agent's hump looks somewhat nicer with slowly increasing consumption at the beginning of life and decreasing consumption shortly before retirement.

⁴³Because power utility eliminates the possibility of negative or zero consumption and because of the small possibility of zero income in all future periods, the agents will never find it optimal to borrow. Moreover, power utility implies prudence such that all agents have a standard precautionary-savings motive. However, this motive is rather weak because the standard agent's consumption begins to decrease rather early in life. Moreover, in a model with only transitory shocks and no zero-income state, the precautionary savings motive is so weak that the standard agent's consumption is flat throughout. Attanasio (1999) criticizes this weak motive as lacking realism.

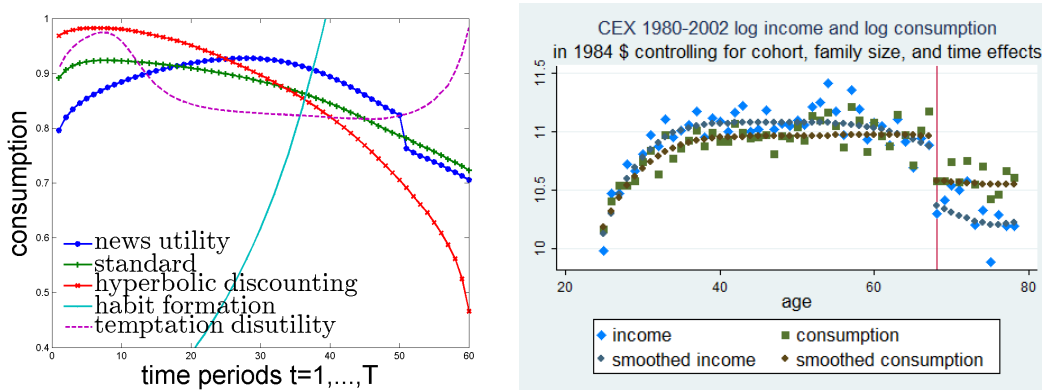


FIGURE II:
POWER-UTILITY LIFE-CYCLE PROFILES AND CEX CONSUMPTION AND INCOME DATA

The drop in consumption at retirement. For the exponential-utility model, Figure I shows a substantial drop in consumption at retirement. Such a drop can also be observed in Figure II for both the power-utility model and the CEX consumption data. Thus, I conclude that the news-utility agent’s lifetime consumption profile looks very similar to the average consumption profile from the CEX data, which I explain in greater detail in the next section.

V.3 Structural estimation

CEX data. I structurally estimate the news-utility parameters following Gourinchas and Parker (2002). I use data from the Consumer Expenditure Survey (CEX) for the years 1980 to 2002 as provided by the NBER. I generate a pseudo panel, which averages individual observations at each age, because the CEX does not survey households consecutively. To control for cohort, family size, and time effects, I employ average cohort techniques (see, e.g., Verbeek (2007), Attanasio (1998), and Deaton (1985)) and a fifth-order polynomial methodology. Because cohort and time effects are not separately identifiable, I proxy time effects with the regional unemployment rate. I deflate the data to 1984 dollars and assume that all agents retire at 68 and die after age 78. Figure II displays the average empirical income and average empirical consumption profiles.

Identification. Theoretically, the functional form of Λ_t , or the agent's first-order condition in the power-utility model, implies that news utility introduces such specific variation in consumption growth that all preference parameters are identified in the finite-horizon model, i.e., η , λ , γ , β , and θ , because the Jacobian has full rank.⁴⁴ Roughly speaking, the shape of the consumption profile identifies β and θ . Because consumption tracks income too closely and peaks too early in the standard model, $\eta > 0$ and $\lambda > 1$ can be identified. Finally, the drop in consumption at retirement identifies $\gamma < 1$.

Structural estimation results. I employ a two-stage method-of-simulated-moments procedure. In the first stage, I estimate all of the structural parameters governing the environment $\hat{\mu}_P$, $\hat{\sigma}_P$, $\hat{\mu}_T$, $\hat{\sigma}_T$, \hat{p} , \hat{G} , \hat{r} , \hat{a}_0 , \hat{R} , and \hat{T} and estimate $\hat{\mu}_P = -0.002$, $\hat{\mu}_T = -0.0031$, $\hat{\sigma}_P = 0.18$, $\hat{\sigma}_T = 0.16$, and $\hat{p} = 0.0031$ in accordance with the literature. The mean of Moody's municipal bond index is $r = 3.1\%$. Moreover, because 25 is chosen as the beginning of life by Gourinchas and Parker (2002), 68 is the average retirement age in the US according to the OECD, and 78 is the average life expectancy in the US according to the UN list, I choose $\hat{R} = 11$ and $\hat{T} = 54$. At age 25, I estimate the mean ratio of liquid wealth to income as 0.0096 under the assumption that $P_0 = 1$. In the second stage, I estimate the preference parameters β , θ , η , λ , and γ and obtain $\hat{\theta} = 0.79$, $\hat{\beta} = 0.97$, $\hat{\eta} = 1.1$, $\hat{\lambda} = 2.4$, and $\hat{\gamma} = 0.53$.⁴⁵ I display all first- and second-stage structural parameter estimates in Table III.⁴⁶ The preference parameters are estimated very tightly, and I cannot reject the overidentification test, which is a surprisingly positive result given the number of moments T

⁴⁴Numerically, I confirm this result in a Monte Carlo simulation and estimation exercise. Moreover, because previous studies cannot separately identify η and λ , I confirm that I obtain similar estimates when I assume $\eta = 1$ and only estimate the other parameters.

⁴⁵I estimate the second stage as follows. Let $\ln\tilde{C}_a = \frac{1}{n_a} \sum_{c=1}^{n_a} \ln(\tilde{C}_{i,a})$ be the average consumption at each age $a \in [25, 78]$ across all observations i , with $\ln(\tilde{C}_{i,a})$ being the household consumption data uncontaminated by cohort, time, number of earners, and family size effects. I minimize the sum of squares of the 65 data points $\ln\tilde{C}_a$ and the simulated equivalent $\ln\tilde{C}_a(\beta, \theta, \lambda, \gamma, \hat{\Xi})$, which depends on the parameters β , θ , η , λ , and γ and the estimated first-stage parameters $\hat{\Xi}$. I use a Nelder-Mead minimization algorithm with an optimal weighting matrix, i.e., the inverse of the variance of each point of $\ln\tilde{C}_a$. I obtain standard errors by numerically estimating the gradient of the moment function at its optimum neglecting first-stage estimation errors for the present.

⁴⁶Alternatively, I use a more complex set of moments to estimate the preference parameters, namely the degree of excess smoothness in consumption, the extent of the drop in consumption at retirement, and four other points of the life-cycle consumption profile. The resulting estimates and their standard errors are quantitatively very similar to the original ones.

**TABLE III:
FIRST-STAGE PARAMETER ESTIMATION RESULTS**

$\hat{\mu}_P$	$\hat{\sigma}_P$	$\hat{\mu}_T$	$\hat{\sigma}_T$	\hat{p}	\hat{G}_t	\hat{r}	P_0	$\frac{\hat{A}_0}{\hat{P}_0}$	\hat{R}	\hat{T}
0	0.19	0	0.15	0.0031	$e^{Y_{t+1}-Y_t}$	3.1%	1	0.0096	11	54
SECOND-STAGE PARAMETER ESTIMATION RESULTS										
news-utility model						standard model				
	$\hat{\beta}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\theta}$			
estimate	0.97	0.77	0.97	2.33	0.59	0.9	2.01			
standard error	(0.0002)	(0.007)	(0.039)	(0.006)	(0.008)	(0.0035)	(0.079)			
$\chi(\cdot)$	9.74			75.7						

The overidentification test's critical value at 5% is 67.5.

and the number of parameters, which is only five. In contrast, for the standard model, the standard errors are considerably larger and I reject the overidentification test, as do Gourinchas and Parker (2002). Finally, although I am hesitant to overinterpret this evidence, I obtain suggestive evidence for one of the new comparative statics generated by news utility; the excess-smoothness ratio in the CEX data increases from 0.68 at age 25 to 0.82 at the start of retirement.⁴⁷

VI EXTENSIONS AND WELFARE

In the following, I briefly outline three extensions of the basic life-cycle model that I have developed separately and the preferences' welfare implications.

Extensions. As a first extension, I introduce both illiquid savings and credit-card borrowing to demonstrate that the beliefs-based time inconsistency generates simultaneous demand for illiquid retirement savings and excessive credit-card borrowing. I assume that the agent can borrow against his illiquid savings up to his natural⁴⁸ borrowing constraint, which is determined by the discounted

⁴⁷For comparison, Figure III in Appendix A displays the consumption and income data of Gourinchas and Parker (2002) as well as the authors' fitted consumption profile (i.e. the standard model) and the fitted consumption of the news-utility model using the authors' baseline estimation results, which are displayed in Table V in Appendix A, as well as $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$ for the news-utility model. As noted by Gourinchas and Parker (2002), the standard agent's consumption peaks somewhat too early and increases too steeply with income growth. News utility causes consumption to peak later and to increase less steeply at the beginning of life.

⁴⁸Following Carroll (2001), I call this borrowing constraint natural because power utility and the possibility of zero income in all future periods induce the agent to never want to borrow beyond this constraint.

value of his accumulated illiquid savings. I again find that only the news-utility model is able to robustly generate the collection of life-cycle consumption facts. My findings differ from those of Laibson et al. (2012) because I do not assume the existence of non-natural borrowing constraints. Absent such constraints, only those hyperbolic agents at the margin of zero liquid asset holdings would delay consumption adjustments to shocks or tolerate a drop in consumption at retirement.

As a second extension, I let the agent endogenously determine his work hours in response to fluctuations in wages. In the event of an adverse shock, he can maintain high consumption by working more instead of consuming his savings. Thus, if the agent's labor supply is relatively elastic, his consumption becomes more excessively smooth and less excessively sensitive.

As a third extension, I allow the agent to invest in a risky asset in addition to his risk-free asset. I obtain three main implications for portfolio choice. First, the agent chooses a low portfolio share or does not participate in the stock market, as he is first-order risk averse. Second, his optimal portfolio share decreases in the return realization. In the event of a bad return realization, the agent chooses a higher portfolio share to avoid realizing all of the feelings of loss associated with future consumption. Third, the agent exhibits a time-inconsistency for risk. Given his beliefs, the agent is inclined to opt for a positive surprise with additional risk to enjoy the prospect of high future consumption, as he resides on a low-risk path. All of these predictions smooth the agent's risky asset holdings relative to the standard model. Thus, I obtain a novel prediction of stickiness in portfolio choice, which has been observed in household portfolio data by Calvet et al. (2009) or Brunnermeier and Nagel (2008).

Welfare. Beyond the observation that the news-utility agent is unable to follow his expected-utility maximizing path, the news-utility implications for welfare and the costs of business cycle fluctuations differ from those of the standard model. In Section IV, I demonstrate that income uncertainty has a first-order effect on savings and thus welfare in the news-utility model; i.e., the news-utility agent dislikes fluctuations in consumption much more than the standard agent. In the spirit of Lucas (1978), I compute the share λ_W of initial wealth A_1 that the agent would be willing

to give up for a risk-free consumption path. In the power-utility model for the calibration given in Table I, I obtain a share of approximately 47.83% for the news-utility agent, whereas the standard agent's share is 8.65%.

VII CONCLUSION

This paper demonstrates that expectations-based reference-dependent preferences can not only explain micro evidence, such as the endowment effect or consumer pricing and promotions, but they can also provide a unified explanation for three major life-cycle consumption facts. Loss aversion, a robust risk preference analyzed in experimental research and a popular explanation for the equity premium puzzle, generates excess smoothness and sensitivity in consumption, two of the most widely analyzed macro consumption puzzles. Intuitively, the agent wants to allow his expectations-based reference point to decrease or increase prior to adjusting consumption. Moreover, the interplay of news-utility risk and time preferences generate other predictions that are consistent with the evidence: a hump-shaped consumption profile and a drop in consumption at retirement, the first of which results from the net of two preference features. Loss aversion generates an additional precautionary-savings motive, which accumulates more rapidly than the standard precautionary-savings motive in the agent's horizon. Thus, the news-utility agent's consumption path is steeper at the beginning of life. But, the expectations-based reference point introduces what I term beliefs-based present bias: expected utility is higher on an optimal pre-committed consumption path in which the agent simultaneously optimizes over consumption and beliefs. Lacking an appropriate commitment device, however, the pre-committed path is non-feasible because the agent would deviate when taking his beliefs as given. Time consistency in equilibrium induces the agent to choose an overconsumption equilibrium path relative to the pre-committed one. However, once the retirement period begins, time-inconsistent overconsumption is associated with a certain loss in future consumption. Thus, the agent is suddenly able to behave himself, and the consumption path exhibits a drop at retirement. I explore the intuition for the model's results in depth by

solving an exponential-utility model in closed form in addition to numerically solving the standard life-cycle model. Moreover, I structurally estimate the model and obtain news-utility preference parameter estimates in line with the existing micro evidence.

