Rising Indebtedness and Hyperbolic Discounting: A Welfare Analysis*

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Abstract

Is the observed large increase in consumer indebtedness since the 1980s beneficial for U.S. consumers? This paper quantitatively studies the macroeconomic and welfare implications of relaxing borrowing constraints when consumers exhibit a hyperbolic discounting preference. The model can capture two contrasting views: the positive view, which links increased indebtedness to financial innovation and thus better insurance, and the negative view, which is associated with consumers' *over-borrowing*. I find that the latter is sizable: the calibrated model implies a social welfare *loss* equivalent to a 0.2% decrease in per-period consumption from the relaxed borrowing constraint consistent with the observed increase in indebtedness. The welfare implication is strikingly different from the model with the standard exponential discounting preference, which implies a welfare *gain* of 0.6%, even though the two models are observationally similar. Naturally, according to the hyperbolic discounting model, there is a welfare gain from restricting consumer borrowing in the current U.S. economy.

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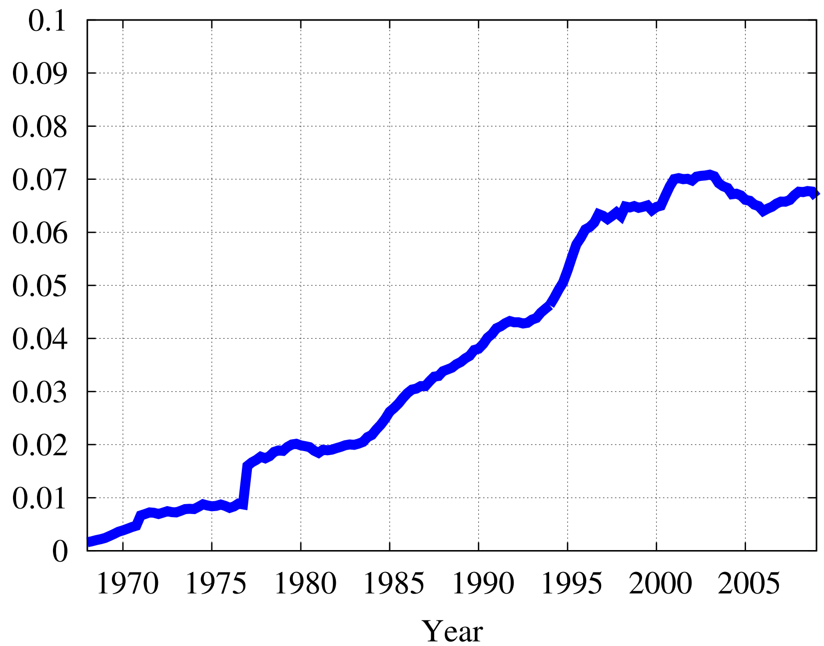


Figure 1: Total unsecured consumer debt over GDP

1 Introduction

Since the early 1980s, there has been a substantial increase in the indebtedness of U.S. consumers, although that trend might reverse as a result of the ongoing deep recession. Total household debt in the U.S. increased from 43% of GDP in 1982 to 62% in 2000.[1] Both unsecured and secured debt increased. Figure 1 shows the trend of unsecured consumer debt relative to GDP.[2] It was close to zero before 1970, rose to 2% by 1980, and has stabilized around 7% since 2000. While an increase in indebtedness is often seen as a result of an innovation in the financial sector and thus is linked to a gain in social welfare, there are two issues. First, increased indebtedness might induce under-saving, which slows down capital accumulation. Second, there is a popular perception that consumers might be *over-borrowing* and *over-consuming*. While the first issue is studied, among others, by Campbell and Hercowitz (2009) and Obiols-Homs (forthcoming), the second issue has not been studied, since it cannot be systematically captured by models with a time-consistent preference. To the best of my knowledge, this paper is the first to quantitatively investigate the issue.

In order to clarify the statement above, let me define the terms *over-consuming* and *over-*

[1] Smith (2009).

[2] Unsecured consumer debt is measured as the revolving consumer credit in the G.19 series of the Federal Reserve Board (FRB). In the FRB data, total consumer credit consists of non-revolving and revolving credit. Revolving credit mainly consists of loans for automobiles, mobile homes, and boats but also includes some unsecured credit. Livshits et al. (2010) constructed an unsecured consumer credit data series that includes not only revolving credit but also a part of non-revolving credit. However, the difference between the revolving credit and the unsecured consumer credit they constructed is small (less than one percentage point as a percentage of disposable income) for the period where more reliable data are available (after 1989).

borrowing. Over-consuming (over-borrowing) refers to the situation where a consumer consumes (borrows) more than he would have consumed (borrowed) if he could have committed to a level of consumption (borrowing) before. Alternatively, if one interprets the preference of the consumer with hyperbolic discounting as the preference featuring temptation and self-control, as in Krusell et al. (2009), over-consuming (over-borrowing) refers to the situation where a consumer consumes (borrows) more than he would have consumed (borrowed) if he had perfect self-control and thus is not affected by the temptation to consume (borrow) more today. According to the latter interpretation, over-consuming and over-borrowing could incur welfare loss when the borrowing constraint is relaxed. Quantitatively how important are these? That is one of the main questions investigated in this paper.

Notice that there is the possibility that an increase in borrowing is associated with lower welfare, even without over-borrowing as defined here. One way this happens is due to the general equilibrium effect: higher borrowing is associated with capital deccumulation, which reduces the aggregate output of the economy. This is exactly the case where the negative welfare effect of a relaxed borrowing constraint occurs in Campbell and Hercowitz (2009) and Obiols-Homs (forthcoming). The decline in welfare due to the general equilibrium effect associated with the relaxed borrowing constraint occurs in the previous literature as well as in the current paper, but the decrease in welfare due to over-borrowing is a unique feature of the current paper. I will quantify the relative importance of the two channels of welfare loss associated with increased indebtedness.

This paper examines the macroeconomic and welfare implications of a relaxed borrowing constraint in a general equilibrium life-cycle model with hyperbolic discounting consumers. In particular, I assume that the increased indebtedness is due to a relaxed borrowing limit that consumers face, calibrate the borrowing limits so that the induced indebtedness matches the observed aggregate debt level, and study the macroeconomic and welfare implications associated with the relaxed borrowing limit. The hyperbolic discounting model has become popular for analyzing consumers' behavior, especially behavior associated with borrowing and defaulting, since it is argued that the hyperbolic discounting model not only is consistent with experimental evidence but also is able to replicate some dimensions of consumers' behavior that the standard exponential discounting model cannot. By introducing the hyperbolic discounting preference into a standard macroeconomic model, and calibrating the model, I can systematically and quantitatively investigate the welfare implications associated with increased indebtedness in the hyperbolic discounting model. I employ both the steady-state comparison and the analysis of the equilibrium transition path.

The model developed here is built on a general equilibrium model with incomplete markets initially developed by Huggett (1996) and Aiyagari (1994). Since this class of models allows researchers to explicitly study heterogeneity either due to ex-ante heterogeneity or due to market incompleteness within a standard general equilibrium macroeconomic framework, the models have been extended in a variety of ways and widely used to analyze a variety of issues, including business cycles, wealth inequality, optimal taxation, Social Security reform, and asset pricing. The model developed in the current paper introduces a hyperbolic discounting preference into the standard general equilibrium model with market incompleteness. The model is closest to the one by İmrohoroğlu et al. (2003), but they do not focus on intra-generational heterogeneity, while the heterogeneous welfare effect on different types of consumers is a key aspect of the analysis in the current paper.

The model in the current paper also builds on recent developments in the quantitative model with hyperbolic discounting consumers. The pioneer work is Laibson (1997). I will review the literature on hyperbolic discounting in Section 2.1. Most existing work with hyperbolic discounting consumers is built on the partial equilibrium consumption-savings model of Deaton (1991) and Carroll (1997), while general equilibrium effect plays an important role in the main results of the paper here.[3] In addition, the current paper is one of the few that focus not only on the implications of the allocation of hyperbolic discounting, but also on the welfare implications.[4]

Although the borrowing constraint is taken as exogenous in the current paper, there is a recent development in quantitative macro models with consumer bankruptcy, which endogenously generate a borrowing constraint for unsecured loans. The seminal works are Athreya (2002), Livshits et al. (2007), and Chatterjee et al. (2007). Li and Sarte (2006) argue that the general equilibrium effect associated with the recent reform of the consumer bankruptcy law, which makes filing for bankruptcy more difficult, relaxes borrowing constraints and thus reduces aggregate savings, is sizable.

There are five main findings. First, when calibrated using the same strategy, models with exponential discounting and hyperbolic discounting are observationally similar in terms of average life-cycle profiles of consumption, savings, and borrowing. The finding echoes what Angeletos et al. (2001) find and is closely related to the observational equivalence result of Barro (1999) for the neoclassical growth model. Even though a hyperbolic discounting preference induces consumers to over-consume through a low short-term discount factor compared with the exponential discounting model, the long-term discount factor must be calibrated to be higher in the hyperbolic discounting model to match the aggregate capital stock in the data, which basically nullifies the effect of the low short-term discount factor. Second, although average life-cycle profiles are observationally similar, the two models have some different cross-sectional implications: the model with hyperbolic discounting consumers generates (i) more borrowing-constrained consumers, (ii) higher dispersion of wealth, and (iii) higher consumption volatility. Third, not only are the models observationally similar, in particular the average life-cycle profiles, in the steady-state equilibrium, the aggregate response to a relaxed borrowing limit is both qualitatively and quantitatively similar between the two models. This result is valid both in the steady-state comparison and in the equilibrium transition analysis. Because of the observational similarity, it is hard to distinguish the two models by the response in terms of macroeconomic aggregates. Fourth, even though the macroeconomic implications are similar, the two models have strikingly different welfare implications. More specifically, while a relaxed borrowing constraint has a positive effect on social welfare in the model with exponential discounting consumers, hyperbolic discounting consumers on average suffer from a relaxed borrowing constraint mainly because of over-borrowing and over-consumption. In particular, both the steady-state comparison and the analysis of the equilibrium transition path confirm that the consumers who start their working life in 2000 are worse off than those in 1980 in terms of ex-ante expected life-time utility. The problem is serious from a policy perspective because the two models are hard to distinguish but

[3]Notable exceptions are İmrohoroğlu et al. (2003) and Krusell et al. (2009).

