Model and State Uncertainty

Eric R. Young

University of Virginia

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Uncertainty

- Standard economic theory endows agent with a lot of information:
  - Complete preferences over goods
  - Complete knowledge of data-generating process (model)
  - Complete knowledge of current situation (state)
- Recent work abandons (2) and (3)
- I will discuss these in the context of a portfolio choice problem (based on work with Yulei Luo)

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Entropy

- Key idea will be entropy:

$$\mathcal{H}(X) = -\int \ln(f(x)) f(x)\, dx$$

- Relative entropy of two distributions is a measure of distance between them (Kullbeck-Leibler divergence)
- Conditional entropy of two distributions is a measure of difference of information content

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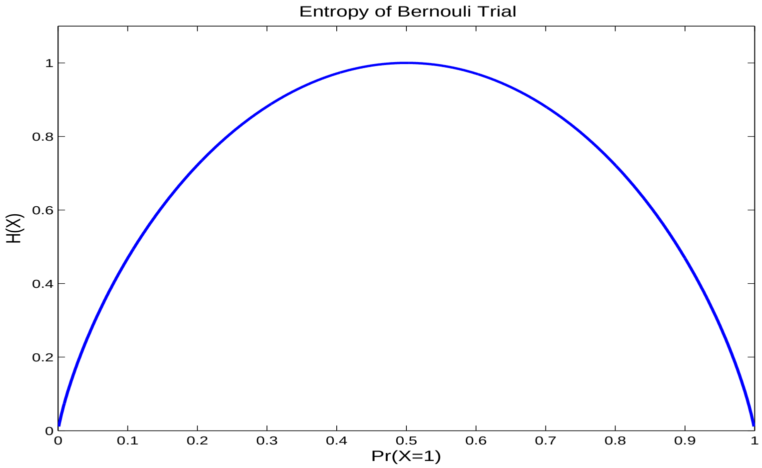
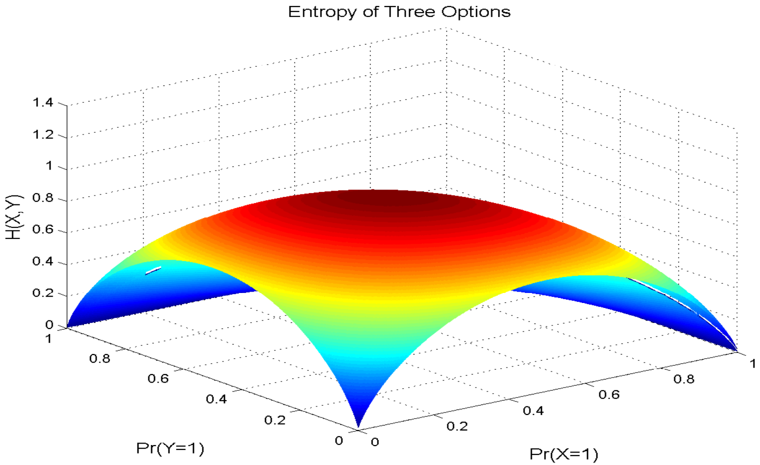

Entropy of Bernouli Trial
H(X)
Pr(X=1)
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Entropy


Entropy of Three Options
H(X,Y)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Pr(Y=1)
1
0.8
0.6
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0.2
0
Pr(X=1)
0
0.2
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1

Model Uncertainty

- Three models of interest:
  - "True" model – unknown and unknowable
  - "Approximating" model – point estimate of true model
  - "Alternative" models – models that cannot be statistically distinguished from approximating model
- Think of alternative models as a "cloud" of models that surround the approximating one
- Size of cloud determines amount of uncertainty aversion (measured by difference in entropy between distributions implied by approximating and alternative models)

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Equivalence Result

- Take agent with time-separable log preferences and constant discounting
- Endow agent with a fear of model misspecification, leads to utility recursion

$$V(x) = \log(c) + \beta \min_{g \in G} \int V(x') g(x'|x) dx'$$

- g is chosen from the set of alternative models, f is the approximating model

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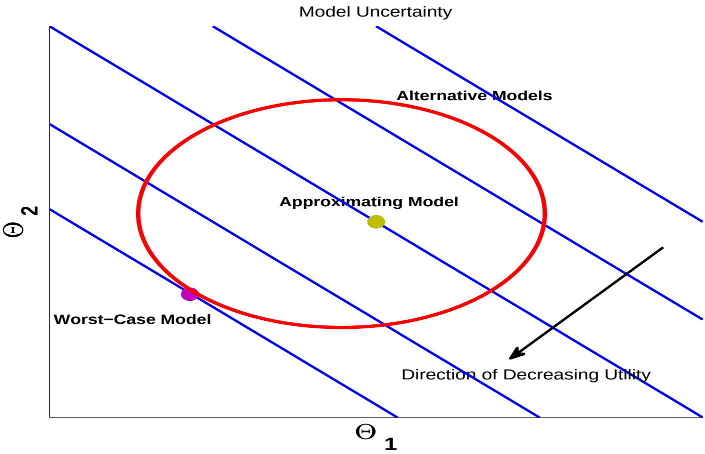