In the future, I wish to further explore expectations-based reference dependence as a potential micro foundation for behavioral biases that have been widely documented. For instance, all of my life-cycle results support the notion that fluctuations in beliefs about consumption are painful. If people have some discretion in choosing how much information to gather, they might choose to “stick their head into the sand” occasionally to avoid fluctuations in beliefs that are painful on average; i.e., people are myopically loss averse. For instance, a long-term investor might choose to not check on his portfolio, particularly when he suspects that it might have decreased in value; this behavior has been termed the ostrich effect. Similarly, a CEO might choose to not evaluate a project when he suspects that it is performing poorly. An outsider, who acquires all information he does not have a stake in, will perceive the investor’s or CEO’s behavior as overconfident and extrapolative because their expectations are based on an overly favorable and outdated information set whenever they have received adverse but only incomplete information.

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A MORE FIGURES AND TABLES

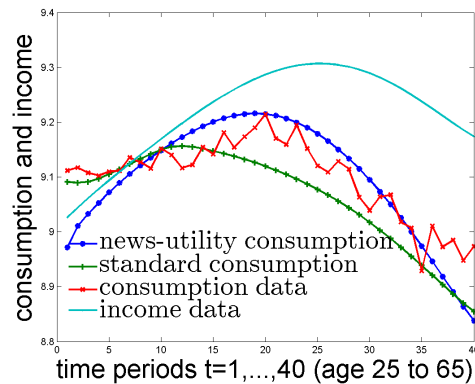
**TABLE IV:
RISK ATTITUDES OVER SMALL AND LARGE WEALTH BETS**

Loss (L)	standard	news-utility		habit-formation
		contemp.	prospective	
10	10	15	22	10
200	200	300	435	200
1000	1000	1500	2166	1000
5000	5000	7500	10719	5000
50000	50291	75000	105487	52502
100000	100406	150000	2066770	112040

For each loss L , the table's entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at a wealth level of 300,000 and a permanent income of 100,000 (power-utility model).

**TABLE V:
BASELINE ESTIMATION RESULTS OF GOURINCHAS AND PARKER (2002)**

μ_n	σ_n	μ_u	σ_u	p	r	β	θ	γ_0	γ_1	P_0	A_0	T
0	$\sqrt{0.044}$	0	$\sqrt{0.0212}$	0.00302	0.0344	0.9598	0.514	0.0701	0.071	1	0.3	40



**FIGURE III:
CONSUMPTION AND INCOME PROFILES AND THE FITTED MODEL'S CONSUMPTION
FROM GOURINCHAS AND PARKER (2002)**

The news-utility consumption follows the same specification except for the choice of news-utility parameters $\eta = 1$, $\lambda = 2$, and $\gamma = 0.85$.

B DERIVATIONS AND PROOFS

B.1 Summary of utility functions under consideration

I briefly summarize the lifetime utility of all preference specifications that I consider. I define the “news-utility” agent’s lifetime utility in each period $t = \{0, \dots, T\}$ as

$$u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t,t-1}) + E_t \left[\sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau} \right]$$

with $\beta \in [0, 1]$, $u(\cdot)$ a HARA⁴⁹ utility function, $\eta \in (0, \infty)$, $\lambda \in (1, \infty)$, and $\gamma \in [0, 1]$. Additionally, I first consider standard preferences as analyzed by Carroll (2001), Gourinchas and Parker (2002), and Deaton (1991), among many others. The “standard” agent’s lifetime utility is given by

$$u(C_t^s) + E_t \left[\sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^s) \right].$$

Second, I consider internal, multiplicative habit-formation preferences as assumed in Michaelides (2002). The “habit-forming” agent’s lifetime utility is given by

$$u(C_t^h) - hu(C_{t-1}^h) + E_t \left[\sum_{\tau=1}^{T-t} \beta^\tau (u(C_{t+\tau}^h) - hu(C_{t+\tau-1}^h)) \right]$$

with $h \in [0, 1]$.

Third, I consider $\beta\delta$ – or hyperbolic-discounting preferences as developed by Laibson (1997). The “ $\beta\delta$ –” or “hyperbolic-discounting” agent’s lifetime utility is given by

$$u(C_t^b) + bE_t \left[\sum_{\tau=1}^{T-t} \beta^\tau u(C_{t+\tau}^b) \right]$$

with $b \in [0, 1]$ corresponding to the $\beta\delta$ –agent’s β .

Fourth, I consider temptation-disutility preferences as developed by Gul and Pesendorfer (2004) following the specification of Bucciol (2012). The “tempted” agent’s lifetime utility is given by

$$u(C_t^{td}) - \lambda^{td} (u(\tilde{C}_t^{td}) - u(C_t^{td})) + E_t \left[\sum_{\tau=1}^{T-t} \beta^\tau (u(C_{t+\tau}^{td}) - \lambda^{td} (u(\tilde{C}_{t+\tau}^{td}) - u(C_{t+\tau}^{td}))) \right]$$

with \tilde{C}_t^{td} being the most tempting alternative consumption level and $\lambda^{td} \in [0, \infty)$.

⁴⁹A utility function $u(c)$ is said to exhibit hyperbolic absolute risk aversion (HARA) if the level of risk tolerance, $-\frac{u''(c)}{u'(c)}$ is a linear function of c .

B.2 Derivation of the exponential-utility model

B.2.1 The finite-horizon model

A simple derivation of the second-to-last period can be found in the text. The exponential-utility model can be solved through backward induction. In the following, I outline the model's solution for period $T - i$ in which the agent chooses how much to consume C_{T-i} and how much to invest in the risk-free asset A_{T-i} . I guess and verify the model's consumption function

$$C_{T-i} = \frac{(1+r)^i}{f(i)} (1+r)A_{T-i-1} + P_{T-i-1} + s_{T-i}^P + \left(1 - \frac{f(i-1)}{f(i)}\right) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}$$

with

$$\Lambda_{T-i} = \frac{1}{\theta} \log \left(\frac{(1+r)^i \psi_{T-i} + \gamma Q_{T-i} (\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)))}{f(i-1) \left(1 + \eta F(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T)) \right)} \right),$$

and $f(i) = \sum_{j=0}^i (1+r)^j = (1+r)^i \frac{1+r - (\frac{1}{1+r})^i}{r}$ (in the text $a(i) = \frac{f(i-1)}{f(i)}$). Then, the budget constraint $A_{T-i} = (1+r)A_{T-i-1} + Y_{T-i} - C_{T-i}$ determines end-of-period asset holdings

$$A_{T-i} = \frac{f(i-1)}{f(i)} (1+r)A_{T-i-1} + \frac{f(i-1)}{f(i)} s_{T-i}^T + \frac{f(i-1)}{f(i)} \Lambda_{T-i}.$$

Λ_{T-i} is a function independent of A_{T-i-1} and P_{T-i-1} but dependent on s_{T-i}^P and s_{T-i}^T . In the last period the agent consumes everything such that $\Lambda_T = 0$. As a first step to verify the solution guess, I sum up the expectation of the discounted consumption function utilities from period $T - i$ to T

$$\begin{aligned} \beta E_{T-i-1} \left[\sum_{\tau=0}^i \beta^\tau u(C_{T-i+\tau}) \right] &= u \left(P_{T-i-1} + \frac{(1+r)^i}{f(i)} (1+r)A_{T-i-1} \right) Q_{T-i-1} \\ &= -\frac{1}{\theta} \exp \left\{ -\theta \left(P_{T-i-1} + \frac{(1+r)^i}{f(i)} (1+r)A_{T-i-1} \right) \right\} Q_{T-i-1}, \end{aligned}$$

with Q_{T-i-1} given by

$$Q_{T-i-1} = \beta E_{T-i-1} \left[\exp \left\{ -\theta \left(s_{T-i}^P + \left(1 - \frac{f(i-1)}{f(i)}\right) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i} \right) \right\} + \exp \left\{ -\theta \left(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T + \frac{(1+r)^i}{f(i)} \Lambda_{T-i} \right) \right\} Q_{T-i} \right]$$

Q_{T-i-1} is a constant if Λ_{T-i} depends only on s_{T-i}^P and s_{T-i}^T . To derive the above sum, I simply plug in the asset-holding function into each future consumption function. For instance, C_{T-i+1} is given by

$$C_{T-i+1} = \frac{(1+r)^{i-1}}{f(i-1)} (1+r)A_{T-i} + P_{T-i} + s_{T-i+1}^P + \left(1 - \frac{f(i-2)}{f(i-1)}\right) s_{T-i+1}^T - \frac{f(i-2)}{f(i-1)} \Lambda_{T-i+1}$$

$$= \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1} + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i} + P_{T-i-1} + s_{T-i}^P + s_{T-i+1}^P + (1 - \frac{f(i-2)}{f(i-1)})s_{T-i+1}^T - \frac{f(i-2)}{f(i-1)}\Lambda_{T-i+1}.$$

The consumption function and it's sum allows me to write down the agent's continuation utility in period $T-i-1$ as follows

$$u(P_{T-i-1} + \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})\psi_{T-i-1} = -\frac{1}{\theta} \exp\{-\theta(P_{T-i-1} + \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1})\}\psi_{T-i-1}$$

with ψ_{T-i-1} given by

$$\begin{aligned} \psi_{T-i-1} = & \beta E_{T-i-1}[\exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i})\} + \omega(\exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i})\}) \\ & + \gamma Q_{T-i} \omega(\exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i})\}) + \exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i})\}]\psi_{T-i} \end{aligned}$$

and $\omega(x)$ for any random variable $X \sim F_X$, where the realization is denoted x , is given by

$$\omega(x) = \eta \int_{-\infty}^x (x-y)dF_X(y) + \eta\lambda \int_x^{\infty} (x-y)dF_X(y).$$

The above expression for ψ_{T-i-1} can be easily inferred from the agent's utility function. The first component in ψ_{T-i-1} corresponds to the expectation of consumption utility in period $T-i$, the second to contemporaneous gain-loss in period $T-i$, the third to prospective gain-loss in period $T-i$ that depends on the sum of future consumption utilities Q_{T-i} , and the last to the agent's continuation value. Moreover, for any random variable $Y \sim F_Y = F_X$ note that

$$\begin{aligned} \int_{-\infty}^{\infty} \omega(g(x))dF_X(x) &= \int_{-\infty}^{\infty} \left\{ \eta \int_{-\infty}^x \underbrace{(g(x) - g(y))}_{<0 \text{ if } g'(\cdot) < 0} dF_Y(y) + \eta\lambda \int_x^{\infty} \underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot) < 0} dF_Y(y) \right\} dF_X(x) > 0 \\ \int_{-\infty}^{\infty} \left\{ \eta \int_{-\infty}^x \underbrace{(g(x) - g(y))}_{<0 \text{ if } g'(\cdot) < 0} dF_Y(y) + \eta \int_x^{\infty} \underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot) < 0} dF_Y(y) + \eta(\lambda - 1) \int_x^{\infty} \underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot) < 0} dF_Y(y) \right\} dF_X(x) &> 0 \\ &= \int_{-\infty}^{\infty} \left\{ \eta(\lambda - 1) \int_x^{\infty} \underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot) < 0} dF_Y(y) \right\} dF_X(x) > 0 \end{aligned}$$

if $\lambda > 1$, $\eta > 0$, and $g'(\cdot) < 0$. The above consideration implies that $\psi_{T-i-1} > Q_{T-i-1}$ necessarily if $\theta > 0$ such that $u(\cdot)$ is concave. Now, I turn to the agent's maximization problem in period $T-i$, which is given by

$$u(C_{T-i}) + n(C_{T-i}, F_{C_{T-i}}^{T-i-1}) + \gamma \sum_{\tau=1}^i \beta^\tau n(F_{C_{T-i+\tau}}^{T-i, T-i-1}) + u(P_{T-i} + \frac{(1+r)^{i-1}}{f(i-1)}A_{T-i})\psi_{T-i}.$$