[4]Notable exceptions are Krusell et al. (2009) and Petersen (2004).

have contrasting welfare implications. Barro (1999) argues that we can largely keep relying on the neoclassical growth model with exponential discounting consumers as the workhorse framework even though there is some evidence in favor of a hyperbolic discounting preference, because the growth models with the two different preference specifications are observationally equivalent. The case I study in this paper shows that one needs to be careful even if the hyperbolic discounting model is observationally similar to the exponential discounting counterpart, because the two models could have very different welfare implications. Finally, the *optimal* level of the borrowing limit is substantially lower, at about 15% of average income, in the model with hyperbolic discounting compared with the standard exponential discounting model, whose optimal borrowing limit is about 37%. Even in an exponential discounting model like those in Campbell and Hercowitz (2009) and Obiols-Homs (forthcoming), there is a level of the borrowing limit at which the gain from a relaxed borrowing limit (better consumption insurance) is dominated by the negative general equilibrium effect (capital deccumulation). The reason why the optimal borrowing limit is substantially lower in the model with hyperbolic discounting is over-borrowing and over-consumption. When consumers exhibit a hyperbolic discounting preference, there is an extra welfare gain from restricting borrowing by consumers. It is also interesting that while the exponential discounting model indicates that the current level of indebtedness in the U.S. is associated with the borrowing limit close to the optimal one, the hyperbolic discounting model indicates that the current borrowing limit is too high compared with the optimal level.

I review the related literature, particularly on hyperbolic discounting models and on the increased indebtedness of U.S. consumers since the early 1980s, in Section 2. Section 3 develops the model. Section 4 describes how the model is calibrated for quantitative exercises. Since the model is solved numerically, Section 5 gives an overview of the computational algorithm. Section 6 presents the main results of the paper, using steady-state analysis. Section 7 conducts an analysis explicitly taking into account the equilibrium transition path from an initial steady state to a new one. The analysis of the equilibrium transition confirms the results obtained using the steady-state analysis in Section 6. Section 8 discusses some of the assumptions. Section 9 concludes.

2 Related Literature

I will provide a brief review of the two literatures to which the current paper is closely related.

2.1 Hyperbolic Discounting

Strotz (1956) first argued that people are more impatient with respect to short-run trade-offs than long-run trade-offs and formalized the dynamic inconsistency problem using a game played against future selves.[5] Phelps and Pollak (1968) use the quasi-hyperbolic discounting function in the context of intergenerational time preferences; the consumer in the current generation discounts the utility of the next generation with a higher discount rate than the rate applied to subsequent generations. The quasi-hyperbolic discounting used in the current paper was developed by Laibson (1997) in the context of a life-cycle model with a time-inconsistent preference.[6]

[5] Angeletos et al. (2001) provide a good summary of the literature.

[6] The actual discount factor function used in the "hyperbolic discounting" models are not precisely hyperbolic, but it is called by that name because originally a hyperbolic function was used. Krusell and Smith (2000) call

Following Laibson (1997), there has been a surge in research on the hyperbolic discounting preference. Laibson (1997) studies the role of an illiquid asset like housing in providing an imperfect commitment device for time-inconsistent consumers. Laibson et al. (2003) use a hyperbolic discounting model to explain why many people carry a balance on credit cards with a high interest rate even if they also carry a positive balance of a liquid asset simultaneously. Angeletos et al. (2001) compare the implications of models with exponential and hyperbolic discounting consumers and argue that the hyperbolic discounting model replicates various dimensions of consumption and savings behavior better than the standard exponential discounting model. Laibson et al. (2007) use a simulated method of moments to jointly estimate key parameters associated with the hyperbolic discounting model. Malin (2008) studies the role of a savings floor when consumers exhibit a hyperbolic discounting preference. İmrohoroğlu et al. (2003) study the macroeconomic and welfare effects of having an unfunded Social Security program in the model with hyperbolic discounting consumers. Barro (1999) studies the neoclassical growth model with a hyperbolic discounting preference. Tobacman (2009) investigates the wealth distribution of such a model.

Although the hyperbolic discounting preference potentially has welfare implications very different from those in the standard exponential discounting preference, not many papers quantitatively study the welfare implications of the macroeconomic model with a hyperbolic discounting preference. Notable exceptions are İmrohoroğlu et al. (2003), Krusell et al. (2009) and Petersen (2004). İmrohoroğlu et al. (2003) study how the welfare implications of Social Security reform are different between the standard exponential discounting model and hyperbolic discounting model. Krusell et al. (2009) show that in the model with temptation and self-control, which can be considered as the generalization of the hyperbolic discounting preference studied in the current paper, a savings subsidy (or negative capital income tax) is optimal. This is because a savings subsidy prevents consumers from over-consuming and under-saving and thus helps consumers overcome the temptation to overindulge in current consumption. Petersen (2004) studies the welfare implications of various tax policies in the life-cycle general equilibrium model.

2.2 Increasing Indebtedness in the U.S.

The two papers most closely related to the current paper are Campbell and Hercowitz (2009) and Obiols-Homs (forthcoming). Both papers investigate the welfare consequences associated with rising debt in the U.S., but both use the standard exponential discounting preference. Campbell and Hercowitz (2009) study the welfare implications of the observed increase in consumer debt, in particular, secured debt. They use the calibrated general equilibrium model of impatient borrowers (who have a perpetually lower discount factor) and patient savers (with a perpetually higher discount factor) and argue that the welfare effect of relaxing a down-payment constraint is negative for borrowers and positive for savers. Even though borrowers enjoy a welfare gain from relaxation of the down-payment constraint, a negative effect from a higher equilibrium interest rate dominates the welfare gain from better consumption smoothing. Naturally, savers gain substantially from the higher interest rate. As discussed by Smith (2009), since the model is highly stylized, including the assumption of the different discount factor, it is not clear if

the preference *quasi-geometric* discounting, which offers a more precise description of the actual discount factor function typically used in the literature. The preference is also characterized as exhibiting a *present bias*.

the quantitative result of the paper is robust in a more general environment. Obiols-Homs (forthcoming) studies the cross-section of the welfare effect of a relaxed borrowing limit, as in the current paper, in the general equilibrium model with infinitely lived consumers. The author finds that the welfare effect is U-shaped with respect to individual productivity. In the current paper, the U-shape is obtained as well for the exponential discounting model, but the hyperbolic discounting preference adds interesting implications to the heterogeneity of the welfare effect.

Livshits et al. (2010) investigate jointly the reasons behind the increase in unsecured loans and in consumer bankruptcies. They find that a combination of a decline in the transaction cost of lending and the cost of filing for bankruptcy replicates what has occurred since the early 1980s quite well. Benton et al. (2007) study over-borrowing from the point of view of behavioral economics using survey evidence.

3 Model

The model is based on the general equilibrium life-cycle model of Huggett (1996), with the quasi-hyperbolic discounting preference of Laibson (1996). Below I will describe the model as the one with hyperbolic discounting consumers. Then, in Section 3.8, I will provide an alternative interpretation of the same problem of hyperbolic discounting consumers, developed by Krusell et al. (2009). I will use the latter as it allows a straightforward welfare analysis by avoiding the problem of a consumer with multiple selves.

3.1 Demographics

Time is discrete and starts from 0. In each period, the economy is populated by I overlapping generations of consumers. In period t, a measure $(1+\nu)^t$ of consumers are born. ν is the constant population growth rate. Each generation is populated by a mass of consumers, each of whom is measure zero. Consumers are born at age 1 and could live up to age I. There is a probability of early death. Specifically, s_i is the probability with which an age i consumer survives to age $i+1$. With probability $(1 - s_i)$, an age i consumer does not survive to age $i + 1$. I is the maximum possible age, which implies $s_I = 0$.

Consumers retire at age $1 < I_R < I$. Consumers with age $i < I_R$ are called workers, and those with age $i \geq I_R$ are called retirees. I_R is fixed: there is no retirement decision.

3.2 Preference

The preference of consumers is time separable and characterized by an instantaneous utility function and two discount factors. The instantaneous utility function $u(c)$ is standard: it is strictly increasing and strictly concave in c.

I use a quasi-hyperbolic discounting preference, which was first analyzed by Phelps and Pollak (1968) and used in a quantitative macroeconomic model in Laibson (1996) and Laibson et al. (2007). According to their set-up, in period t, instantaneous utility in period t, $t + 1$, $t + 2$, $t + 3$, $t + 4$,..., is discounted by 1, $\beta\delta$, $\beta\delta^2$, $\beta\delta^3$,... Since β is used only to discount utility from the current period and the next, while δ is used to discount future utility every period, β and δ are called *short-term* and *long-term* discount factors, respectively. Notice that the standard exponential discounting is a special case with $\beta = 1$: in this case, future utility is discounted at the constant factor of δ.

For an age i consumer, the expected life-time utility U_i can be defined as follows:

$$U_i = u(c_i) + \beta \, \mathbb{E} \sum_{j=i+1}^{I} \delta^{j-i} u(c_j) \tag{1}$$

The important feature of this class of preference is that the preference exhibits time inconsistency. For example, the discount factor applied between period $t+1$ and $t+2$ in period t is δ, while the discount factor between the same periods changes to $\beta\delta$ in period $t+1$. When $\beta \in (0,1)$, the preference implies a present bias: if there is no binding constraint or commitment device, consumers over-consume and under-save or over-borrow from the perspective in previous periods.