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- Suppose agent has Epstein-Zin preferences with IES $= 1$ and RRA $= 1 - \gamma$:

$$\begin{aligned} U_t &= c_t \left(E_t U_{t+1}^{\gamma}\right)^{\frac{\beta}{\gamma}} \\ \log(U_t) &= \log(c_t) + \frac{\beta}{\gamma} \log\left(E_t U_{t+1}^{\gamma}\right) \\ V_t &= \log(c_t) + \frac{\beta}{\gamma} \log\left(E_t \exp\left(\gamma V_t\right)\right) \end{aligned}$$

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Estimating γ

- Model misspecification that is easy to detect would simply lead to a change in approximating model
- Require that measures be absolutely continuous with respect to each other in finite samples (same zero probability events)
    - Discrete random variables – implies that realizations are not altered, only probabilities
    - Continuous random variables – pretty weak condition

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  - Simulate approximating model and test whether worst-case model is preferred by data (sample size matters!)
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  - Repeat lots of times and count number of "mistakes" – this is the detection error probability p
- Pick the γ that yields $p = 0.1$, for example

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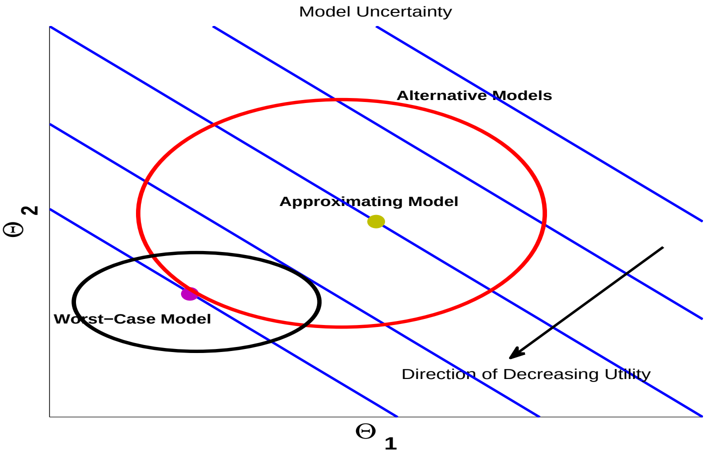

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- Want to model agents who are uncertain of the current state
- Underlying microfoundation: rational inattention
  - Information-processing capacity (Shannon channel) is finite
  - Agents must allocate limited attention to observing state of the world

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Modeling Information Flow

- Entropy measures uncertainty about a random variable
- Conditional entropy: entropy of X given an observation on Y:

$$H(X|Y) = -E[\ln(f(X|Y))]$$

- Change in conditional entropy can be used to measure information flow

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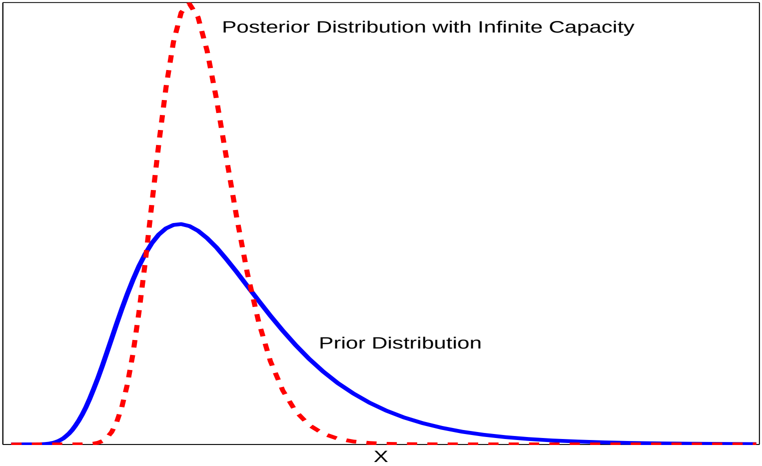
Signal Processing



Rational Inattention

- Agents face information-processing constraint

$$H\left(x_{t+1}|\mathcal{I}_t\right) - H\left(x_{t+1}|\mathcal{I}_{t+1}\right) \leq \kappa$$