I want to find the agent's first-order condition. I begin by explaining the first derivative of contemporaneous gain-loss utility $n(C_{T-i}, F_{C_{T-i}}^{T-i-1})$. The agent takes his beliefs about period $T-i$

consumption $F_{C_{T-i}}^{T-i-1}$ as given such that

$$\frac{\partial n(C_{T-i}, F_{C_{T-i}}^{T-i-1})}{\partial C_{T-i}} = \frac{\partial(\eta \int_{-\infty}^{C_{T-i}} (u(C_{T-i}) - u(c)) dF_{C_{T-i}}^{T-i-1}(c)) + \eta \lambda \int_{C_{T-i}}^{\infty} (u(C_{T-i}) - u(c)) dF_{C_{T-i}}^{T-i-1}(c)}{\partial C_{T-i}}$$

$$= u'(C_{T-i})(\eta F_{C_{T-i}}^{T-i-1}(C_{T-i}) + \eta \lambda (1 - F_{C_{T-i}}^{T-i-1}(C_{T-i}))) = u'(C_{T-i})(\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)))$$

the last step results from the guessed consumption function and the assumption that admissible consumption functions are increasing in both shocks. Here, I abuse notation somewhat by writing $F(\cdot) = F_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T}(\cdot)$. The first derivative of the agent's prospective gain-loss utility

$\sum_{\tau=1}^i \beta^\tau n(F_{C_{T-i+\tau}}^{T-i, T-i-1})$ over the entire stream of future consumption utilities $u(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}) Q_{T-i}$ can be inferred in a similar manner. Recall that Q_{T-i} is a constant under the guessed consumption function; thus, the agent only experiences gain-loss utility over the realized uncertainty in period $T-i$, i.e.,

$$\begin{aligned} \frac{\partial \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{T-i+\tau}}^{T-i-1, T-i})}{\partial A_{T-i}} &= \sum_{\tau=1}^{\infty} \beta^\tau \frac{\partial}{\partial A_{T-i}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(c) - u(r)) dF_{C_{T-i+\tau}}^{T-i-1, T-i}(c, r) \\ &= \frac{\partial}{\partial A_{T-i}} \int_{-\infty}^{\infty} \mu(u(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}) Q_{T-i} - u(x) Q_{T-i}) dF_{P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i}}^{T-i-1}(x) \\ &= \frac{(1+r)^i}{f(i-1)} \exp\{-\theta(P_{T-i} + \frac{(1+r)^i}{f(i-1)} A_{T-i})\} Q_{T-i} (\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))) \end{aligned}$$

and again, $F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)$ results from the solution guess for A_{T-i} times $\frac{(1+r)^i}{f(i-1)}$ and the fact that future admissible consumption is increasing in both shocks. The derivative of the agent's continuation utility with respect to A_{T-i} is simply given by

$$\frac{(1+r)^i}{f(i-1)} \exp\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\} \psi_{T-i}.$$

In turn, in any period $T-i$ the news-utility agent's first-order condition (normalized by P_{T-i}) is given by

$$\begin{aligned} \underbrace{\exp\{-\theta((1+r)A_{T-i-1} + s_{T-i}^T - A_{T-i})\}}_{=-\theta(C_{T-i} - P_{T-i}) \text{ budget constraint}} (1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))) \\ = \frac{(1+r)^i}{f(i-1)} \exp\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\} (\psi_{T-i} + \gamma Q_{T-i} (\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))). \end{aligned}$$

The first-order condition can be rewritten to obtain the optimal consumption and end-of-period

asset holdings functions and the function Λ_{T-i}

$$\Lambda_{T-i} = \frac{1}{\theta} \log \left(\frac{(1+r)^i \Psi_{T-i} + \gamma Q_{T-i} (\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)))}{f(i-1) 1 + \eta F(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T))} \right)$$

and the guessed consumption function can be verified.

B.2.2 The infinite-horizon model

Suppose $\sigma_{P_t} = \sigma_P$ and $\sigma_{T_t} = \sigma_T$ for all t and $T, i \rightarrow \infty$. I use a simple guess and verify procedure to find the infinite-horizon recursive equilibrium; alternatively, the solution can be obtained by simple backward induction taking T and i to infinity. The infinite-horizon model consumption and asset-holding functions are given by

$$C_t = Y_t + rA_{t-1} - \frac{1}{1+r} s_t^T - \Lambda_t = P_{t-1} + s_t^P + rA_{t-1} + \frac{r}{1+r} s_t^T - \Lambda_t \text{ and } A_t = A_{t-1} + \frac{1}{1+r} s_t^T + \Lambda_t.$$

The first-order condition normalized by P_t is given by

$$\begin{aligned} & \exp\{-\theta(1+r)A_{t-1} - \theta s_t^T + \theta A_t\} (1 + \eta F(s_t^P + \frac{r}{1+r} s_t^T) + \eta \lambda (1 - F(s_t^P + \frac{r}{1+r} s_t^T))) \\ & = \text{rexp}\{-\theta r A_t\} (\psi + \gamma Q (\eta F(s_t^P + \frac{r}{1+r} s_t^T) + \eta \lambda (1 - F(s_t^P + \frac{r}{1+r} s_t^T)))). \end{aligned}$$

Solving for optimal end-of-period asset holdings yields

$$A_t = A_{t-1} + \frac{1}{1+r} s_t^T + \underbrace{\frac{1}{\theta(1+r)} \log \left(r \frac{\psi + \gamma Q (\eta F(s_t^P + \frac{r}{1+r} s_t^T) + \eta \lambda (1 - F(s_t^P + \frac{r}{1+r} s_t^T)))}{1 + \eta F(s_t^P + \frac{r}{1+r} s_t^T) + \eta \lambda (1 - F(s_t^P + \frac{r}{1+r} s_t^T))} \right)}_{=\Lambda_t}.$$

Consumption is then determined by the budget constraint

$$C_t = Y_t + rA_{t-1} - \frac{1}{1+r} s_t^T - \Lambda_t = P_{t-1} + s_t^P + rA_{t-1} + \frac{r}{1+r} s_t^T - \Lambda_t.$$

Q and ψ are constant in an i.i.d. environment and given by

$$\begin{aligned} Q &= \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T - \Lambda_{t+1})\}]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T + r\Lambda_{t+1})\}]} \\ \psi &= \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T - \Lambda_{t+1})\}] + \omega(\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T - \Lambda_{t+1})\}) + \gamma Q \omega(\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T + r\Lambda_{t+1})\})]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T + r\Lambda_{t+1})\}]} \end{aligned}$$

B.2.3 The optimal pre-committed equilibrium

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any gain-loss utility. Thus, in period zero the agent chooses optimal consumption in period t in each possible contingency jointly with his beliefs, which of course coincide with the agent's optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for $C(Y^*)$ when income Y^* has been realized

$$\begin{aligned} & \frac{\partial}{\partial C(Y^*)} \left\{ \int \int \mu(u(C(Y)) - u(C(Y'))) dF_Y(Y') dF_Y(Y) \right\} \\ = & \frac{\partial}{\partial C(Y^*)} \int \eta \int_{-\infty}^Y \{ (u(C(Y)) - u(C(Y'))) dF_Y(Y') + \eta \lambda \int_Y^{\infty} (u(C(Y)) - u(C(Y'))) dF_Y(Y') \} dF_Y(Y) \\ & = u'(C(Y^*)) (\eta F_Y(Y^*) + \eta \lambda (1 - F_Y(Y^*))) - u'(C(Y^*)) (\eta (1 - F_Y(Y^*)) + \eta \lambda F_Y(Y^*)) \\ & = u'(C(Y^*)) \eta (\lambda - 1) (1 - 2F_Y(Y^*)) \text{ with } \eta (\lambda - 1) (1 - 2F_Y(Y^*)) > 0 \text{ for } F_Y(Y^*) < 0.5. \end{aligned}$$

Consider the difference to the term in the initial first-order condition $u'(C_t)(\eta F_{C_t}^{t-1}(C_t) + \eta \lambda (1 - F_{C_t}^{t-1}(C_t)))$: when choosing the pre-committed plan the additional utility of increasing consumption a little bit is no longer only made up of the additional step in the probability distribution; instead the two additional negative terms consider that in all other states of the world the agent experiences less gain feelings and more loss feelings because of increasing consumption in that contingency. The equation says that the marginal utility of state Y^* will be increased by news utility if the realization is below the median. For realizations above the median the marginal utility will be decreased and the agent will consume relatively less.

Unfortunately there is a problem arising in the pre-commitment optimization problem that has been absent in the non-pre-committed one: When beliefs are taken as given the agent optimizes over two concave functions consumption utility and the first part of gain-loss utility, accordingly the first-order condition pins down a maximum. In contrast, when the agent chooses his beliefs simultaneously to his consumption additionally the second convex part of gain-loss utility is optimized over. The additional part determining marginal utility $-u'(C_t)(\eta (1 - F_{C_t}^{t-1}(C_t))) + \eta \lambda F_{C_t}^{t-1}(C_t))$ is largest for particular good income realizations, since increasing consumption in these states implies additional loss feelings in almost all other states of the world. It can be easily shown that the sufficient condition of the optimization problem holds if the parameters satisfy following simple condition: $\eta (\lambda - 1) (2F_{C_t}^{t-1}(C_t) - 1) < 1$. Accordingly, for $\eta (\lambda - 1) < 1$, which is true for a range of commonly used parameter combinations, the first-order condition pins down the optimum.

From the above consideration it can be easily inferred that the optimal pre-committed consumption function in the exponential-utility model is thus given by

$$\Lambda_{T-i}^c = \frac{1}{\theta} \log \left(\frac{(1+r)^i \psi_{T-i}^c + \gamma Q_{T-i}^c \eta (\lambda - 1) (1 - 2F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))}{f(i-1) \left(1 + \eta (\lambda - 1) (1 - 2F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)) \right)} \right)$$

with

$$Q_{T-i-1}^c = E_{T-i-1}[\beta \exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^c)\} + \beta \exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i}^c)\} Q_{T-i}^c]$$

and

$$\begin{aligned} \psi_{T-i-1}^c &= \beta E_{T-i-1}[\exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^c)\} + \omega(\exp(-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^c)))] \\ &+ \gamma Q_{T-i}^c \omega(\exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i}^c)\}) + \frac{1}{\theta} \exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i}^c)\} \psi_{T-i}^c. \end{aligned}$$

B.3 The other agent's exponential-utility consumption functions

By the same arguments as for the derivation of the news-utility model, the “standard” agent's consumption function in period $T - i$ is

$$\begin{aligned} A_{T-i}^s &= \frac{f(i-1)}{f(i)}(1+r)A_{T-i-1}^s + \frac{f(i-1)}{f(i)}s_{T-i}^T + \frac{f(i-1)}{f(i)}\Lambda_{T-i}^s \\ C_{T-i}^s &= \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1}^s + P_{T-i-1} + s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^s \\ \Lambda_{T-i}^s &= \frac{1}{\theta} \log\left(\frac{(1+r)^i}{f(i-1)}Q_{T-i}^s\right) \end{aligned}$$

$$Q_{T-i-1}^s = \beta E_{T-i-1}[\exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^s)\} + \beta \exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i}^s)\} Q_{T-i}^s].$$

In the infinite-horizon equilibrium $Q^s = \psi^s$ in an i.i.d. environment with $\sigma_{P_t} = \sigma_P$ and $\sigma_{T_t} = \sigma_T$ for all t

$$\begin{aligned} A_t^s &= A_{t-1}^s + \frac{1}{1+r}s_t^T + \underbrace{\frac{1}{\theta(1+r)}\log(r\psi^s)}_{=\Lambda^s} \\ \psi^s &= Q^s = \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T - \Lambda^s)\}]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r}s_{t+1}^T + r\Lambda^s)\}]} \\ C_t^s &= Y_t + rA_{t-1}^s - \frac{1}{1+r}s_t^T - \Lambda^s = P_t + s_t^P + rA_{t-1}^s + \frac{r}{1+r}s_t^T - \Lambda^s. \end{aligned}$$