3.3 Technology

There is a representative firm that has access to the following constant returns to scale production technology:

$$Y = ZF(K, L) \tag{2}$$

where Y is output, Z is the level of total factor productivity, K is capital stock, and L is labor supply. Capital depreciates at a constant rate κ per period.

3.4 Endowment

Consumers are born with zero assets. Each consumer is endowed with one unit of time each period and inelastically supplies labor, since leisure is not valued. Labor productivity of a consumer is characterized by $e(i, p)$, where i captures the life-cycle profile of labor productivity, and p is the uninsured shock to labor productivity. p is assumed to have finite support: $p \in \{p_1, p_2, ..., p_N\}$. Each newborn consumer draws its initial p from an i.i.d. distribution where π_p^0 is the probability attached to each p. After the initial p is drawn, p follows a first-order Markov process with $\pi_{p,p'}$ as the transition probability from p to p'.

3.5 Market Arrangements

Capital and labor are traded competitively. Consumers are not allowed to trade state-contingent securities but can borrow or save using asset a, subject to a borrowing limit $\underline{a}_t$.

3.6 Government

The government has three roles in the model: (i) running the Social Security program, (ii) collecting a proportional income tax, and (iii) collecting accidental bequests using estate taxes and redistributing the proceeds with a lump-sum transfer.

The government runs a simple pay-as-you-go Social Security program. The government imposes a flat payroll tax with the tax rate of τ_S on all workers and uses the proceeds to finance Social Security benefits $b_{t,i}$ of current retirees. It is assumed that all retirees receive the same amount of benefits regardless of their contribution, and the government budget associated with the Social Security program balances each period. Formally, $b_{t,i} = 0$ for $i < I_R$ and $b_{t,i} = \overline{b}_t$ for $i \geq I_R$. $\overline{b}_t$ is the amount of Social Security benefit in period t and is the same for all retirees regardless of their age or contribution.

The government collects a proportional general income tax with tax rate τ_I. Both capital and labor income are taxed at the same rate. The proceeds are not redistributed or valued by consumers.

Because of the stochastic death, there are accidental bequests in the model. I assume that the government collects all of the accidental bequests using 100% of the estate taxes and redistributes the proceeds equally to the surviving consumers every period. d_t denotes the lump-sum transfer under the program in period t.

3.7 Consumer's Problem

I define the problem of a consumer recursively. The problem of an age i consumer with the current productivity shock p and asset position a in period t can be characterized by the following Bellman equation:

$$\widetilde{V}_t(i,p,a) = \max_{a'} \left[u(c) + \beta\ \delta\ s_i\ \sum_{p'} \pi_{p,p'} V_{t+1}(i+1,p',a') \right] \tag{3}$$

subject to

$$c + a' = (a + d_t)(1 + r_t(1-\tau_I)) + e(i,p)(1-\tau_I-\tau_S)w_t + b_{t,i} \tag{4}$$

$$a' \geq \underline{a}_t \tag{5}$$

$a' = g_t^a(i,p,a)$ is the optimal decision rule associated with the Bellman equation above. Notice that the value function on the left-hand side, $\widetilde{V}_t(i,p,a)$, is different from the one on the right-hand side, which is $V_t(i,p,a)$. This is due to the time-inconsistency problem associated with the quasi-hyperbolic discounting preference. The value function is updated with the following equation:

$$V_t(i,p,a) = \left[u(c) + \delta\ s_i\ \sum_{p'} \pi_{p,p'} V_{t+1}(i+1,p',a') \right] \tag{6}$$

where

$$a' = g_t^a(i,p,a) \tag{7}$$

$$c = (a + d_t)(1 + r_t(1-\tau_I)) + e(i,p)(1-\tau_I-\tau_S)w_t + b_{t,i} - g_t^a(i,p,a) \tag{8}$$

Mechanically, the consumer chooses the optimal asset level a' with the discounting factor $\beta\delta$ but the actual value function is evaluated with the discount factor δ. İmrohoroğlu et al. (2003) distinguish the two cases in terms of what hyperbolic discounting consumers expect about their own future decisions. According to their classification, a *naive* consumer wrongly thinks that future selves make decisions in a time-consistent manner (using only the discount factor δ). On the other hand, a *sophisticated* consumer thinks that future selves are time-inconsistent (using both β and δ). The way I formalize the consumer's problem, which is the same as in Laibson (1996) and Laibson et al. (2007), can be classified as sophisticated consumers according to their classification. Angeletos et al. (2001) find that naive and sophisticated hyperbolic discounting consumers behave similarly in a life-cycle model.

3.8 Consumer's Problem with Temptation and Self-Control

Krusell et al. (2009) provide an alternative interpretation of the problem of a hyperbolic-discounting consumer, based on the preference exhibiting temptation and self-control (Gul and Pesendorfer (2001)). In their setup, a consumer is *tempted* to choose consumption and saving based on the discount factor $\beta\delta$. There is another parameter in consumer's preference, γ, which determines the strength of the temptation. Let me state the problem of the consumer using this setup below.

$$\begin{aligned} V_t(i,p,a) = \max_{a'} \Bigg[& u(c) + \delta \ s_i \ \sum_{p'} \pi_{p,p'} V_{t+1}(i+1,p',a') \\ & + \gamma \left(W_t(i,p,a,a') - \max_{a'} W_t(i,p,a,a') \right) \Bigg] \end{aligned} \tag{9}$$

where

$$W_t(i,p,a,a') = u(c) + \beta \ \delta \ s_i \ \sum_{p'} \pi_{p,p'} V_{t+1}(i+1,p',a') \tag{10}$$

and subject to (4) and (5). The maximand of the Bellman equation consists of two parts, corresponding to the two lines of (9). The first part is the standard Bellman equation, with the discount factor δ. If the consumer only cares about this first part, the optimal choice is the one associated with the discount factor δ. However, there is the second part, which represents temptation. Notice that $W_t(.)$ is a maximand of the standard Bellman equation with the discount factor $\beta\delta$, and that the second term of the second part ($\max_{a'} W_t(i,p,a,a')$) is the optimal value associated with the discount factor $\beta\delta$. If the consumer only cares about the second part, the consumer chooses a' that maximizes $W_t(.)$ because that would maximize the second part. Therefore, the consumer is torn by two opposing forces. On the one hand, the consumer wants to choose a' that maximizes the standard Bellman equation with the discount factor δ (first part). On the other hand, the consumer also wants to choose a' that maximizes the standard Bellman equation with the discount factor $\beta\delta$ (second part). And it is easy to see that γ controls the relative strength of the second part. What happens if γ goes to ∞? In this particular case, the consumer succumbs to the temptation to and basically chooses a' that maximizes $W_t(.)$. Besides, since the second part of the Bellman equation becomes zero, the value is evaluated based on the discount factor δ. This is exactly the problem described in the previous section.

Two remarks are worth making here. First, the value function $V_t(i,p,a)$ and the optimal decision rule $g_t^a(i,p,a)$ obtained from (9) are equivalent to those obtained from (3). Second, either $\beta = 1$ or $\gamma = 0$ collapses the problem back to the one with the standard exponential discounting.

3.9 Equilibrium

I will first define the recursive competitive equilibrium where the demographic structure is stationary, even though the size of the population is growing at a constant rate ν. Then I will move on to define the steady-state recursive competitive equilibrium, where prices $\{r_t, w_t\}_{t=0}^{\infty}$, and government policy variables $\{b_{t,i}, d_t\}_{t=0}^{\infty}$ are constant over time, although the aggregate variables are growing at the population growth rate.

Let $\mathbf{M}$ be the space of an individual state, i.e., $(i, p, a) \in \mathbf{M}$. Let $\mathcal{M}$ be the Borel σ-*algebra* generated by $\mathbf{M}$, and μ the probability measure defined over $\mathcal{M}$. I will use a probability space $(\mathbf{M}, \mathcal{M}, \mu)$ to represent a type distribution of consumers.

Definition 1 (Recursive competitive equilibrium) *Given a sequence of total factor productivity $\{Z_t\}_{t=0}^{\infty}$, a sequence of borrowing limit $\{\underline{a}_t\}_{t=0}^{\infty}$, and the initial type distribution of consumers μ_0, a recursive competitive equilibrium is a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$, government policy variables $\{b_{t,i}, d_t\}_{t=0}^{\infty}$, aggregate capital stock $\{K\}_{t=0}^{\infty}$, aggregate labor supply $\{L\}_{t=0}^{\infty}$, value functions $V_t(i, p, a)$ and $\widetilde{V}_t(i, p, a)$, optimal decision rule $g_t^a(i, p, a)$, and the measure after normalization with respect to population growth, $\{\mu_t\}_{t=0}^{\infty}$, such that:*

1. *In each period t, given the prices and policy variables, $V_t(i, p, a)$ and $\widetilde{V}_t(i, p, a)$ are a solution to the consumer's optimization problem defined in Section 3.7, and $g_t^a(i, p, a)$ is the associated optimal decision rule.*

2. *The prices $\{r_t, w_t\}_{t=0}^{\infty}$ are determined competitively, i.e.,*

$$r_t = Z_t F_K(K_t, L_t) - \kappa \tag{11}$$

$$w_t = Z_t F_L(K_t, L_t) \tag{12}$$

where

$$K_{t+1} = \frac{1}{1+\nu} \int_{\mathbf{M}} g_t^a(i, p, a)\ d\mu_t \tag{13}$$

$$L_t = \int_{\mathbf{M}} e(i, p)\ d\mu_t \tag{14}$$

3. *Given the initial measure μ_0, the sequence of measure of consumers $\{\mu_t\}_{t=0}^{\infty}$ is consistent with the demographic transition, the stochastic process of shocks, and the optimal decision rules, after normalization with respect to population growth in each period t.*