- $\mathcal{I}_t$ is the consumer's *observed* and *processed* information at time t
- κ is the upper limit on information flow
- $H\left(x_{t+1}|\mathcal{I}_t\right)$: entropy of state before signal at time $t+1$ (prior)
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Decision-Making with Rational Inattention

- Decision variable is *joint distribution* of states and controls
- Prior distribution over states (so-called belief state) combines with joint distribution to produce posterior
- Decision process:
  - Agent chooses joint distribution of states and controls, given prior
  - Nature selects controls from distribution given actual state (which is unknown to agent)
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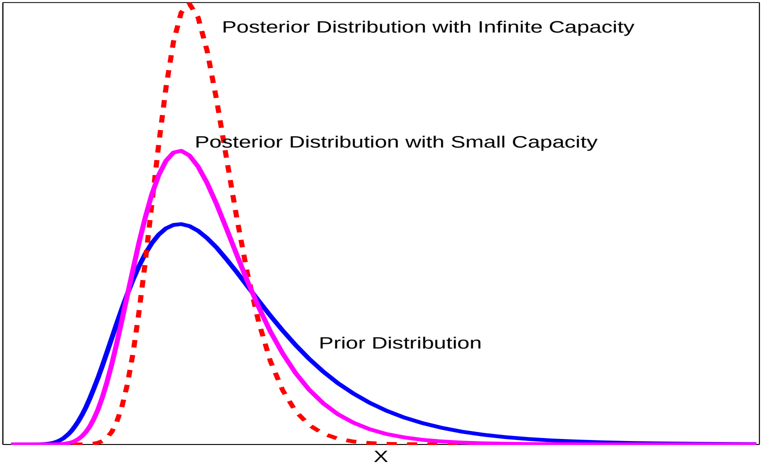
Signal Processing with Limited Information Flow



Equivalence Result

- Suppose world is linear-quadratic-Gaussian (including priors)
- Can prove **optimal** posterior is also Gaussian (therefore use Kalman filter to construct posterior)
- Entropy constraint reduces to change in variances of distributions:

$$\frac{1}{2}\sigma_t - \frac{1}{2}\sigma_{t+1} \leq \kappa$$

- Interpretation – observe state with error, but distribution of error chosen optimally

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Portfolio Choice Model

- Risky asset e with random return $r_{e,t+1}$
- Riskless asset f with constant return r_f
- Let $\mu = E_t\left[r_{e,t+1}\right]$ and $\text{var}\left(r_{e,t+1}\right) = \omega^2$
- Intertemporal budget constraint (log-linearized):

$$\Delta a_{t+1} = \left(1 - \frac{1}{\phi}\right)\left(c_t - a_t\right) + \psi + r_{p,t+1}$$

$$r_{p,t+1} = \alpha_t\left(r_{e,t+1} - r_f\right) + r_f + \frac{1}{2}\alpha\left(1-\alpha\right)\omega^2$$

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Portfolio Choice with Model Uncertainty

- Assume agent fears model uncertainty – use the Epstein-Zin representation:

$$W_t = c_t - \frac{1}{2}(\gamma - 1) c_t^2 - \frac{\beta}{\gamma - 1} \log\left(E_t\left[\exp\left((1-\gamma) W_{t+1}\right)\right]\right)$$

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- More uncertainty leads to smaller α

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Portfolio Choice with Model and State Uncertainty

- Exploit separation principle:

$$c_t = b_0 + \widehat{a}_t$$

$$\widehat{a}_{t+1} = (1-\theta)\,\widehat{a}_t + \theta\left(a_{t+1} + \xi_{t+1}\right) + \Omega$$

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- $\theta = 1 - 1/\exp(2\kappa)$ is optimal weight on new signal $a_{t+1} + \xi_{t+1}$
- $\widetilde{\gamma}$ is *effective* risk aversion, and is decreasing in θ
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- $\widetilde{\gamma}$ is *effective* risk aversion, and is decreasing in θ
- Lower processing capacity leads to smaller α

Portfolio Choice with Model and State Uncertainty

- Exploit separation principle:

$$c_t = b_0 + \widehat{a}_t$$

$$\widehat{a}_{t+1} = (1-\theta)\,\widehat{a}_t + \theta\left(a_{t+1} + \xi_{t+1}\right) + \Omega$$

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Quantitative Analysis

- Long-run effect of r_e on consumption:

$$\varsigma = \theta \sum_{i=0}^{\infty} \left(\frac{1-\theta}{\beta} \right)^{i} = \frac{\theta}{1-(1-\theta)\beta} \geq 1$$