The “tempted” agent's maximization problem is given by

$$\max_{C_t^{td}} \{u(C_t^{td}) - \lambda^{td}(u(\tilde{C}_t^{td}) - u(C_t^{td})) + E_t[\sum_{\tau=1}^{T-t} \beta^\tau (u(C_{t+\tau}^{td}) - \lambda^{td}(u(\tilde{C}_{t+\tau}^{td}) - u(C_{t+\tau}^{td})))]\}$$

with $\tilde{C}_{t+\tau}^{td}$ being the most tempting alternative. In period T as the agent cannot die in debt the most tempting alternative is $\tilde{C}_T^{td} = X_T^{td}$ but the agent will consume X_T anyway thus temptation disutility

is zero and $Q_{T-1}^{td} = Q_{T-1}^s$. In period $T - 1$ the agent's consumption is then given by

$$A_{T-1}^{td} = \frac{1+r}{2+r} A_{T-2}^{td} + \frac{1}{2+r} s_{T-1}^T + \frac{1}{2+r} \Lambda_{T-1}^{td}$$

$$C_{T-1}^{td} = \frac{1+r}{2+r} (1+r) A_{T-2}^{td} + P_{T-2} + s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T - \frac{1}{2+r} \Lambda_{T-1}^{td}$$

$$\text{with } \Lambda_{T-1}^{td} = \frac{1}{\theta} \log\left((1+r) \frac{1}{1+\lambda^{td}} Q_{T-1}^{td}\right) \text{ and } Q_{T-1}^{td} = \beta E_{T-1}[\exp\{-\theta(s_T^P + s_T^T)\}].$$

What's the agent's most tempting alternative in period $T - 1$? The value of cash-on-hand is X_{T-1}^{td} but the most tempting alternative is $\tilde{C}_T^{td} \rightarrow \infty$ as consumption could be negative in the last period $C_T \rightarrow -\infty$, which would yield $\lim_{C_T^{td} \rightarrow -\infty} u(C_T^{td}) = \lim_{C_T^{td} \rightarrow -\infty} -\frac{1}{\theta} e^{-\theta C_T^{td}} \rightarrow -\infty$. Accordingly, Q_{T-2}^{td} enters $\lim_{\tilde{C}_T^{td} \rightarrow -\infty} u'(\tilde{C}_T^{td}) = \lim_{\tilde{C}_T^{td} \rightarrow -\infty} e^{-\theta \tilde{C}_T^{td}} \rightarrow 0$

$$Q_{T-2}^{td} = \beta E_{T-2}[\exp\{-\theta(s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T - \frac{1}{2+r} \Lambda_{T-1}^{td})\} - \lambda^{td} (\exp\{-\theta(s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T - \frac{1}{2+r} \Lambda_{T-1}^{td})\}) \\ - \underbrace{\exp\{\theta \frac{1+r}{2+r} (1+r) A_{T-2}^{td} - \theta \tilde{C}_T^{td}\}}_{\rightarrow 0} + \exp\{-\theta(s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T + \frac{1+r}{2+r} \Lambda_{T-1}^{td})\} Q_{T-1}^{td}]$$

$$Q_{T-2}^{td} = E_{T-2}[\beta \exp\{-\theta(s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T - \frac{1}{2+r} \Lambda_{T-1}^{td})\} (1 - \lambda^{td}) + \beta \exp\{-\theta(s_{T-1}^P + \frac{1+r}{2+r} s_{T-1}^T + \frac{1+r}{2+r} \Lambda_{T-1}^{td})\} Q_{T-1}^{td}].$$

And in period $T - i$

$$A_{T-i}^{td} = \frac{f(i-1)}{f(i)} (1+r) A_{T-i-1}^{td} + \frac{f(i-1)}{f(i)} s_{T-i}^T + \frac{f(i-1)}{f(i)} \Lambda_{T-i}^{td}$$

$$C_{T-i}^{td} = \frac{(1+r)^i}{f(i)} (1+r) A_{T-i-1}^{td} + P_{T-i-1} + s_{T-1}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^{td}$$

$$\Lambda_{T-i}^{td} = \frac{1}{\theta} \log\left(\frac{(1+r)^i}{f(i-1)} \frac{1}{1+\lambda^{td}} Q_{T-i}^{td}\right)$$

$$Q_{T-i-1}^{td} = \beta E_{T-i-1}[\exp\{-\theta(s_{T-1}^P + (1 - \frac{f(i-1)}{f(i)}) s_{T-i}^T - \frac{f(i-1)}{f(i)} \Lambda_{T-i}^{td})\} (1 - \lambda^{td}) + \beta \exp\{-\theta(s_{T-1}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T + \frac{(1+r)^i}{f(i)} \Lambda_{T-i}^{td})\} Q_{T-i}^{td}].$$

And for $T \rightarrow \infty$

$$A_t^{td} = A_{t-1}^{td} + \frac{1}{1+r} n_t + \underbrace{\frac{1}{\theta(1+r)} \log\left(r \frac{1}{1+\lambda^{td}} Q^{td}\right)}_{=\Lambda^{td}}$$

$$C_t^{td} = Y_t + r A_{t-1}^{td} - \frac{1}{1+r} n_t - \Lambda^{td} = P_{t-1} + s_t^P + r A_{t-1}^{td} + \frac{r}{1+r} s_t^T - \Lambda^{td}$$

$$Q^{td} = \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T - \Lambda^{td})\} (1 - \lambda^{td})]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T + r \Lambda^{td})\}]}$$

The “hyperbolic-discounting” agent’s consumption in period $T - i$ is

$$A_{T-i}^b = \frac{f(i-1)}{f(i)}(1+r)A_{T-i-1}^b + \frac{f(i-1)}{f(i)}s_{T-i}^T + \frac{f(i-1)}{f(i)}\Lambda_{T-i}^b$$

$$C_{T-i}^b = \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1}^b + P_{T-i-1} + s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^b$$

$$\Lambda_{T-i}^b = \frac{1}{\theta} \log\left(\frac{(1+r)^i}{f(i-1)} b Q_{T-i}^b\right)$$

$$Q_{T-i-1}^b = \beta E_{T-i-1}[\exp\{-\theta(s_{T-i}^P + (1 - \frac{f(i-1)}{f(i)})s_{T-i}^T - \frac{f(i-1)}{f(i)}\Lambda_{T-i}^b)\}] + \beta \exp\{-\theta(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T + \frac{(1+r)^i}{f(i)}\Lambda_{T-i}^b)\} Q_{T-i}^b].$$

and for $T \rightarrow \infty$

$$A_t^b = A_{t-1}^b + \frac{1}{1+r} s_t^T + \underbrace{\frac{1}{\theta(1+r)} \log(rbQ^b)}_{=\Lambda^b}$$

$$C_t^b = Y_t + rA_{t-1}^b - \frac{1}{1+r} s_t^T - \Lambda^b = P_{t-1} + s_t^P + rA_{t-1}^b + \frac{r}{1+r} s_t^T - \Lambda^b$$

$$Q^b = \frac{\beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T - \Lambda^b)\}]}{1 - \beta E_t[\exp\{-\theta(s_{t+1}^P + \frac{r}{1+r} s_{t+1}^T + r\Lambda^b)\}]}$$

B.4 Proofs of Section IV:

B.4.1 Proof of Proposition 1

If the consumption function derived in Section B.2.1 belongs to the class of admissible consumption functions then the equilibrium exists and is unique as the equilibrium solution is obtained by maximizing the agent’s objective function, which is globally concave, and there is a finite period that uniquely determines the equilibrium. Please refer to Section B.2.1 for the derivation of the consumption function. σ_t^* is implicitly defined by the two admissible consumption function restrictions $\frac{\partial C_{T-i}}{\partial s_{T-i}^P} > 0$ and $\frac{\partial C_{T-i}}{\partial s_{T-i}^T} > 0$ as

$$C_{T-i} = \frac{(1+r)^i}{f(i)}(1+r)A_{T-i-1} + P_{T-i-1} + s_{T-i}^P + (1 - a(i))s_{T-i}^T - a(i)\Lambda_{T-i}$$

the restrictions are equivalent to $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P} < 1$ and $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^T} < 1 - a(i)$ as $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P}, \frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^T} > 0$ (since $\Psi_{T-i} > \gamma Q_{T-i}$ (for any concave utility function which I have shown in Section B.2)). Recall that

$a(i) = 1 - \frac{(1+r)^i}{f(i)} = \frac{f(i-1)}{f(i)}$. Then, σ_{T-i}^* is implicitly defined by the two restrictions

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P} = \frac{a(i)}{\theta\left(\frac{1-a(i)}{a(i)}\right)} \frac{(\psi_{T-i} - \gamma Q_{T-i})\eta f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)(\lambda-1)}{1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))} < 1$$

and

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^T} = \frac{a(i)}{\theta\left(\frac{1-a(i)}{a(i)}\right)} \frac{(\psi_{T-i} - \gamma Q_{T-i})\eta f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)(\lambda-1)}{1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))} < 1 - a(i).$$

Here, the normal pdf of any random variable X is denoted by f_X . Increasing σ_{P_t} and σ_{T_t} unambiguously decreases $f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)$ and thereby $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P}$ and $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^T}$. Thus, there exists a condition $\sigma_{P_t}^2 + \left(\frac{(1+r)^i}{f(i)}\right)^2 \sigma_{T_t}^2 \geq \sigma_t^*$ for all t which ensures that an admissible consumption function exists that uniquely determines the equilibrium (given the admissible consumption functions in each future period until the final period) because the optimization problem is globally concave.

B.4.2 Proof of Proposition 2

Please refer to the derivation of the exponential-utility model Section B.2 for a detailed derivation of Λ_{T-i} . According to Definition 5 consumption is excessively smooth if $\frac{\partial C_t}{\partial s_t^P} < 1$ and excessively sensitive if $\frac{\partial \Delta C_{t+1}}{\partial s_t^P} > 0$. Consumption growth is

$$\Delta C_{T-i} = s_{T-i}^P + (1 - a(i))s_{T-i}^T - a(i)\Lambda_{T-i} + \Lambda_{T-i-1}$$

so that $\frac{\partial C_{T-i}}{\partial s_{T-i}^P} < 1$ iff $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P} > 0$ and $\frac{\partial \Delta C_{T-i}}{\partial s_{T-i-1}^P} > 0$ iff $\frac{\partial \Lambda_{T-i-1}}{\partial s_{T-i-1}^P} > 0$. Since $\psi_{T-i} > \gamma Q_{T-i}$ (for any concave utility function which I have shown in Section B.2) it can be easily seen that $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P} > 0$, i.e.

$$\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P} = \frac{1}{\theta\left(\frac{1-a(i)}{a(i)}\right)} \frac{(\psi_{T-i} - \gamma Q_{T-i})\eta f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)(\lambda-1)}{1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))} > 0.$$

The same holds true for the infinite-horizon model

$$\Delta C_t = s_t^P + \frac{r}{1+r} s_t^T - \Lambda_t + (1+r)\Lambda_{t-1}$$

as Λ_t is increasing in the permanent shock

$$\frac{\partial \Lambda_t}{\partial s_t^P} = \frac{1}{\theta(1+r)r\psi + \gamma Q(\eta F(s_t^P + \frac{r}{1+r}s_t^T) + \eta\lambda(1 - F(s_t^P + \frac{r}{1+r}s_t^T)))} \frac{(\psi - \gamma Q)\eta f_{s_t^P + \frac{r}{1+r}s_t^T}(s_t^P + \frac{r}{1+r}s_t^T)(\lambda - 1)}{1 + \eta F(s_t^P + \frac{r}{1+r}s_t^T) + \eta\lambda(1 - F(s_t^P + \frac{r}{1+r}s_t^T))} > 0.$$

Accordingly, $\frac{\partial \Lambda_t}{\partial s_t^P} > 0$ as $\psi > \gamma Q$. Thus, if $s_t^P \uparrow$ then $\Lambda_t \uparrow$ and the shock induced change in consumption is less than one and the period t shock induced change in one-period ahead consumption ΔC_{t+1} is larger than zero.

The difference to the standard, tempted, and quasi-hyperbolic discounting agents is that $\frac{\partial \Lambda_t^{s,t,b}}{\partial s_t^P} = 0$ for all t such that consumption is neither excessively sensitive nor excessively smooth.