4. *Government satisfies the period-by-period budget balance with respect to the Social Security program in each period t, i.e.,*

$$\int_{\mathbf{M}} b_{t,i}\ d\mu_t = \int_{\mathbf{M}} e(i, p)\ w_t\ \tau_S\ d\mu_t \tag{15}$$

5. *Government satisfies the period-by-period budget constraint with respect to the estate tax and lump-sum transfer in each period t, i.e.,*

$$\int_{\mathbf{M}} d_{t+1}\ d\mu_{t+1} = \frac{1}{1+\nu} \int_{\mathbf{M}} (1 - s_i)\ g_t^a(i, p, a)\ d\mu_t \tag{16}$$

Definition 2 (Steady-state recursive competitive equilibrium) *A steady-state recursive competitive equilibrium is a recursive competitive equilibrium where total factor productivity, borrowing limit, type distribution, prices, government policy variables, aggregate capital stock, aggregate labor supply, value functions, and optimal decision rules are constant over time, after normalizing the type distribution of consumers by the population growth rate.*

Notice that although I use the word *steady state*, the model is on a balanced growth path with a constant population growth rate and the type distribution (after normalization) of heterogeneous consumers is stationary. The measure of consumers is normalized to be a probability measure (total measure is one) each period, which makes all the aggregate variables constant over time instead of growing at the population growth rate.

4 Calibration

This section describes how the steady-state model (the model in a steady-state recursive competitive equilibrium) is calibrated. Consequently, the time script t is dropped throughout the section. Each of the subsections below corresponds to those in Section 3.

4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. I is set at 81, meaning that the maximum actual age is 100. I_R is set at 45, implying that consumers retire at the actual age of 65. The population growth rate, ν, is set at 1.2% annually. This is the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{s_i\}_{i=1}^{I}$ are taken from the life table in Social Security Administration (2007).[7] In order to guarantee the maximum age of $I = 81$, $s_{81} = 0$ is imposed. Figure 12 in Appendix A.1 shows the conditional survival probabilities used.

4.2 Preference

For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{17}$$

σ is set at 1.5, which is the commonly assumed value. It is also the point estimate of Laibson et al. (2007).

Discount factors β and δ are calibrated to be different for different model economies, but the calibration strategy is common. For all cases, I set the short-term discount factor β at first and calibrate the long-term discount factor δ so that the capital-output ratio of the economy in the baseline case is 3.0, which is the historical average value of the U.S. economy. In other words, different model economies have different short-term discount factors β, but they have the same aggregate capital stock in equilibrium.[8]

In the model with the standard exponential discounting consumers, $\beta = 1$ by assumption. I found that with $\delta = 0.9740$ the steady-state equilibrium of the model generates a capital-output ratio of 3.0. For the model with hyperbolic discounting consumers, I use $\beta = 0.70$ as the baseline value of the short-term discount factor and calibrate δ such that the model achieves the same capital-output ratio of 3.0. The short-term discount factor of 0.70 is the one-year discount factor

[7]Table 4.C6 of Social Security Administration (2007). An average of the survival probabilities of males and females is used.

[8]Both Angeletos et al. (2001) and Tobacman (2009) calibrate the long-term discount factor δ for the model with exponential discounting consumers such that the average wealth holding at age 63 (the age just before retirement) is the same as in the model with hyperbolic discounting consumers where β and δ are jointly estimated.

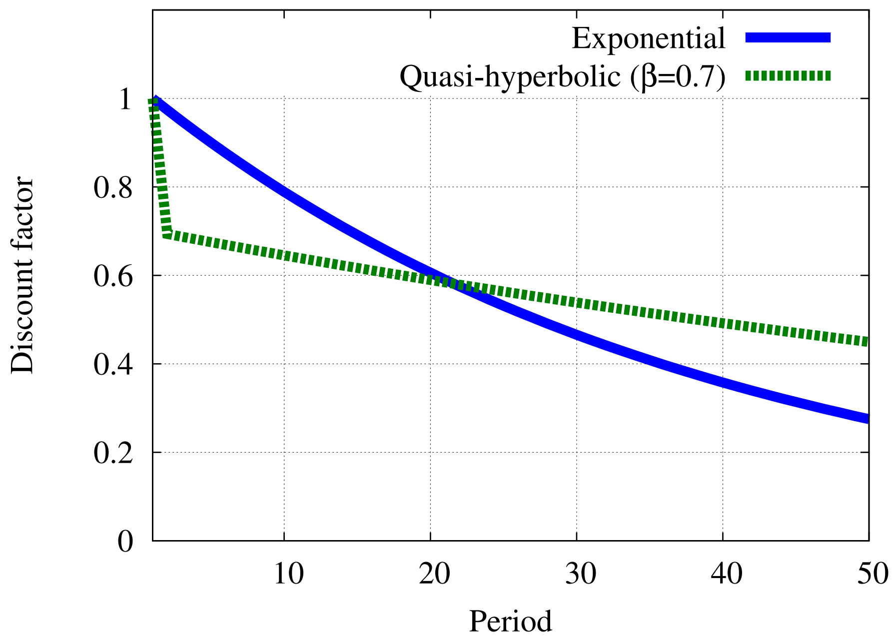


Figure 2: Comparison of discount factors

typically obtained from laboratory experiments. Moreover, the benchmark point estimate of Laibson et al. (2007) is $\beta = 0.703$, or the annual short-term discount rate of about 40%. The same calibration strategy generates $\delta = 0.9910$. The calibrated value of δ is higher than 0.958, which is the value that Laibson et al. (2007) estimate jointly with β. A large part of the difference is due to the existence of the mortality shock in the current model, which Laibson et al. (2007) do not have. If δ is adjusted by being multiplied by the average survival probability (0.9828), the resulting *effective* long-term discount factor is 0.974. Figure 2 compares the discount factors of the standard exponential discounting and hyperbolic discount for periods 1 to 50. The calibrated β and δ are used. Notice that the discount factor function drops substantially more from period 1 to 2 in the case of the hyperbolic discounting preference. On the other hand, the discount factor applied to utility in the distant future is higher for the hyperbolic discounting model. Laibson (1997) argues that housing, from which inhabitants can enjoy utility as long as they own it and live in it, has an extra value for hyperbolic discounting consumers, since the dividends can be enjoyed for a long period of time.

I also investigate the case when the discount rate is 80% annually, which is twice as high as in the baseline hyperbolic discounting case. An 80% annual discount rate implies a short-term discount factor δ of 0.56. Using the same calibration strategy, the economy with a low short-term discount factor yields $\delta = 1.0005$. Even though the long-term discount factor is above unity, the effective discount factor becomes less than one if the survival probability is taken into account.

4.3 Technology

The following standard Cobb-Douglas production function is assumed:

$$Y = ZF(K, L) = ZK^{\theta}L^{1-\theta} \tag{18}$$

Z is pinned down such that, in the baseline steady state, the equilibrium wage is normalized to one. The procedure implies $Z = 0.896$. θ is set at 0.36. This value is consistent with the average capital share of income of the U.S. economy. Capital is assumed to depreciate at the constant rate of $\kappa = 0.06$ per year. Huggett (1996) calibrates $\kappa = 0.06$ by matching the depreciation-output ratio of the model economy to its U.S. counterpart.

4.4 Endowment

I assume the following standard multiplicative form of individual productivity.

$$e(i, p) = e_i \; p \tag{19}$$

e_i represents the average age-earnings profile and p is the individual productivity shock. Since retirement age is fixed at I_R, $e_i = 0$ for $i \geq I_R$. To calibrate $\{e_i\}_{i=1}^{I_R-1}$, I follow Huggett (1996) and use the data on the median earnings of male workers of different age groups from Social Security Administration (2007).[9] The median earnings data are multiplied by the employment to population ratio of males in each age group. The employment to population ratio for each age group is obtained from McGrattan and Rogerson (2004).[10] Finally, the resulting age-productivity profile is smoothed out by fitting the age profile of the product of median earnings and the employment to population ratio to a quadratic function of age. The resulting earnings profile is shown in Figure 13 in Appendix A.1.

In order to calibrate the stochastic process for p, first, following Huggett (1996), I assume that the logarithm of p is initially drawn from a normal distribution $N(0, \sigma_0^2)$ and follows an AR(1) process with the persistence parameter ρ_p and the standard deviation of the innovation term σ_ϵ. These assumptions imply that the earnings for each age group is log-normally distributed, which captures the empirical distribution well. In sum, the stochastic process for p is characterized by a triplet $(\rho_p, \sigma_0^2, \sigma_\epsilon^2)$. Following Huggett (1996), I set $(\rho_p, \sigma_0^2, \sigma_\epsilon^2) = (0.96, 0.38, 0.045)$. These parameter values are consistent with existing estimates of the stochastic process of individual earnings shocks and jointly replicate the empirical earnings Gini coefficient of 0.42.

The AR(1) process obtained above is approximated using the algorithm of Tauchen (1986) with 18 abscissas. Among the 18 possible realizations of p, 17 are equally spaced between $-4\sigma_p$ and $4\sigma_p$ where σ_p is the standard deviation of the unconditional distribution of p. In order to capture to a certain extent the observed extreme concentration of earnings, the last abscissa is set at $6\sigma_p$.

4.5 Market Arrangements

In the baseline case, the borrowing limit $\underline{a}$ is set at zero, i.e., no borrowing is allowed. In experiments, I will relax the borrowing limit to the extent such that the aggregate amount of

[9] The earnings data are taken from Table 4.B6 of Social Security Administration (2007).