- Long-run adjustment to risk aversion:

$$\Gamma = \frac{\gamma - 1}{1-\sigma} (\varsigma - 1) \geq 0$$

- Optimal portfolio share:

$$\alpha = (\gamma + \Gamma)^{-1} \frac{\mu - r_f + 0.5\omega^2}{\gamma\omega^2}$$

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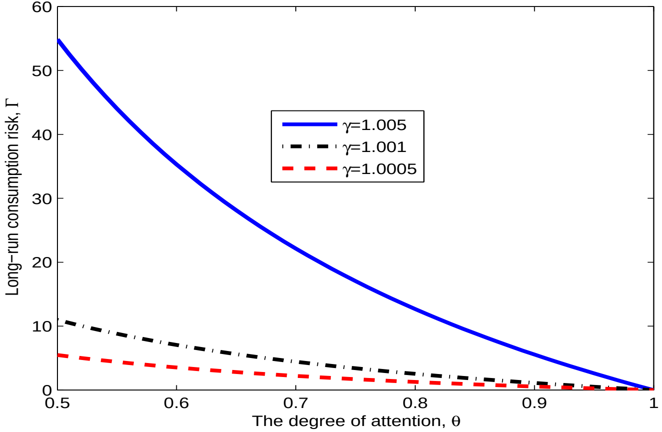
Size of Γ



60
50
40
30
20
10
0
0.5
0.6
0.7
0.8
0.9
1
Long−run consumption risk, Γ
The degree of attention, θ
γ=1.005
γ=1.001
γ=1.0005

A Rough Calibration

- Gabaix and Laibson (2002): $\alpha = 0.22$
- Campbell (2003): $\omega = 0.16$, $\mu - r_f = 0.06$
- Choose $\beta = 0.91$, $\sigma = 0.99999$, $\gamma = 1.001$, set $\theta = 0.48$ to match $\alpha = 0.22$
- If $\theta = 1$, then need $\gamma = 13$ to match $\alpha = 0.22$
- Open question: how to measure θ outside a model?

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Extending the Basic Model

- Adding nontraded labor income changes nothing
- What about abandoning linear-quadratic framework?
  - Really hard to do, because optimal posterior is not Gaussian
  - Matjeka and Sims (2011) show posterior is discrete if support is bounded (conserves on information flow)
- Show a single picture from a two-period portfolio choice problem without model uncertainty

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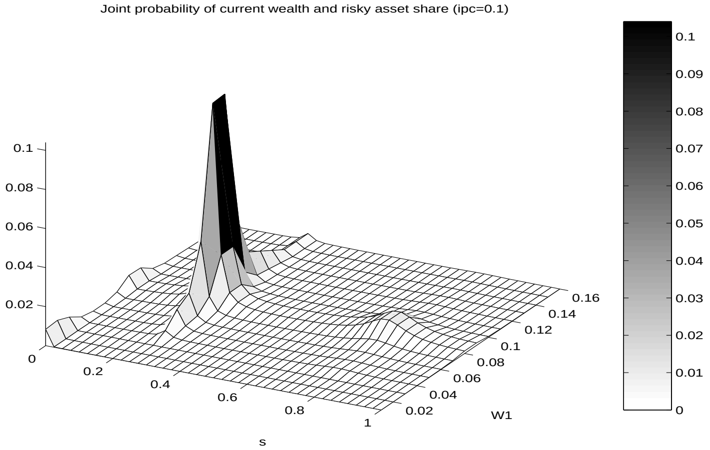
Risky Asset Share



Concluding Comments

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Table: Implications of Different Models ($p = 0.1$)

	Data	RE	RB
$\sigma(\Delta c)/\sigma(\Delta y)_{em}$	0.71	0.21	0.77
$\sigma(\Delta c)/\sigma(\Delta y)_{de}$	0.59	0.31	0.62
$\rho(ca, y)_{em}$	-0.17	1.00	0.60
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Dynamics of Debt and Taxes

Table: VAR Test for Predicted Path of Debt

Model	λ_1	λ_2	$\chi^2_W(2)$	p-value
RE	$-0.09(0.21)$	$0.34(0.21)$	20.04	0.00
$\Sigma = 0.1$	$-0.08(0.21)$	$0.37(0.21)$	19.48	0.00
$\Sigma = 0.5$	$-0.06(0.22)$	$0.51(0.18)$	11.36	0.00
$\Sigma = 0.9$	$-0.05(0.23)$	$0.71(0.18)$	2.78	0.25
$\Sigma = 0.95$	$-0.06(0.23)$	$0.73(0.18)$	2.23	0.33

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