B.4.3 Proof of Lemma 1

I start with the first part of the lemma, the precautionary-savings motive. In the second-to-last period of the simple model outlined in the text, the first-order condition is given by

$$\begin{aligned} & u'(C_{T-1}) + u'(C_{T-1})(\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))) \\ &= (1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P) \underbrace{\gamma \beta E_{T-1}[u'(S_T^P)]}_{Q_{T-1}} (\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))) \\ &+ (1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P) \underbrace{\beta E_{T-1}[u'(S_T^P) + \eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(y)) dF_P(y)]}_{\psi_{T-1}}. \end{aligned}$$

From Section B.2 I know that $\psi_{T-1} > Q_{T-1}$ because for any two random variables $X \sim F_X$ and $Y \sim F_Y$ with $F_X = F_Y$ in equilibrium

$$\int_{-\infty}^{\infty} \left\{ \eta \int_{-\infty}^x \underbrace{(g(x) - g(y))}_{<0 \text{ if } g'(\cdot) < 0} dF_Y(y) + \eta\lambda \int_x^{\infty} \underbrace{(g(x) - g(y))}_{>0 \text{ if } g'(\cdot) < 0} dF_Y(y) \right\} dF_X(x) > 0$$

if $\lambda > 1$, $\eta > 0$, and $g'(\cdot) < 0$. Thus, $\beta E_{T-1}[\eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(y)) dF_P(y)] > 0$ if $u''(\cdot) > 0$

the agent is risk averse or $u(\cdot)$ is concave. Moreover, it can be easily seen that $\frac{\partial \beta E_{T-1}[\eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(y)) dF_P(y)]}{\partial \eta} > 0$

0 and $\frac{\partial \beta E_{T-1}[\eta(\lambda - 1) \int_{S_T^P}^{\infty} (u'(S_T^P) - u'(y)) dF_P(y)]}{\partial \lambda} > 0$. Then, for any value of savings $A_{T-1} = s_{T-1}^P - C_{T-1}$ the right hand side of the first-order condition is increased by the presence of expected gain-loss disutility if $\sigma_P > 0$ whereas if $\sigma_P = 0$ then $\psi_{T-1} = Q_{T-1}$. The increase of the agent's marginal value of savings by the presence of expected gain-loss disutility depends on $\sigma_P > 0$, but does not go to zero as $\sigma_P \rightarrow 0$ so that the additional precautionary savings motive is first-order $\frac{\partial (s_{T-1}^P - C_{T-1})}{\partial \sigma_P} |_{\sigma_P=0} > 0$ as can be easily shown for any normally distribution random variable

$$X \sim F_X = N(\mu, \sigma^2)$$

$$\begin{aligned} & E_{T-1}[\eta(\lambda - 1) \int_X (u'(X) - u'(y)) dF_X(y)] \\ &= e^{-\theta\mu} \int_{-\infty}^{\infty} (\eta \int_{-\infty}^z (e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon}) dF_{01}(\varepsilon) + \eta\lambda \int_z^{\infty} (e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon}) dF_{01}(\varepsilon)) dF_{01}(z) \text{ with } z, \varepsilon \sim F_{01} = N(0, 1) \\ &= e^{-\theta\mu} \eta(\lambda - 1) \int_z^{\infty} (e^{-\theta\sigma z} - e^{-\theta\sigma\varepsilon}) dF_{01}(\varepsilon) dF_{01}(z) \\ &= e^{-\theta\mu} \eta(\lambda - 1) \int_{-\infty}^{\infty} \{(1 - F_{01}(z))e^{-\theta\sigma z} - e^{\frac{1}{2}\theta^2\sigma^2} F_{01}(-\theta\sigma - z)\} dF_{01}(z) \\ \frac{\partial(\cdot)}{\partial\sigma}\Big|_{\sigma=0} &= e^{-\theta\mu} \eta(\lambda - 1) \int_{-\infty}^{\infty} \{-\theta z(1 - F_{01}(z))e^{-\theta\sigma z} - \theta\sigma e^{\frac{1}{2}\theta^2\sigma^2} F_{01}(-\theta\sigma - z) + \theta e^{\frac{1}{2}\theta^2\sigma^2} F_{01}(-\theta\sigma - z)\} dF_{01}(z)\Big|_{\sigma=0} \\ &= e^{-\theta\mu} \theta \eta(\lambda - 1) \int_{-\infty}^{\infty} \{-z + zF_{01}(z) + F_{01}(-z)\} dF_{01}(z) \approx e^{-\theta\mu} \theta \eta(\lambda - 1) 0.7832 > 0. \end{aligned}$$

Thus, news-utility introduces a first-order precautionary-savings motive.

In the second part of the lemma the implications for consumption can be immediately seen by comparing the agents' first-order conditions. The standard agent's first-order condition in period $T - 1$ is given by

$$u'(C_{T-1}) = Ru'((s_{T-1}^P - C_{T-1})R + s_{T-1}^P)Q_{T-1}.$$

The difference to the news-utility model can be seen easily: First, $\frac{\Psi_{T-1}}{Q_{T-1}} > 1$ implies that

$$\frac{\frac{\Psi_{T-1}}{Q_{T-1}} + \gamma(\eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta\lambda(1 - F_P(s_{T-1}^P))} > 1$$

for γ high enough such that the news-utility agent consumes less than the standard agent if he does not discount prospective gain-loss utility very highly. Moreover, as $\frac{\Psi_{T-1}}{Q_{T-1}}$ is increasing in σ_P the threshold value for γ , i.e., $\bar{\gamma}$, in each comparison is decreasing in σ_P .

B.4.4 Proof of Proposition 3

The agent optimally chooses consumption and asset holdings in periods $T - i = 1, \dots, T$ for any horizon T . I defined a hump-shaped consumption profile as characterized by increasing consumption and asset holdings in the beginning of life $C_1 < C_2$ and decreasing consumption in the end of life $C_T < C_{T-1}$ (note that, I derive the thresholds $\underline{\sigma}_P$ and $\bar{\sigma}_P$ for $s_t^P = 0$ and $s_t^T = 0$ in all periods, since Λ_{T-i} is skewed this is not exactly the average consumption path but the difference is minor). The first characteristic requires $C_1 < C_2$ which implies that

$$\frac{(1+r)^{T-1}}{f(T-1)}(1+r)A_0 + P_0 - \frac{f(T-2)}{f(T-1)}\Lambda_1 < \frac{(1+r)^{T-2}}{f(T-2)}(1+r)A_1 + P_0 - \frac{f(T-3)}{f(T-2)}\Lambda_2$$

so that $\Lambda_1 > \frac{f(T-3)}{f(T-2)}\Lambda_2$ and since $\frac{f(T-3)}{f(T-2)} < 1$ this holds always if $\Lambda_1 > 0$ as T becomes large since in the limit $\Lambda_1 = \Lambda_2$. Recall that if $\lambda > 1$ and $\eta > 0$ then $\Psi_{T-i} > Q_{T-i}$, $\Psi_{T-i} > \Psi_{T-i+1}$ and

$Q_{T-i} > Q_{T-i+1}$ and $\psi_{T-i} - Q_{T-i} > \psi_{T-i+1} - Q_{T-i+1}$ for all i and $\frac{\psi_{T-i}}{Q_{T-i}}$ approaches its limit $\frac{\psi}{Q}$ as i and T become large. $\underline{\sigma}_P$ is then implicitly defined by the requirement $\Lambda_1 > 0$ which is equivalent to

$$\frac{(1+r)^{T-1} \psi_1 + \gamma Q_1 \eta^{\frac{1}{2}}(1+\lambda)}{f(T-2) \left(1 + \eta^{\frac{1}{2}}(1+\lambda)\right)} = \frac{r}{1 - \left(\frac{1}{1+r}\right)^{T-1}} \frac{\psi_1 + \gamma Q_1 \eta^{\frac{1}{2}}(1+\lambda)}{1 + \eta^{\frac{1}{2}}(1+\lambda)} > 1.$$

Accordingly, if $\frac{\psi_1}{Q_1}$ (which is determined by expected marginal gain-loss utility) is large enough relative to γ the agent chooses an increasing consumption path. For $T \rightarrow \infty$ the condition boils down to

$$r \frac{\psi + \gamma Q \eta^{\frac{1}{2}}(1+\lambda)}{1 + \eta^{\frac{1}{2}}(1+\lambda)} > 1 \Rightarrow r(\psi + \gamma Q \eta^{\frac{1}{2}}(1+\lambda)) > 1 + \eta^{\frac{1}{2}}(1+\lambda)$$

for which a sufficient condition is $\gamma Q > \frac{1}{r}$.

The second characteristic requires $C_T < C_{T-1}$ which implies that

$$(1+r)A_{T-1} < \frac{1+r}{2+r}(1+r)A_{T-2} - \frac{1}{2+r}\Lambda_{T-1}$$

and is equivalent to $\Lambda_{T-1} < 0$. Thus, $\overline{\sigma}_P$ is implicitly defined by $\Lambda_{T-1} < 0$

$$\frac{1}{\theta} \log\left((1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} \eta^{\frac{1}{2}}(1+\lambda)}{1 + \eta^{\frac{1}{2}}(1+\lambda)}\right) < 0 \Rightarrow (1+r) \frac{\psi_{T-1} + \gamma Q_{T-1} \eta^{\frac{1}{2}}(1+\lambda)}{1 + \eta^{\frac{1}{2}}(1+\lambda)} < 1.$$

Note that, because $\beta(1+r) \approx 1$ the standard agent will choose an almost flat consumption path such that $(1+r)Q_{T-1} \approx 1$. Thus, the news-utility agent chooses a mean falling consumption path in the end of life as long as $\frac{\psi_{T-1}}{Q_{T-1}}$ is not too large or γ is not too close to one.

B.4.5 Proof of Proposition 4

In the deterministic setting, $s_t^P = s_t^T = 0$ for all t such that the news-utility agent will not experience actual news utility in a subgame-perfect equilibrium because he cannot fool himself and thus $\psi_t = Q_t$ for all t . Thus, the expected-utility maximizing path corresponds to the standard agent's one which is determined in any period $T-i$ by the following first-order condition

$$\exp\{-\theta(1+r)A_{T-i-1} + \theta A_{T-i}\} = \frac{(1+r)^i}{f(i-1)} \exp\left\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\right\} Q_{T-i}^s.$$

If the agent believes he follows the above path then the consistency constraint (increasing consumption is not preferred) has to hold

$$\exp\{-\theta(1+r)A_{T-i-1} + \theta A_{T-i}\}(1+\eta) < \frac{(1+r)^i}{f(i-1)} \exp\left\{-\theta \frac{(1+r)^i}{f(i-1)} A_{T-i}\right\} Q_{T-i}^s (1+\gamma\eta\lambda).$$

Thus, if $\eta < \gamma\eta\lambda \Rightarrow \gamma > \frac{1}{\lambda}$ the agent follows the expected-utility maximizing path. Whereas for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption is characterized by equality of the consistency constraint, because the agent will choose the lowest consumption level that just satisfies it. Then, the first-order condition becomes equivalent to a $\beta\delta$ -agent's first-order condition with $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

In the infinite-horizon model, a simple perturbation argument gives the following consistency constraint

$$\exp(-\theta(1+r)A_{t-1} + \theta A_t)(1 + \eta) < \exp(-\theta r A_t)Q(1 + \gamma\eta\lambda),$$

because $\psi = Q$. However, if $\gamma > \frac{1}{\lambda}$ the news-utility agent finds it optimal to follow the expected-utility-maximizing standard agent's path

$$\exp(-\theta(1+r)A_{t-1} + \theta A_t) = \exp(-\theta r A_t)Q^s \Rightarrow A_t = A_{t-1} + \Lambda^s \Rightarrow C_t = rA_{t-1} + Y_t - \Lambda^s$$

$$\Lambda^s = \frac{1}{\theta(1+r)} \log(rQ^s) \text{ with } Q^s = \frac{\beta \exp(-\theta(-\Lambda^s))}{1 - \beta \exp(-\theta r \Lambda^s)}.$$

Whereas for $\gamma \leq \frac{1}{\lambda}$ news-utility consumption will choose the lowest consumption level that just satisfies his consistency constraint. Then, the first-order condition becomes equivalent to a $\beta\delta$ -agent's first-order condition with $b = \frac{1+\gamma\eta\lambda}{1+\eta} < 1$.