[10] Table 3, 4, and 5 of McGrattan and Rogerson (2004).

debt is the same between the model and the corresponding U.S. economy. In other words, I will back up the degree of relaxation of the borrowing constraint from the observed increase in indebtedness.

4.6 Government

The payroll tax rate for the Social Security contribution τ_S is set at 0.10, which is the average contribution to the Social Security program as a fraction of labor income in the U.S. The proportional income tax rate of $\tau_I = 0.2378$ is set to match the ratio of total (federal, state, and local) government consumption over total income. The historical average of the ratio of government consumption over total income for the U.S. is 0.195.

5 Computation

Since there is no analytical solution to the model, the model is solved numerically. The solution algorithm is standard: both the value functions and the optimal decision rule are approximated using piecewise linear functions. The optimal decision with respect to saving is solved using a golden section search. The equilibrium prices (wage and interest rate) and the government policy variables (transfer and Social Security benefits) are found using iteration. Details about the numerical procedure are found in Appendix A.2.

6 Results: Steady-State Analysis

This section presents the main results, based on the steady-state analysis. In the next section, I explicitly consider the equilibrium transition path, but the basic messages of the transition analysis are the same as those presented in this section. In Section 6.1, I compare the life-cycle profiles of economies with standard exponential discounting and hyperbolic discounting. The purpose is to investigate the (non-)difference in the life-cycle profiles generated by the time-inconsistent preference. In Section 6.2, I investigate the cross-sectional distribution of wealth in both economies. In Sections 6.3 and 6.4, I investigate the effect of the changes in the borrowing limit in the model with exponential discounting and with hyperbolic discounting. For the hyperbolic discounting model, two economies with different short-term discount factors (0.70 and 0.56) are studied. I will analyze the changes in the macroeconomic aggregates and distribution in Section 6.3 and study the welfare implications of changes in the borrowing limit in Section 6.4.

6.1 Life-Cycle Profiles

Figure 3 compares the life-cycle profiles of the model economies with standard exponential discounting on the left and with hyperbolic discounting (with the short-term discount factor of $\beta = 0.70$) on the right. The top two panels compare the average profiles of total income, consumption, and savings over the life-cycle. The middle two panels compare the average profile of asset holdings in two model economies. What is most striking is that there is little difference between the two model economies in terms of the average life-cycle profile. In both economies, the average consumption profile is flatter than the income profile. Consumers save during the working period and dissave during the retirement period.

The bottom two panels of Figure 3 compare the cross-sectional variance of earnings and consumption over the life-cycle in two model economies. Since there is no labor-leisure decision, the profile for earnings is common between the two figures. The variance of consumption increases

Table 1: Wealth distribution

Economy	Gini	Mean/ Median	Prop ≤ 0	Proportion of wealth held by 1st	2nd	3rd	4th	5th	top 1%
U.S.[1]	0.803	4.03	0.099	−0.3	1.3	5.0	12.2	81.7	34.7
Exponential	0.726	3.61	0.234	0.0	0.7	5.8	19.1	74.4	11.9
Hyperbolic ($\beta = 0.70$)	0.733	4.02	0.284	0.0	0.4	5.4	19.1	75.1	11.9
Hyperbolic ($\beta = 0.56$)	0.734	4.24	0.308	0.0	0.3	5.1	19.5	75.0	11.9

[1] Source: Survey of Consumer Finances 1998, statistics computed by Budría et al. (2002).

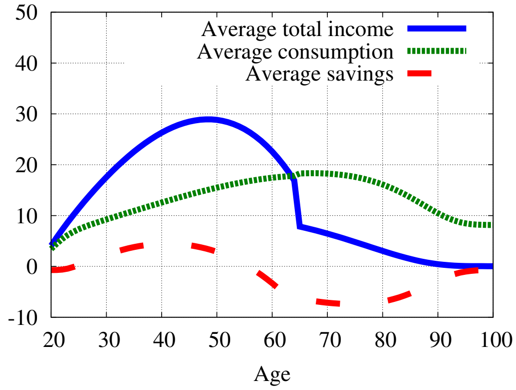
with age in both model economies, which is a feature of the life-cycle model with a persistent earnings shock. The only noticeable difference between the two figures is that the consumption variance increases faster in the model with hyperbolic discounting consumers especially toward the end of the working period. The feature is due to the extreme assumption of the constant Social Security benefit. If the amount of benefit is positively but imperfectly correlated with the contribution, which is the case in the U.S. economy, the spike of the consumption variance at the time of retirement substantially weakens.

6.2 Wealth Inequality

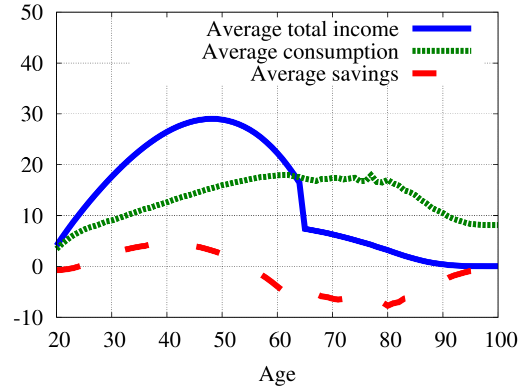
Table 1 compares the statistics related to the wealth inequality of the models and the data. Figure 4 compares the Lorenz curves for the U.S. economy and model economies with exponential and hyperbolic discounting. The statistics are based on the Survey of Consumer Finances (SCF) 1998 wave, computed by Budría et al. (2002). When the parameters are calibrated such that the macroeconomic aggregates are similar across different models, the hyperbolic discounting preference generates a slightly higher wealth inequality. The Gini coefficient for total wealth in the baseline hyperbolic discounting model is 0.733, which is slightly higher than 0.726, the Gini coefficient for the exponential discounting model. In terms of the skewness of the wealth distribution, the hyperbolic discounting model generates a higher skewness: the mean/median ratio of wealth is 3.61 for the exponential discounting model, while it is 4.02 in the hyperbolic discounting model. The value for the hyperbolic discounting model is close to the empirical value of 4.03.

The reason behind the higher wealth inequality is that more consumers are consuming all of their income and saving nothing in the hyperbolic discounting model. The proportion of consumers with non-positive assets (which means zero, because borrowing is prohibited for now) is 0.284 in the hyperbolic discounting model, compared with 0.234 for the exponential discounting model. In terms of the top end of the wealth distribution, both the exponential discounting model and the hyperbolic discounting model fail to replicate the extreme wealth concentration at the top of the distribution: the proportion of wealth held by the top 1% wealthiest is the same across the models at 11.9%, and it is far below the empirical value of 34.7%.[11] The features of the

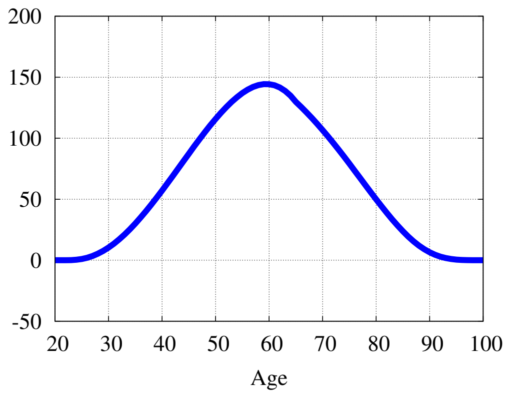
[11]The failure to replicate the extreme concentration of wealth arises partly because the individual productivity



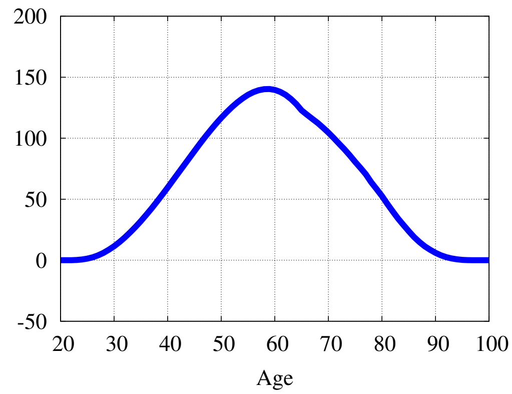
(a) Exponential: income consumption, and savings



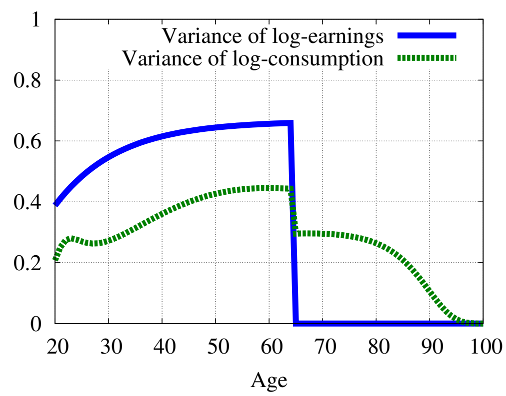
(b) Hyperbolic: income consumption, and savings



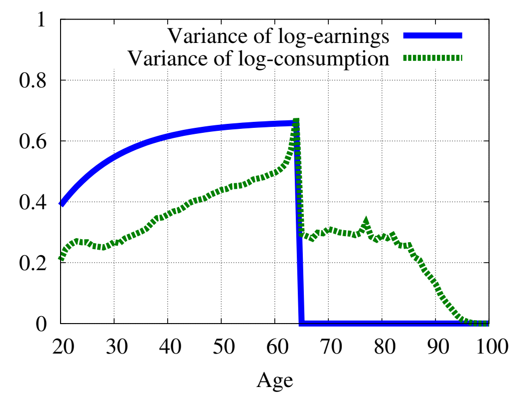
(c) Exponential: asset holdings



(d) Hyperbolic: asset holdings



(e) Exponential: cross-sectional variance of earnings and consumption



(f) Hyperbolic: cross-sectional variance of earnings and consumption

Figure 3: Comparison between exponential discounting and hyperbolic discounting models