B.4.6 Proof of Proposition 5

I say that the news-utility agent's consumption path features a drop in consumption at retirement, if the change in consumption at retirement is negative and smaller than it is after the start of retirement, i.e., ΔC_{T-R} is negative and smaller than ΔC_{T-R+1} . In general, after the start of retirement the news-utility agent's consumption growth follows the standard model or the hyperbolic-discounting model. Thus, the news-utility agent's implied hyperbolic-discount factor after retirement is $b^R \in \{\frac{1+\gamma\eta\lambda}{1+\eta}, 1\}$, which is larger than the news-utility agent's implied hyperbolic-discount factor at retirement. In the at-retirement period the weight on future marginal value versus current marginal consumption is between $\{\frac{1+\gamma\eta\lambda}{1+\eta\lambda}, \frac{1+\gamma\eta}{1+\eta}\}$ and since $\frac{1+\gamma\eta\lambda}{1+\eta\lambda} < \frac{1+\gamma\eta}{1+\eta} < \frac{1+\gamma\eta\lambda}{1+\eta} < 1$ the hyperbolic-discount factor implied by at-retirement consumption growth is necessarily lower than the hyperbolic-discount factor implied by post-retirement consumption growth. Thus, consumption growth at retirement will necessarily be less than consumption growth after retirement. Moreover, if consumption growth after retirement is approximately zero, because $\log((1+r)\beta) \in [-\Delta, \Delta]$ with Δ small then consumption growth at retirement will be negative.

Let me formalize the agent's consumption growth at and after retirement. After retirement the news-utility agent's consumption growth is $\Delta C_{T-R+1} = C_{T-R+1} - C_{T-R} = -a(R-1)\Lambda_{T-R+1} + \Lambda_{T-R}$ and will correspond to a hyperbolic-discounting agent's consumption with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ such that the agent's continuation utilities in period $T-R$ and $T-R+1$ (which determine Λ_{T-R} and Λ_{T-R+1}) correspond to

$$Q_{T-R} = \psi_{T-R} = Q_{T-R}^b \text{ and } Q_{T-R+1} = \psi_{T-R+1} = Q_{T-R+1}^b$$

such that

$$\Lambda_{T-R} = \frac{1}{\theta} \log\left(\frac{(1+r)^R}{f(R-1)} b Q_{T-R}^b\right) \text{ and } \Lambda_{T-R+1} = \frac{1}{\theta} \log\left(\frac{(1+r)^{R-1}}{f(R-2)} b Q_{T-R+1}^b\right)$$

thus if $\log((1+r)\beta) \in [-\Delta, \Delta]$ with Δ small then consumption growth after retirement will be approximately zero (if $b = 1$ and the news-utility agent follows the standard agent's path) or negative

if $b < 1$ (if the news-utility agent follows a hyperbolic-discounting path but $\log((1+r)\beta) \approx 0$). Consumption growth at retirement is $\Delta C_{T-R} = C_{T-R} - C_{T-R-1} = -a(R)\Lambda_{T-R} + \Lambda_{T-R-1}$. Λ_{T-R} will correspond to a hyperbolic-discounting agent's value with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ as above. But, Λ_{T-R-1} will correspond to a hyperbolic-discounting agent's value with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta\lambda}, \frac{1+\eta\gamma}{1+\eta}\}$ and it can be easily seen that $\frac{1+\eta\gamma\lambda}{1+\eta\lambda} < \frac{1+\eta\gamma}{1+\eta} < \frac{1+\eta\gamma\lambda}{1+\eta} < 1$. Thus, from above

$$\Lambda_{T-R} = \frac{1}{\theta} \log\left(\frac{(1+r)^R}{f(R-1)} b Q_{T-R}^b\right)$$

and if the news-utility agent would continue this hyperbolic path implied by the past retirement $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ then

$$\Lambda_{T-R-1}^b = \frac{1}{\theta} \log\left(\frac{(1+r)^{R+1}}{f(R)} b Q_{T-R-1}^b\right)$$

whereas in fact his Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} \log\left(\frac{(1+r)^{R+1} \psi_{T-R-1} + \gamma Q_{T-R-1} (\eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))}{f(R) \left(1 + \eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))\right)}\right)$$

with $\psi_{T-R-1} = Q_{T-R-1} = Q_{T-R-1}^b$ because there is no uncertainty from period $T-R$ on. As can be easily seen iff $\gamma < 1$ then $\Lambda_{T-R-1} < \Lambda_{T-R-1}^b$ because

$$\frac{Q_{T-R-1}^b + \gamma Q_{T-R-1}^b (\eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))}{1 + \eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))} < Q_{T-R-1}^b$$

for instance, if $F(\cdot) = 0.5$ then $\frac{1+\gamma\frac{1}{2}\eta(1+\lambda)}{1+\frac{1}{2}\eta(1+\lambda)} < 1$ iff $\gamma < 1$. Thus, news-utility consumption growth is smaller at retirement than after retirement. Moreover, it is negative because it is either approximately zero after retirement (if $b^R = 1$) or negative after retirement (if $b^R = \frac{1+\eta\gamma\lambda}{1+\eta} < 1$).

B.4.7 Proof of Corollary 1

After retirement the news-utility agent's consumption from period $T-R$ on will correspond to a hyperbolic-discounting agent's consumption with $b \in \{\frac{1+\eta\gamma\lambda}{1+\eta}, 1\}$ such that the agent's continuation utilities correspond to

$$Q_{T-R-1} = \psi_{T-R-1} = Q_{T-R-1}^b$$

thus Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} \log\left(\frac{(1+r)^{R+1} \psi_{T-R-1} + \gamma Q_{T-R-1} (\eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))}{f(R) \left(1 + \eta F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T) + \eta\lambda(1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))\right)}\right)$$

as can be easily seen iff $\gamma < 1$ then $\frac{\partial \Lambda_{T-R-1}}{\partial s_{T-R-1}^P} > 0$ and consumption is excessively smooth and

sensitive in the pre-retirement period.

B.4.8 Proofs of the new predictions about consumption (Section IV.4)

The new predictions can be easily inferred from the above considerations.

1. Consumption is more excessively sensitive for permanent than for transitory shocks in an environment with permanent shocks. In an environment with transitory shocks only, however, news-utility consumption is excessively sensitive with respect to transitory shocks. Λ_t varies more with the permanent shock than with the transitory shock, because the agent is consuming only the per-period value $\frac{r}{1+r}s_t^T$ of the period t transitory shock, such that $F_{C_t}^{t-1}(C_t)$ varies little with s_t^T . However, in the absence of permanent shocks Λ_t would vary with $F_{S_{T-i}^T}(s_{T-i}^T)$ which fully determines $F_{C_t}^{t-1}(C_t)$ even though consumption itself will increase only by the per-period value $\frac{r}{1+r}s_t^T$ of the transitory shock. Thus, consumption is excessively sensitive for transitory shocks when permanent shocks are absent. With permanent shocks, however, consumption is excessively sensitive for transitory shocks only when the horizon is very short or the permanent shock has very little variance such that the transitory shock actually moves $F_{S_{T-i}^P + \frac{(1+r)^i}{f(i)}S_{T-i}^T}(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T)$ despite the fact that it is discounted by $\frac{(1+r)^i}{f(i)}$.
2. The degree of excess smoothness and sensitivity is decreasing in the amount of economic uncertainty σ_P . If σ_P is small, the agent's beliefs change more quickly relative to the change in the realization of the shock; hence, the consumption function is more flat for realizations around μ_P . The consumption function C_{T-i} is less increasing in the realizations of the shocks $s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T$ if $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P}$ is relatively high. As can be seen easily, $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P}$ is increasing in $f_{S_{T-i}^P + \frac{(1+r)^i}{f(i)}S_{T-i}^T}(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T)$ which is high if $f_{S_{T-i}^P + \frac{(1+r)^i}{f(i)}S_{T-i}^T}$ is very high at $s_{T-i}^P = \mu_P$ which happens if $f_{S_{T-i}^P + \frac{(1+r)^i}{f(i)}S_{T-i}^T}$ is a very narrow distribution, i.e., σ_P is small.
3. Any bell-shaped shock distribution induces the variation in Λ_{T-i} and thereby the amount of excess sensitivity to be bounded. If the agent is hit by an extreme shock, the actual value of the low probability realization matters less because neighboring states have very low probability. The expression $\eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)}s_{T-i}^T))$ is bounded if the two shocks' distributions are bell shaped. Thus, the variation in Λ_{T-i} is bounded.
4. Consumption is more excessively sensitive and excessively smooth when the agent's horizon increases, because the marginal propensity to consume out of permanent shocks declines when the additional precautionary-savings motive accumulates. $\frac{\Psi_{T-i}}{Q_{T-i}}$ is increasing in i and approaches a constant $\frac{\Psi}{Q}$ when $T \rightarrow \infty$ and $i \rightarrow \infty$. Then, the variation in Λ_{T-i} is increasing in i . And since consumption growth ΔC_{T-i} is determined by $-a(i)\Lambda_{T-i} + \Lambda_{T-i-1}$ on average the larger variation in Λ_{T-i} translates into a higher coefficient in the OLS regression. This

can be seen by looking at $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P}$, i.e.,

$$\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P} = \frac{a(i)}{\theta\left(\frac{1-a(i)}{a(i)}\right)} \frac{(\psi_{T-i} - Q_{T-i})\eta f_{s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T} (s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T)(\lambda - 1)}{1 + \eta F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T) + \eta \lambda (1 - F(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T))} > 0.$$

As $a(i) = \frac{f(i-1)}{f(i)}$ is increasing in i because $f(i) = \sum_{j=0}^i (1+r)^j$ and thus $\frac{(a(i))^2}{1-a(i)}$ is increasing in i and approaching a constant and $\frac{\psi_{T-i}}{Q_{T-i}}$ is increasing in i and approaching a constant it can be easily seen that $\frac{\partial a(i)\Lambda_{T-i}}{\partial s_{T-i}^P}$ is increasing in i which means that consumption becomes more excessively smooth as the agent's horizon increases. Moreover, as $\frac{a(i)}{1-a(i)}$ is increasing in i too $\frac{\partial \Lambda_{T-i}}{\partial s_{T-i}^P}$ is increasing in i which means that consumption becomes more excessively sensitive as the agent's horizon increases.

B.4.9 Proof of Proposition 6

In the following, I assume that the following parameter condition (which ensures that the agent's maximization problem is globally concave) holds $\eta(\lambda - 1) < 1$. All of the following proofs are direct applications of the prior proofs for the monotone-personal equilibrium just using Λ_{T-i}^c instead of Λ_{T-i} . Thus, I make the exposition somewhat shorter.

1. The personal and pre-committed consumption functions are different in each period as can be seen in Section B.2. But, if there's no uncertainty and $\gamma > \frac{1}{\lambda}$ then the personal and pre-committed consumption functions both correspond to the standard agent's consumption function as shown in the proof of Proposition 4.
2. Please refer to the derivation of the exponential-utility pre-committed model in Section B.2 for a detailed derivation of Λ_{T-i}^c . According to Definition 5 consumption is excessively smooth if $\frac{\partial C_{t+1}^c}{\partial s_{t+1}^P} < 1$ and excessively sensitive if $\frac{\partial \Delta C_{t+1}^c}{\partial s_t^P} > 0$. Consumption growth is

$$\Delta C_{T-i}^c = s_{T-i}^P + (1 - a(i))s_{T-i}^T - a(i)\Lambda_{T-i}^c + \Lambda_{T-i-1}^c$$

so that $\frac{\partial C_{T-i}^c}{\partial s_{T-i}^P} < 1$ iff $\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}^P} > 0$ and $\frac{\partial \Delta C_{T-i}^c}{\partial s_{T-i-1}^P} > 0$ iff $\frac{\partial \Lambda_{T-i-1}^c}{\partial s_{T-i-1}^P} > 0$. Since $\psi_{T-i}^c > \gamma Q_{T-i}^c$ it can

be easily seen that $\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}^P} > 0$, i.e.

$$\frac{\partial \Lambda_{T-i}^c}{\partial s_{T-i}^P} = \frac{1}{\theta \left(\frac{1-a(i)}{a(i)} \right)} \frac{(\psi_{T-i} - \gamma Q_{T-i}) \eta (\lambda - 1) 2f \left(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T \right)}{1 + \eta (\lambda - 1) (1 - 2F \left(s_{T-i}^P + \frac{(1+r)^i}{f(i)} s_{T-i}^T \right))} > 0.$$

Thus, optimal pre-committed consumption is excessively smooth and sensitive.