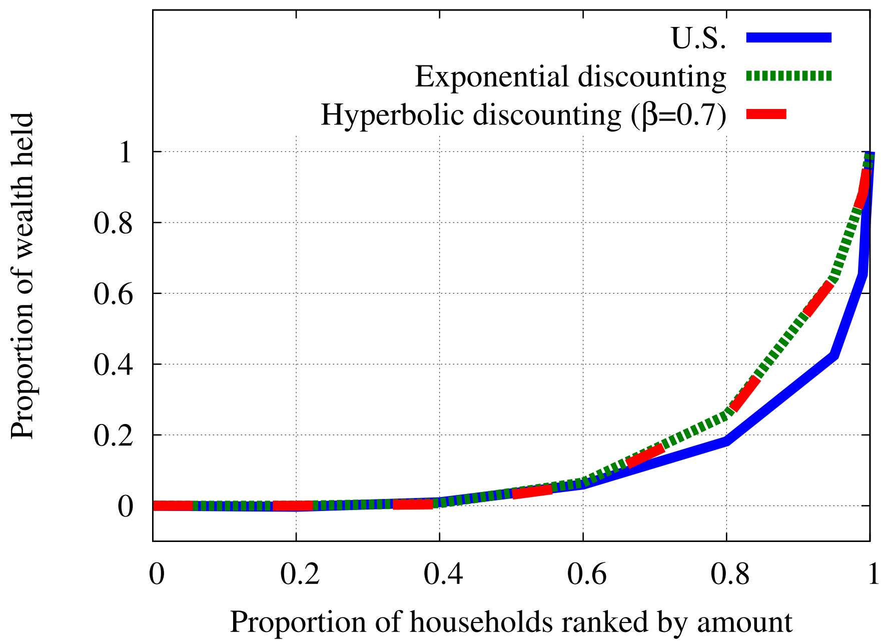


Figure 4: Lorenz curve for the wealth distribution

hyperbolic discounting model become stronger in the model with a very low short-term discount factor ($\beta = 0.56$), but the additional gain in the Gini coefficient is small (from 0.733 to 0.734). Tobacman (2009) also compares the wealth inequality implied by models with exponential and hyperbolic discounting. In the baseline case with both liquid and illiquid assets, the model with hyperbolic discounting exhibits a Gini coefficient of 0.508, which is slightly higher than the value for the exponential discounting model (0.488). The magnitude of the difference is comparable to what is obtained here.

6.3 Rising Indebtedness: Macroeconomic Implications

I will investigate the macroeconomic and welfare implications of the increased indebtedness in the models with exponential and hyperbolic discounting. In particular, I assume that the increased indebtedness is due to a relaxed borrowing constraint that consumers face, calibrate the borrowing limits so that the induced indebtedness in the model matches the observed aggregate debt level, and analyze the macroeconomic and welfare implications associated with the relaxed borrowing limits. Relaxing the borrowing constraint is a parsimonious way to capture various types of innovation in the consumer credit market that happened over the last three decades.

This section studies the macroeconomic implications and Section 6.4 analyzes the welfare implications. The focus is on the difference between the implications of the standard exponential discounting model and those of the hyperbolic discounting models. I show in the previous section that when calibrated to the same set of targets, in particular aggregate wealth, the steady-state implications are similar between the models with different preference specifications.

process is calibrated based on the Panel Study of Income Dynamics (PSID), where very rich households are substantially under-represented. For more on models of wealth distribution, see Quadrini and Ríos-Rull (1997).

Table 2: Macroeconomic implications of rising debt

Economy[1]	$\underline{a}$[2]	D/Y	K[3]	Y[3]	r%	wage	Var(c)[4]
Exponential discounting model ($\beta = 1.00$)							
1970	0.000	0.000	1.000	1.000	6.01	1.000	0.481
1980	−0.103	−0.020	0.986	0.995	6.11	0.995	0.465
2000	−0.337	−0.070	0.959	0.985	6.33	0.985	0.455
Hyperbolic discounting model ($\beta = 0.70$) **with** $\underline{a}$ **of exponential discounting model**							
1970	0.000	0.000	1.000	1.000	5.99	1.000	0.489
1980	−0.103	−0.023	0.989	0.996	6.09	0.996	0.474
2000	−0.337	−0.082	0.960	0.985	6.32	0.985	0.468
Hyperbolic discounting model ($\beta = 0.70$)							
1970	0.000	0.000	1.000	1.000	5.99	1.000	0.489
1980	−0.090	−0.020	0.988	0.996	6.09	0.996	0.475
2000	−0.290	−0.070	0.964	0.987	6.29	0.987	0.467
Hyperbolic discounting model ($\beta = 0.56$)							
1970	0.000	0.000	1.000	1.000	5.95	1.002	0.504
1980	−0.080	−0.020	0.988	0.996	6.04	0.998	0.492
2000	−0.271	−0.070	0.965	0.987	6.22	0.990	0.484

[1] 1970: Economy with no borrowing. 1980: Economy calibrated to debt-to-output ratio of 2%. 2000: Economy calibrated to debt-to-output ratio of 7%. For the second panel, borrowing limits that are obtained in the exponential discounting model are used in the hyperbolic discounting model with $\beta = 0.70$.

[2] Borrowing limit relative to total income.

[3] Level in the 1970 economy normalized to one.

[4] Average of cross-sectional variances of consumption for all age groups.

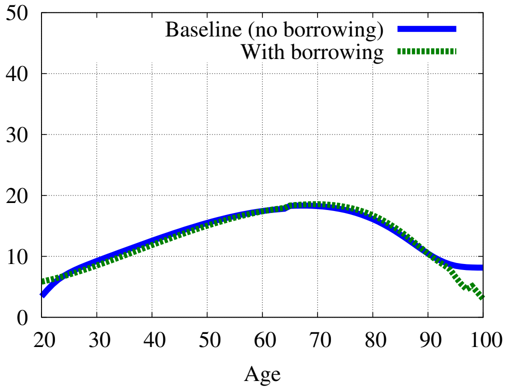
If, in addition, the macroeconomic and welfare implications of increased indebtedness are also similar between the exponential and hyperbolic discounting models, there is no need to use the non-standard hyperbolic discounting preference for an analysis of increased indebtedness. What I will show is that this is not the case: in particular, the welfare implications are very different between the models with different preference specifications.

Table 2 summarizes the macroeconomic implications of rising aggregate debt from 1970 to 1980 and 2000. The first panel summarizes the results of the exponential discounting model. The three rows correspond to the economies in the three time periods: The first one corresponds to the *1970 economy*, where borrowing does not exist, i.e., $\underline{a} = 0$. This is the economy calibrated in Section 4. The second economy is called the *1980 economy*, where the borrowing limit is relaxed such that the aggregate amount of debt in the new steady-state equilibrium is 2% of output. This target corresponds to the aggregate amount of unsecured debt in the U.S. economy in the early 1980s. The third economy is the *2000 economy*, where the borrowing limit is further relaxed such that the economy exhibits an aggregate amount of debt as large as 7% of output. One can see from the column labeled $\underline{a}$ in the first panel that, for the exponential discounting model, a borrowing limit of the size of 10.3% and 33.7% of average income is needed to generate aggregate debt of 2% and 7% of GDP, respectively. Capital stock declines as the borrowing limit is relaxed. In the 2000 economy, equilibrium capital stock is 4% lower than in the 1970 economy without borrowing. Since labor is inelastically supplied, the decline in capital stock generates a decline in output; output in the 2000 economy is about 1.5% lower than in the 1970 economy. The equilibrium interest rate goes up from 6% in 1970 to 6.3% in 2000 as capital becomes more scarce, and wage declines as capital stock declines. Since a relaxed borrowing constraint implies better consumption smoothing, consumption variance declines as the borrowing constraint is relaxed: consumption variance drops from 0.481 in the 1970 economy to 0.455 in the 2000 economy.

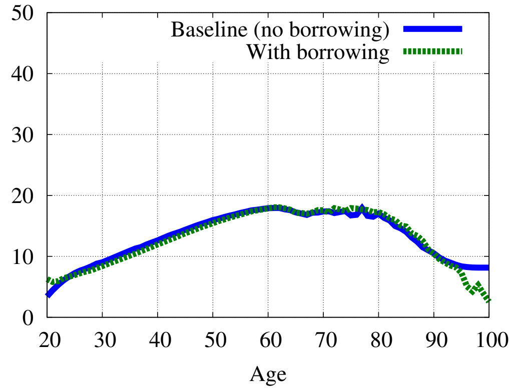
Figure 5 compares the 1970 (no borrowing) and 2000 (7% debt to output ratio) economies with both exponential and hyperbolic discounting consumers. Panels on the left side correspond to the exponential discounting models, and panels on the right side are associated with hyperbolic discounting models. Among the panels on the left side, the most notable change in the figures is the change in the cross-sectional variance of log-consumption. In panel (e), it is easy to see that the variance declines substantially for young consumers, at the expense of an increased variance for later stages of life. Since some young consumers insure themselves better in the economy with borrowing, the aggregate debt for the young on average increases (panel (c)). The average consumption profile becomes flatter with borrowing (panel (a)).

The second panel in Table 2 summarizes the results for the baseline hyperbolic discounting model ($\beta = 0.70$), but with the borrowing limits obtained from the exponential discounting model. One can see in column $\underline{a}$ that the borrowing limits used in the second panel are the same as those used in the first panel. Most changes are quite similar between the first and the second panel. But there is one important difference: the response of aggregate debt is stronger in the hyperbolic discounting model. When the borrowing limit is relaxed such that debt increased by 2% of GDP in the exponential discounting model, the aggregate debt over GDP ratio increased by 2.3% in the hyperbolic discounting model. When the debt over GDP ratio increased to 7% in the exponential discounting model, the same borrowing limit induces an 8.2% debt over GDP ratio in the hyperbolic discounting model.