3. The first-order condition of the second-to-last period in the exemplified model of the text is

$$u'(C_{T-1}^c) = (1+r)u'((s_{T-1}^P - C_{T-1}^c)(1+r) + s_{T-1}^P) \frac{\psi_{T-1} + \gamma Q_{T-1} \eta (\lambda - 1) (1 - 2F_P(s_{T-1}^P))}{1 + \eta (\lambda - 1) (1 - 2F_P(s_{T-1}^P))}.$$

By the exact same argument as above $\psi_{T-1} > Q_{T-1}$ such that news utility introduces a first-order precautionary-savings motive in the pre-committed equilibrium. Compare the above first-order condition with the one for personal-monotone consumption C_{T-1} , i.e.,

$$u'(C_{T-1}) = (1+r)u'((s_{T-1}^P - C_{T-1})(1+r) + s_{T-1}^P) \frac{\psi_{T-1} + \gamma Q_{T-1} (\eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P)))}{1 + \eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P))}.$$

Because $\eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P)) > \eta (\lambda - 1) (1 - 2F_P(s_{T-1}^P))$ for all s_{T-1}^P and $\psi_{T-1} > \gamma Q_{T-1}$ monotone personal consumption is higher $C_{T-1} > C_{T-1}^c$ than pre-committed consumption. Moreover, the difference $\eta F_P(s_{T-1}^P) + \eta \lambda (1 - F_P(s_{T-1}^P)) - \eta (\lambda - 1) (1 - 2F_P(s_{T-1}^P)) = \eta (1 - F_P(s_{T-1}^P)) + \eta \lambda F_P(s_{T-1}^P)$ is increasing in s_{T-1}^P such that the difference in consumption $C_{T-1} - C_{T-1}^c$ is increasing in s_{T-1}^P .

4. Consider the pre-committed first-order conditions before and after retirement. After retirement the news-utility agent's consumption from period $T - R$ on will correspond to the standard agent's one, i.e.,

$$Q_{T-R-1} = \psi_{T-R-1} = Q_{T-R-1}^s$$

thus Λ_{T-R-1} is given by

$$\Lambda_{T-R-1} = \frac{1}{\theta} \log \left(\frac{(1+r)^{R+1} \psi_{T-R-1} + \gamma Q_{T-R-1} \eta (\lambda - 1) (1 - 2F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T))}{f(R) (1 + \eta (\lambda - 1) (1 - F(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T)))} \right)$$

it can be easily seen that

$$\frac{\partial \Lambda_{T-R-1}}{\partial \gamma} < 0 \text{ only if } F \left(s_{T-R-1}^P + \frac{(1+r)^{R+1}}{f(R+1)} s_{T-R-1}^T \right) < 0.5.$$

Thus, $\gamma < 1$ does not necessarily increase or decrease Λ_{T-R-1} and thereby consumption growth at retirement is not necessarily negative and smaller than consumption growth after retirement. There is no systematic underweighting of marginal utility before or after retire-

ment and there does not occur a drop in consumption at retirement for $\gamma < 1$.

B.5 Derivation of the power-utility model

In the following, I outline the numerical derivation of the model with a power-utility specification $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$. I start with the standard model to then explain the news-utility model in detail.

B.5.1 The standard model

The standard agent's maximization problem in any period $T - i$ is

$$\max\{u(C_{T-i}) + \sum_{\tau=1}^i \beta^\tau E_{T-i}[u(C_{T-i+\tau})]\}$$

$$\text{subject to } X_t = (X_{t-1} - C_{t-1})R + Y_t \text{ and } Y_t = P_{t-1}G_t e^{s_t^P} e^{s_t^T} = P_t e^{s_t^T}.$$

The maximization problem can be normalized by $P_{T-i}^{1-\theta}$ and then becomes in normalized terms ($X_t = P_t x_t$ for instance)

$$\max\{u(c_{T-i}) + \sum_{\tau=1}^i \beta^\tau E_{T-i}[\prod_{j=1}^{\tau} (G_{T-i+j} e^{s_{T-i+j}^P})^{1-\theta} u(c_{T-i+\tau})]\}$$

$$\text{subject to } x_t = (x_{t-1} - c_{t-1}) \frac{R}{G_t e^{s_t^P}} + y_t \text{ and } y_t = e^{s_t^T}.$$

The model can be solved by numerical backward induction as done by Gourinchas and Parker (2002) and Carroll (2001). The standard agent's first-order condition is

$$u'(c_{T-i}) = \Phi'_{T-i} = \beta R E_{T-i} \left[\frac{\partial c_{T-i+\tau}}{\partial x_{T-i+1}} (G_{T-i+1} e^{s_{T-i+1}^P})^{-\theta} u'(c_{T-i+\tau}) + (1 - \frac{\partial c_{T-i+1}}{\partial x_{T-i+1}}) (G_{T-i+1} e^{s_{T-i+1}^P})^{-\theta} \Phi'_{T-i-1} \right]$$

with his continuation value

$$\Phi'_{T-i-1} = \beta R E_{T-i-1} \left[\frac{\partial c_{T-i}}{\partial x_{T-i}} (G_{T-i} e^{s_{T-i}^P})^{-\theta} u'(c_{T-i}) + (1 - \frac{\partial c_{T-i}}{\partial x_{T-i}}) (G_{T-i} e^{s_{T-i}^P})^{-\theta} \Phi'_{T-i} \right]$$

where it can be easily noted that

$$\begin{aligned} P_{T-i} \Phi'_{T-i} &= E_{T-i} \left[\frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial \sum_{\tau=1}^i \beta^\tau u(C_{T-i+\tau})}{\partial X_{T-i+1}} \right] = E_{T-i} \left[\frac{\partial X_{T-i+1}}{\partial A_{T-i}} \left(\frac{\partial \beta u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial \sum_{\tau=1}^i \beta^{\tau+1} u(C_{T-i+1+\tau})}{\partial X_{T-i+1}} \right) \right] \\ &= E_{T-i} \left[\frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial \beta u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial X_{T-i+2}}{\partial X_{T-i+1}} \frac{\partial X_{T-i+1}}{\partial X_{T-i+2}} \frac{\partial \sum_{\tau=1}^i \beta^{\tau+1} u(C_{T-i+1+\tau})}{\partial X_{T-i+1}} \right] \end{aligned}$$

$$\begin{aligned}
&= E_{T-i} \left[\frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial \beta u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+1}}{\partial A_{T-i}} \frac{\partial X_{T-i+2}}{\partial X_{T-i+1}} \frac{\partial \sum_{\tau=1}^i \beta^{\tau+1} u(C_{T-i+1+\tau})}{\partial X_{T-i+2}} \right] \\
&= \beta RE_{T-i} \left[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial X_{T-i+2}}{\partial X_{T-i+1}} \frac{\partial \sum_{\tau=1}^i \beta^{\tau} u(C_{T-i+1+\tau})}{\partial X_{T-i+2}} \right] \\
&= \beta RE_{T-i} \left[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial A_{T-i+1}}{\partial X_{T-i+1}} \underbrace{E_{T-i+1} \left[\frac{\partial X_{T-i+2}}{\partial A_{T-i+1}} \frac{\partial \sum_{\tau=1}^i \beta^{\tau} u(C_{T-i+1+\tau})}{\partial X_{T-i+2}} \right]}_{P_{T-i+1} \Phi'_{T-i-1}} \right] \\
&= \beta RE_{T-i} \left[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \frac{\partial (X_{T-i+1} - C_{T-i+1})}{\partial X_{T-i+1}} P_{T-i+1} \Phi'_{T-i-1} \right] = \beta RE_{T-i} \left[\frac{\partial u(C_{T-i+\tau})}{\partial X_{T-i+1}} + \left(1 - \frac{\partial C_{T-i+1}}{\partial X_{T-i+1}}\right) P_{T-i+1} \Phi'_{T-i-1} \right].
\end{aligned}$$

Φ'_{T-i} is a function of savings a_{T-i} thus I can solve for each optimal consumption level c_{T-i}^* on a grid of savings a_{T-i} as $c_{T-i}^* = (\Phi'_{T-i-1})^{-\frac{1}{\theta}} = (f^{\Phi'}(a_{T-i}))^{-\frac{1}{\theta}}$ to then find each optimal level of consumption for each value of the normalized cash-on-hand grid x_{T-i} by interpolation. This endogenous-grid method has been developed by Carroll (2001). Alternatively, I could use the Euler equation instead of the agent's continuation value but this solution illustrates the upcoming solution of the news-utility model of which it is a simple case.

B.5.2 The monotone-personal and pre-committed equilibrium in the second-to-last period

Before starting with the fully-fledged problem, I outline the second-to-last period for the case of power utility. In the second-to-last period the agent allocates his cash-on-hand X_{T-1} between contemporaneous consumption C_{T-1} and future consumption C_T , knowing that in the last period he will consume whatever he saved in addition to last period's income shock $C_T = X_T = (X_{T-1} - C_{T-1})R + Y_T$. According to the monotone-personal equilibrium solution concept, in period $T-1$ the agent takes the beliefs about contemporaneous and future consumption he entered the period with $\{F_{C_{T-1}}^{T-2}, F_{C_T}^{T-2}\}$ as given and maximizes

$$u(C_{T-1}) + n(C_{T-1}, F_{C_{T-1}}^{T-2}) + \gamma \beta n(F_{C_T}^{T-1, T-2}) + \beta E_{T-1} [u(C_T) + n(C_T, F_{C_T}^{T-1})]$$

which can be rewritten as

$$\begin{aligned}
&u(C_{T-1}) + \eta \int_{-\infty}^{C_{T-1}} (u(C_{T-1}) - u(c)) F_{C_{T-1}}^{T-2}(c) + \eta \lambda \int_{C_{T-1}}^{\infty} (u(C_{T-1}) - u(c)) F_{C_{T-1}}^{T-2}(c) \\
&+ \gamma \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u(c) - u(r)) F_{C_T}^{T-1, T-2}(c, r) + \beta E_{T-1} [u(C_T) + \eta(\lambda - 1) \int_{C_T}^{\infty} (u(C_T) - u(c)) F_{C_T}^{T-1}(c)].
\end{aligned}$$

To gain intuition for the model's predictions, I explain the derivation of the first-order condition

$$\begin{aligned}
&u'(C_{T-1})(1 + \eta F_{C_{T-1}}^{T-2}(C_{T-1}) + \eta \lambda (1 - F_{C_{T-1}}^{T-2}(C_{T-1}))) = \gamma \beta RE_{T-1} [u'(C_T)] (\eta F_{A_{T-1}}^{T-2}(A_{T-1}) + \eta \lambda (1 - F_{A_{T-1}}^{T-2}(A_{T-1}))) \\
&+ \beta RE_{T-1} [u'(C_T) + \eta(\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c)].
\end{aligned}$$

The first two terms in the first-order condition represent marginal consumption utility and gain-loss utility over contemporaneous consumption in period $T - 1$. As the agent takes his beliefs $\{F_{C_{T-1}}^{T-2}, F_{C_T}^{T-2}\}$ as given in the optimization, I apply Leibniz's rule for differentiation under the integral sign. This results in marginal gain-loss utility being the sum of states that would have promised less consumption $F_{C_{T-1}}^{T-2}(C_{T-1})$, weighted by η , or more consumption $1 - F_{C_{T-1}}^{T-2}(C_{T-1})$, weighted by $\eta\lambda$,

$$\frac{\partial n(C_{T-1}, F_{C_{T-1}}^{T-2})}{\partial C_{T-1}} = u'(C_{T-1})(\eta F_{C_{T-1}}^{T-2}(C_{T-1}) + \eta\lambda(1 - F_{C_{T-1}}^{T-2}(C_{T-1}))).$$

Note that, if contemporaneous consumption is increasing in the realization of cash-on-hand then I can simplify $F_{C_{T-1}}^{T-2}(C_{T-1}) = F_{X_{T-1}}^{T-2}(X_{T-1})$. Returning to the maximization problem the third term represents prospective gain-loss utility over future consumption C_T experienced in $T - 1$. As before, marginal gain-loss utility is given by the weighted sum of states $u'(C_T)(\eta F_{A_{T-1}}^{T-2}(A_{T-1}) + \eta\lambda(1 - F_{A_{T-1}}^{T-2}(A_{T-1})))$. Note that $F_{C_T}^{T-2}(c)$ is defined as the probability $Pr(C_T < c | I_{T-2})$. Applying a logic similar to the law of iterated expectation

$$Pr(C_T < c | I_{T-2}) = Pr(A_{T-1}R + Y_T < c | I_{T-2}) = Pr(A_{T-1} < \frac{c - Y_T}{R} | I_{T-2})$$

thus if savings are increasing in the realization of cash-on-hand then I can simplify $F_{A_{T-1}}^{T-2}(A_{T-1}) = F_{X_{T-1}}^{T-2}(X_{T-1})$.

The last term in the maximization problem represents consumption and gain-loss utility over future consumption C_T in the last period T , i.e., the first derivative of the agent's continuation value with respect to consumption or the marginal value of savings. Expected marginal gain-loss utility $\eta(\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c)$ is positive for any concave utility function such that

$$\Psi'_{T-1} = \beta RE_{T-1}[u'(C_T) + \eta(\lambda - 1) \int_{C_T}^{\infty} (u'(C_T) - u'(c)) F_{C_T}^{T-1}(c)] > \beta RE_{T-1}[u'(C_T)] = \Phi'_{T-1}.$$

As expected marginal gain-loss disutility is positive, increasing in σ_Y , absent if $\sigma_Y = 0$, and increases the marginal value of savings, I say that news-utility introduces an “additional precautionary-savings motive”. The first-order condition can now be rewritten as

$$u'(C_{T-1}) = \frac{\Psi'_{T-1} + \gamma \Phi'_{T-1} (\eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta\lambda(1 - F_{X_{T-1}}^{T-2}(X_{T-1})))}{1 + \eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta\lambda(1 - F_{X_{T-1}}^{T-2}(X_{T-1}))}.$$

Beyond the additional precautionary-savings motive $\Psi'_{T-1} > \Phi'_{T-1}$ implies that an increase in $F_{X_{T-1}}^{T-2}(X_{T-1})$ decreases

$$\frac{\frac{\Psi'_{T-1}}{\Phi'_{T-1}} + \gamma (\eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta\lambda(1 - F_{X_{T-1}}^{T-2}(X_{T-1})))}{1 + \eta F_{X_{T-1}}^{T-2}(X_{T-1}) + \eta\lambda(1 - F_{X_{T-1}}^{T-2}(X_{T-1}))},$$

i.e., the terms in the first-order condition vary with the income realization X_{T-1} so that consumption is excessively smooth and sensitive.

B.5.3 The monotone-pre-committed equilibrium in the second-to-last-period

The first-order condition for pre-committed consumption in period $T - 1$ is

$$u'(C_{T-1}^c) = \frac{\Psi'_{T-1} + \gamma \Phi'_{T-1} \eta (\lambda - 1) (1 - 2F_{X_{T-1}}^{T-2}(X_{T-1}))}{1 + \eta (\lambda - 1) (1 - 2F_{X_{T-1}}^{T-2}(X_{T-1}))}$$

by the same arguments as in the exponential-utility model derivation of the pre-committed equilibrium.

B.5.4 The monotone-personal equilibrium path in all prior periods

The news-utility agent's maximization problem in any period $T - i$ is given by

$$u(C_{T-i}) + n(C_{T-i}, F_{C_{T-i}}^{T-i-1}) + \gamma \sum_{\tau=1}^i \beta^\tau n(F_{C_{T-i+\tau}}^{T-i, T-i-1}) + \sum_{\tau=1}^i \beta^\tau E_{T-i}[U(C_{T-i+\tau})]$$

again, the maximization problem can be normalized by $P_{T-i}^{1-\theta}$ as all terms are proportional to consumption utility $u(\cdot)$. In normalized terms, the news-utility agent's first-order condition in any period $T - i$ is given by

$$u'(c_{T-i}) = \frac{\Psi'_{T-i} + \gamma \Phi'_{T-i} (\eta F_{c_{T-i}}^{T-i-1}(c_{T-i}) + \eta \lambda (1 - F_{c_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta \lambda (1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

I solve for each optimal value of c_{T-i}^* for a grid of savings a_{T-i} , as Ψ'_{T-i} and Φ'_{T-i} are functions of a_{T-i} until I find a fixed point of c_{T-i}^* , a_{T-i} , $F_{a_{T-i}}^{T-i-1}(a_{T-i})$, and $F_{c_{T-i}}^{T-i-1}(c_{T-i})$. The latter two can be inferred from the observation that each $c_{T-i} + a_{T-i} = x_{T-i}$ has a certain probability given the value of savings a_{T-i-1} I am currently iterating on. However, this probability varies with the realization of permanent income $e^{s_{T-i}^P}$ thus I cannot fully normalize the problem but have to find the right consumption grid for each value of $e^{s_{T-i}^P}$ rather than just one. The first-order condition can be slightly modified as follows

$$u'(e^{s_{T-i}^P} c_{T-i}) = \frac{e^{s_{T-i}^P} \Psi'_{T-i} + \gamma e^{s_{T-i}^P} \Phi'_{T-i} (\eta F_{c_{T-i}}^{T-i-1}(c_{T-i}) + \eta \lambda (1 - F_{c_{T-i}}^{T-i-1}(c_{T-i})))}{1 + \eta F_{a_{T-i}}^{T-i-1}(a_{T-i}) + \eta \lambda (1 - F_{a_{T-i}}^{T-i-1}(a_{T-i}))}$$

to find each corresponding grid value. Note that, the resulting two-dimensional grid for c_{T-i} will be the normalized grid for each realization of s_t^T and s_t^P , because I multiply both sides of the first-order conditions with $e^{s_{T-i}^P}$. Thus, the agent's consumption utility continuation value is

$$\Phi'_{T-i-1} = \beta R E_{T-i-1} \left[\frac{\partial c_{T-i}}{\partial x_{T-i}} (G_{T-i} e^{s_{T-i}^P})^{-\theta} u'(c_{T-i}) + \left(1 - \frac{\partial c_{T-i}}{\partial x_{T-i}}\right) (G_{T-i} e^{s_{T-i}^P})^{-\theta} \Phi'_{T-i} \right].$$

The agent's news-utility continuation value is given by

$$P_{T-i-1}^{-\theta} \Psi'_{T-i-1} = \beta RE_{T-i-1} \left[\frac{dC_{T-i}}{dX_{T-i}} u'(C_{T-i}) + \eta(\lambda - 1) \int_{c_{T-i} < c_{T-i}^{T-i-1}} \left(\frac{dC_{T-i}}{dX_{T-i}} u'(C_{T-i}) - x \right) dF_{\frac{dC_{T-i}}{dX_{T-i}} u'(C_{T-i})}^{T-i-1}(x) \right. \\ \left. + \gamma \eta(\lambda - 1) \int_{c_{T-i} < c_{T-i}^{T-i-1}} \left(\frac{dA_{T-i}}{dX_{T-i}} P_{T-i}^{-\theta} \Phi'_{T-i} - x \right) dF_{\frac{dA_{T-i}}{dX_{T-i}} P_{T-i}^{-\theta} \Phi'_{T-i}}^{T-i-1}(x) + \left(1 - \frac{dC_{T-i}}{dX_{T-i}} \right) P_{T-i}^{-\theta} \Psi'_{T-i} \right]$$

(here, $\int_{c_{T-i} < c_{T-i}^{T-i-1}}$ means the integral over the loss domain) or in normalized terms

$$\Psi'_{T-i-1} = \beta RE_{T-i-1} \left[\frac{dC_{T-i}}{dx_{T-i}} u'(c_{T-i}) (G_{T-i} e^{S_{T-i}^P})^{-\theta} \right. \\ \left. + \eta(\lambda - 1) \int_{c_{T-i} < c_{T-i}^{T-i-1}} \left(\frac{dC_{T-i}}{dx_{T-i}} u'(c_{T-i}) (G_{T-i} e^{S_{T-i}^P})^{-\theta} - x \right) dF_{\frac{dC_{T-i}}{dx_{T-i}} u'(c_{T-i}) (G_{T-i} e^{S_{T-i}^P})^{-\theta}}^{T-i-1}(x) \right. \\ \left. + \gamma \eta(\lambda - 1) \int_{c_{T-i} < c_{T-i}^{T-i-1}} \left(\frac{dA_{T-i}}{dx_{T-i}} \Phi'_{T-i} (G_{T-i} e^{S_{T-i}^P})^{-\theta} - x \right) dF_{\frac{dA_{T-i}}{dx_{T-i}} \Phi'_{T-i} (G_{T-i} e^{S_{T-i}^P})^{-\theta}}^{T-i-1}(x) + \left(1 - \frac{dC_{T-i}}{dx_{T-i}} \right) (G_{T-i} e^{S_{T-i}^P})^{-\theta} \Psi'_{T-i} \right].$$

B.5.5 Risk attitudes over small and large stakes

First, I suppose the agent is offered a gamble about immediate consumption in period t after that period's uncertainty has resolved and that period's original consumption has taken place. I assume that utility over immediate consumption is linear. Then, the agent is indifferent between accepting or rejecting a 50-50 win G or lose L gamble if

$$0.5G - 0.5L + 0.5\eta G - 0.5\eta\lambda L = 0.$$

Second, I suppose the agent is offered a monetary gamble or wealth bet that concerns future consumption. Suppose $T \rightarrow \infty$. I assume that his initial wealth level is $A_t = 100,000$ and $P_t = 300,000$. Let $f^\Psi(A)$ and $f^\Phi(A)$ be the agent's continuation value as a function of the agent's savings A_t . Then, the agent is indifferent between accepting or rejecting a 50-50 win G or lose L gamble if

$$0.5\eta(f^\Phi(A_t + G) - f^\Phi(A_t)) + 0.5\eta\lambda(f^\Phi(A_t - L) - f^\Phi(A_t)) + 0.5f^\Psi(A_t + G) + 0.5f^\Psi(A_t - L) = f^\Psi(A_t - L).$$

B.6 Habit formation

Consider an agent with internal, multiplicative habit formation preferences $u(C_t, H_t) = \frac{(C_t/H_t^h)^{1-\theta}}{1-\theta}$ with $H_t = H_{t-1} + \vartheta(C_{t-1} - H_{t-1})$ and $\vartheta \in [0, 1]$ (Michaelides (2002)). Assume $\vartheta = 1$ such that

$H_t = C_{t-1}$. For illustration, in the second-to-last period his maximization problem is

$$u(C_{T-1}, H_{T-1}) + \beta E_{T-1} [u(R(X_{T-1} - C_{T-1}) + Y_T, H_T)] = \frac{\left(\frac{C_{T-1}}{H_{T-1}^h}\right)^{1-\theta}}{1-\theta} + \beta E_{T-1} \left[\frac{\left(\frac{R(X_{T-1} - C_{T-1}) + Y_T}{H_T^h}\right)^{1-\theta}}{1-\theta} \right]$$

which can be normalized by $P_T^{(1-\theta)(1-h)}$ (then $C_T = P_T c_T$ for instance) and the maximization problem becomes

$$\frac{P_{T-1}^{(1-\theta)(1-h)} \left(\frac{c_{T-1}}{h_{T-1}^h}\right)^{1-\theta}}{1-\theta} + \beta P_{T-1}^{(1-\theta)(1-h)} E_{T-1} \left[\frac{(G_T e^{s_T^p})^{(1-\theta)(1-h)} \left(\frac{(x_{T-1} - c_{T-1}) \frac{R}{G_T e^{s_T^p}} + y_T}{h_T^h}\right)^{1-\theta}}{1-\theta} \right]$$

which results in the following first-order condition

$$c_{T-1}^{-\theta} = h_{T-1}^{-\theta h + h} \beta E_{T-1} [(G_T e^{s_T^p})^{-\theta(1-h)} \left(\frac{c_T}{h_T^h}\right)^{-\theta} (R + \frac{c_T}{h_T} h)] = \Phi'_{T-1}$$

with Φ'_{T-1} being a function of savings $x_{T-1} - c_{T-1}$ and habit h_T . The first-order condition can be solved very robustly by iterating on a grid of savings a_{T-1} assuming $c_{T-1}^* = (\Phi'_{T-1})^{-\frac{1}{\theta}} = (f^{\Phi'}(a_{T-1}, h_T))^{-\frac{1}{\theta}}$ and $h_T = c_{T-1}^* \frac{1}{G_T e^{s_T^p}}$ until a fixed point of consumption and habit has been found. The normalized habit-forming agent's first-order condition in any period $T - i$ is given by

$$c_{T-i}^{-\theta} = h_{T-i}^{-\theta h + h} \Phi'_{T-i} = h_{T-i}^{-\theta h + h} \beta E_{T-i} [(G_{T-i+1} e^{s_{T-i+1}^p})^{-\theta(1-h)} \left(\frac{c_{T-i+1}}{h_{T-i+1}^h}\right)^{-\theta} (R \frac{dc_{T-i+1}}{dx_{T-i+1}} + \frac{c_{T-i+1}}{h_{T-i+1}} h) + (1 - \frac{dc_{T-i+1}}{dx_{T-i+1}}) (G_{T-i+1} e^{s_{T-i+1}^p})^{-\theta(1-h)} \Phi'_{T-i+1}].$$