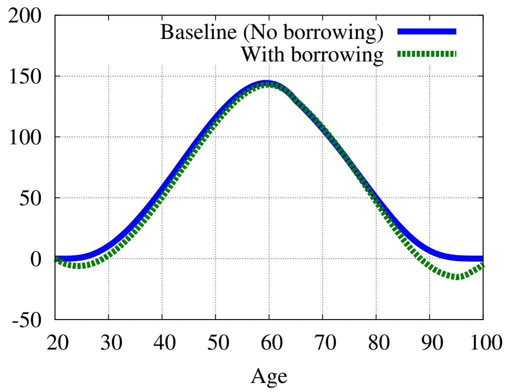
In the third panel, I implement the same procedure as in the first panel for the baseline hyperbolic discounting model ($\beta = 0.70$). Since the response of aggregate debt to a relaxed borrowing limit is stronger in the hyperbolic discounting model, the borrowing limits that induce a 2% and 7% debt to GDP ratio are expected to be tighter than in the exponential discounting model. By applying the same procedure to the hyperbolic discounting model, I am investigating the difference in the macroeconomic implications of an increased indebtedness depending on the model used for the analysis. As expected, the borrowing limits for the 1980 and 2000 economies are tighter than for the exponential discounting model, 9% and 29% of average income, respectively. Naturally, the responses of all the macroeconomic aggregates other than the debt to GDP ratio (which is controlled) are weaker in the third panel. The right panels in Figure 5 exhibit the life-cycle profile of hyperbolic discounting models in 1970 (no debt) and 2000 (7% debt over GDP). The changes by allowing debt are similar to those in the left panels, which are



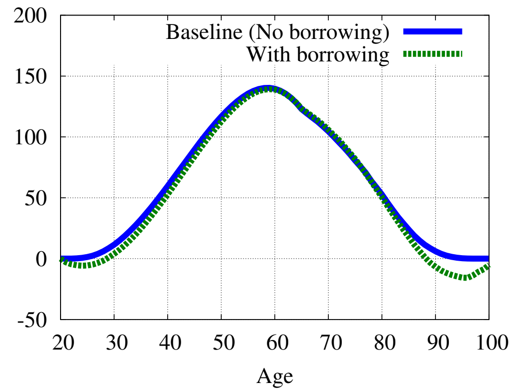
(a) Exponential: consumption



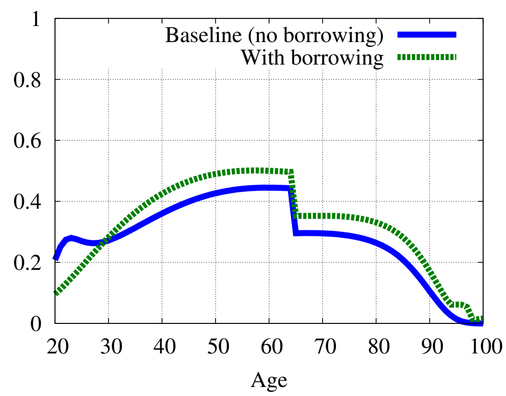
(b) Hyperbolic: consumption



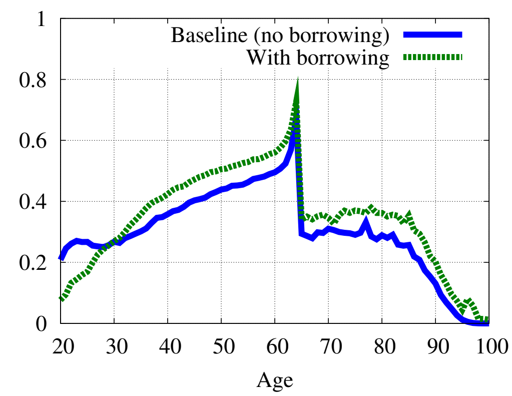
(c) Exponential: asset holdings



(d) Hyperbolic: asset holdings



(e) Exponential: cross-sectional variance of log-consumption



(f) Hyperbolic: cross-sectional variance of log-consumption

Figure 5: Comparison of the models with and without borrowing

associated with exponential discounting models.

The last panel in Table 2 summarizes the macroeconomic implications of an increased indebtedness for the economy with stronger hyperbolic discounting ($\beta = 0.56$). As in the case for the baseline hyperbolic discounting model ($\beta = 0.70$), the borrowing limits are calibrated so that the economy generates a 2% and 7% aggregate debt to GDP ratio in 1980 and 2000 economies, respectively. As the column labeled $\underline{a}$ shows, the borrowing limits have to be even tighter than in the baseline hyperbolic discounting model because of the stronger response of debt to a relaxation of the borrowing constraint. As we saw in the baseline hyperbolic discounting model, the size of the response of macroeconomic aggregates is even weaker than in the hyperbolic discounting model with a higher (lower) discount factor (rate). For example, the cross-sectional log-consumption variance declines by only 2 percentage points, while the consumption variance drops by 2.6 percentage points and 2.2 percentage points in the economies with exponential discounting and baseline hyperbolic discounting, respectively.

What is the role of general equilibrium in shaping the macroeconomic implications discussed above? Table 3 shows the decomposition between the partial and the general equilibrium effect for both the exponential and the hyperbolic discounting models. In Table 3, rows associated with *GE* (General Equilibrium) are the same as in Table 2. Rows associated with *PE* (Partial Equilibrium) show the macroeconomic effects *without* the general equilibrium effect. In particular, for the 1980 economy, prices are fixed at the level of the 1970 economy, and the borrowing limit is relaxed to the 1980 level. For the 2000 economy, prices are fixed at the 1980 level, and the borrowing limit is relaxed to the 2000 level. For all economies, the general equilibrium effect is clear: without the general equilibrium effect, macroeconomic responses are quantitatively stronger. In other words, the general equilibrium effect partly offsets the responses. Without the general equilibrium effect, both capital stock and output decrease even more, debt increases more, and the log-consumption inequality declines to a larger extent, too. There is no difference between the exponential and hyperbolic discounting models in terms of the role of the general equilibrium effect.

6.4 Rising Indebtedness: Welfare Implications

In this section, I will investigate the welfare implications of increased indebtedness. Before starting the analysis, two issues related to the welfare analysis in the current environment need to be addressed. First, since the model used here features a heterogeneous agent model with life-cycle and uninsured idiosyncratic shocks, there is no obvious way to define social welfare. I investigate social welfare in two ways. First, I use the ex-ante expected life-time utility in the steady-state equilibrium as social welfare. The virtue of this welfare criterion is that this naturally takes into account both the welfare gain or loss from changes in aggregate consumption (efficiency effect) and the welfare gain or loss due to changes in the degree of insurance (insurance effect). For this reason, the social welfare function is widely used together with incomplete market models with finitely-lived consumers; for example, Conesa et al. (2009) use it to investigate the optimal capital income taxation. Because of the heterogeneity, it is also important to look at the heterogeneity of the welfare effect for different types of consumers. To that end, I also investigate the expected life-time utility in the steady-state equilibrium for consumers with different initial productivity p. Since the productivity shock is highly persistent, looking at the welfare implications for consumers with different initial p roughly corresponds to studying the heterogeneous effects on

Table 3: Macroeconomic effect of rising debt: partial and general equilibrium effects

Economy[1]	GE[2]	$\underline{a}$[3]	D/Y	K[4]	Y[4]	r%	wage	Var(c)[5]
Exponential discounting model ($\beta = 1.00$)								
1970	–	0.000	0.000	1.000	1.000	6.01	1.000	0.481
1980	PE	−0.125	−0.021	0.973	0.990	6.01	1.000	0.461
1980	GE	−0.125	−0.020	0.986	0.995	6.11	0.995	0.465
2000	PE	−0.337	−0.074	0.931	0.975	6.11	0.995	0.446
2000	GE	−0.337	−0.070	0.959	0.985	6.33	0.985	0.455
Hyperbolic discounting model ($\beta = 0.70$)								
1970	–	0.000	0.000	1.000	1.000	5.99	1.000	0.489
1980	PE	−0.090	−0.021	0.980	0.993	5.99	1.000	0.472
1980	GE	−0.090	−0.020	0.988	0.996	6.09	0.996	0.475
2000	PE	−0.290	−0.073	0.938	0.977	6.09	0.996	0.460
2000	GE	−0.290	−0.070	0.964	0.987	6.29	0.987	0.467
Hyperbolic discounting model ($\beta = 0.56$)								
1970	–	0.000	0.000	1.000	1.000	5.95	1.002	0.504
1980	PE	−0.080	−0.021	0.972	0.990	5.95	1.002	0.488
1980	GE	−0.080	−0.020	0.988	0.996	6.04	0.998	0.492
2000	PE	−0.271	−0.072	0.946	0.980	6.04	0.998	0.479
2000	GE	−0.271	−0.070	0.965	0.987	6.22	0.990	0.484

[1] 1970: Economy with no borrowing. 1980: Economy calibrated to debt-to-output ratio of 2%. 2000: Economy calibrated to debt-to-output ratio of 7%.

[2] GE: general equilibrium. PE: partial equilibrium. For the 1980 economy, prices are fixed at the 1970 level. For the 2000 economy, prices are fixed at the 1980 level.

[3] Borrowing limit relative to total income.

[4] Level in the 1970 economy normalized to one.

[5] Average of cross-sectional variances of consumption for all age groups.

consumers with different productivity potentials. Moreover, in the next section, I will investigate the welfare effect associated with the rising indebtedness taking the equilibrium transition path into account. The analysis enables us to study the heterogeneous welfare effect on consumers in different generations along the transition path.

Second issue stems from the use of hyperbolic discounting preference. When the hyperbolic discounting model is interpreted as the dynamic game between the current and future selves, as in Laibson (1997), welfare of which self should be used? I am not subject to this problem because I use the interpretation of the hyperbolic consumer's problem provided by Krusell et al.

Table 4: Welfare implications of rising debt

		Welfare gain[3]			
			Initial productivity		
Economy[1]	GE[2]	$\mathbb{E}V$	Low	Medium	High
Exponential discounting model ($\beta = 1.00$)					
1970-1980	PE	+1.60	+4.53	+1.40	+0.02
1970-1980	GE	+1.22	+3.96	+1.03	+0.08
1980-2000	PE	+1.46	+4.71	+1.26	+0.04
1980-2000	GE	+0.60	+3.34	+0.41	+0.16
Hyperbolic discounting model ($\beta = 0.70$)					
1970-1980	PE	+0.72	+2.18	+0.60	−0.61
1970-1980	GE	+0.46	+1.69	+0.35	+0.12
1980-2000	PE	+0.25	+2.06	+0.17	−0.32
1980-2000	GE	−0.20	+1.12	−0.28	+0.60
Hyperbolic discounting model ($\beta = 0.56$)					
1970-1980	PE	+0.24	+1.09	+0.11	+1.76
1970-1980	GE	+0.10	+0.58	+0.02	+4.86
1980-2000	PE	−0.17	+0.82	−0.22	−3.34
1980-2000	GE	−0.36	−0.05	−0.41	+0.20

[1] 1970: Economy with no borrowing. 1980: Economy calibrated to debt-to-output ratio of 2%. 2000: Economy calibrated to debt-to-output ratio of 7%.

[2] GE: general equilibrium. PE: partial equilibrium. For the 1980 economy, prices are fixed at the 1970 level. For the 2000 economy, prices are fixed at the 1980 level.

[3] Welfare gain measured by the percentage increase in consumption at all ages and all states of the world associated with relaxing the borrowing limit.

(2009). Specifically, social welfare is defined as follows:

$$\mathbb{E}V = \sum_p \pi_p^0 \, V(1, p, 0) \tag{20}$$

Notice that the important consideration in the current paper is the welfare loss due to the relaxed borrowing constraint and induced over-consumption. The social welfare function naturally captures such welfare loss, because consumers succumb to the temptation of choosing consumption by discounting future value by $\beta\delta$, while the actual value is based on the discount factor δ. In other words, consumers choose consumption that is higher than the level associated with the highest welfare. Malin (2008) uses the same welfare criteria in a three-period model to study the optimality of savings floors.

Table 4 summarizes the welfare implications of rising indebtedness in both the exponential and the hyperbolic discounting models. Figure 6 and Figure 7 show the welfare gain of moving

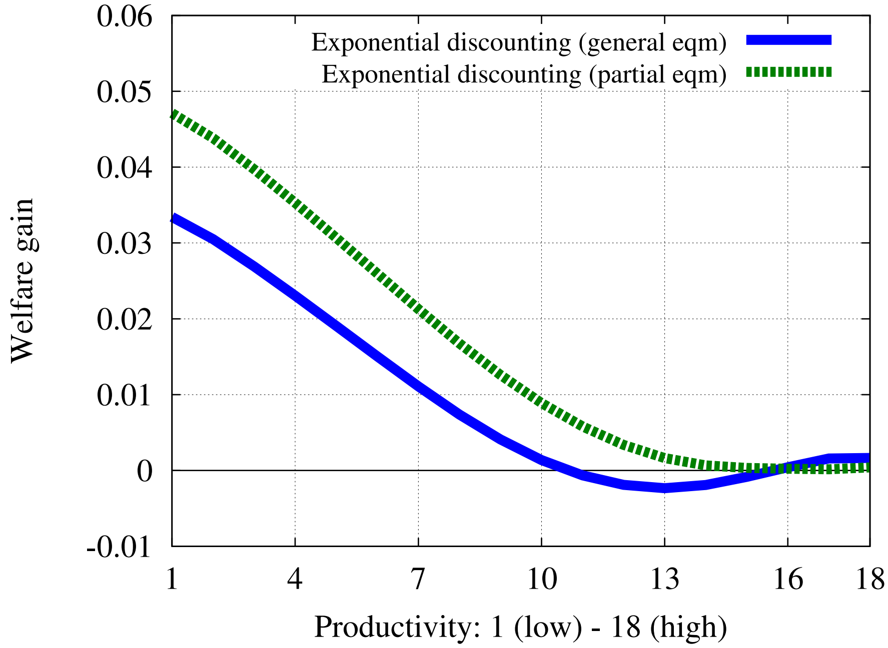


Figure 6: Heterogeneity of welfare gain: Partial and general equilibrium

from the 1980 economy to the 2000 economy for consumers with different initial productivity p. Figure 6 compares the heterogeneous welfare effect with and without a general equilibrium effect in the model with exponential discounting consumers. Figure 7 compares the heterogeneous welfare effect in models with exponential and hyperbolic discounting consumers, taking into account the general equilibrium effect.

First, let us focus on the model with exponential discounting consumers (the top panel of Table 4 and Figure 6). In the exponential discounting model, a relaxed borrowing limit yields a welfare gain in terms of ex-ante expected life-time utility of a consumer. By moving from the 1970 economy (no borrowing) to the 1980 economy (debt-to-output ratio of 2%), there is a gain in social welfare equivalent to as large as a 1.2% increase in per-period consumption. Moving from the 1980 economy to the 2000 economy (debt-to-output ratio of 7%) is associated with a social welfare gain of 0.6%. In both cases, the general equilibrium effect offsets some of the gain, through lower output associated with lower capital stock. With respect to the welfare effect on consumers with different productivity potentials, three groups with different initial productivity are affected very differently. First, those with initial low productivity benefit most from the relaxed borrowing limit. This is because the likelihood that they are constrained by the borrowing limit is highest for this group of consumers. However, they experience a welfare loss from the general equilibrium effect. In Figure 6, the line representing the welfare effect, which takes the general equilibrium effect into account, is located below the line representing the welfare effect without the general equilibrium effect, for consumers with low initial productivity. The reason is a lower equilibrium wage and a higher equilibrium interest rate, caused by a lower capital stock. Since the consumers in the group tend to borrow more often, and the main source of their income is labor income, both price effects hit consumers negatively. Second, the group

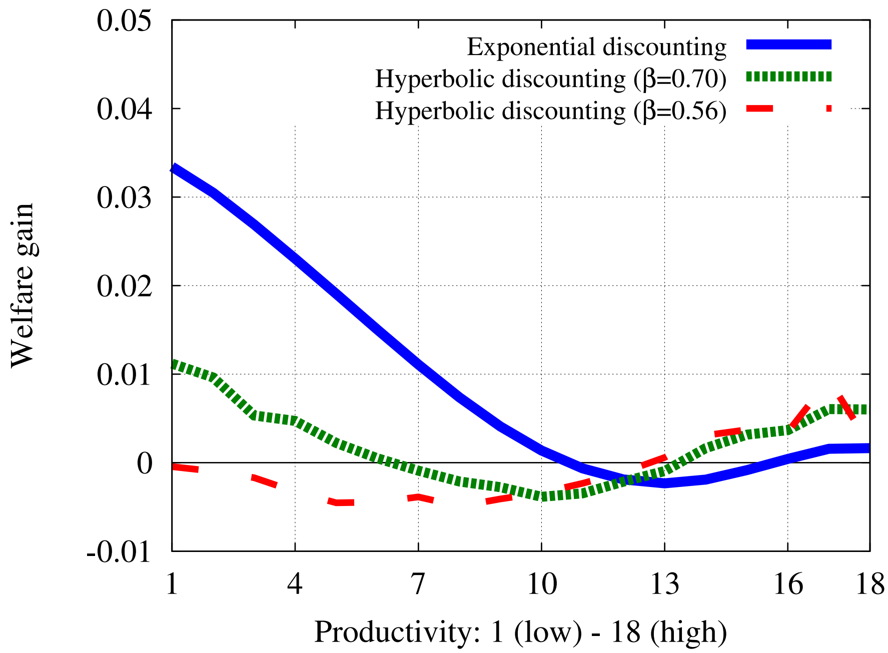


Figure 7: Heterogeneity of welfare gain: Exponential and hyperbolic discounting

with high initial productivity does not gain much from the partial equilibrium effect. For those with the highest initial productivity, the welfare gain without the general equilibrium effect is a mere 0.04% increase in per-period consumption. Since it is not likely that they are constrained by the borrowing limit, they do not gain much from a relaxed borrowing limit. However, they gain from the general equilibrium effect, albeit to a small extent. This is because they most likely remain savers throughout their lives, and they benefit from a higher interest rate, although part of the gain is offset by a lower wage. This contrasting general equilibrium effect for high and low productivity consumers is exactly what Campbell and Hercowitz (2009) emphasize in a different but closely related environment. In the model by Campbell and Hercowitz (2009) with secured credit, high discount rate consumers who remain borrowers lose from the general equilibrium effect, and low discount rate consumers gain from the general equilibrium effect. In their setup, the general equilibrium effect is strong enough to incur welfare loss for consumers with low initial productivity (in their case, high discount rate consumers), but the effect is not strong enough to overturn the welfare gain from the partial equilibrium effect here. Finally, interestingly, consumers with medium initial productivity suffer from the relaxed borrowing limit. This is due to the combination of the weak welfare gain from the relaxed borrowing limit and the stronger negative welfare loss from a lower wage and a higher interest rate. As a result, the solid line in Figure 6, which represents the welfare effect for heterogeneous consumers, exhibits a U-shape. This is the same property that Obiols-Homs (forthcoming) found in a similar environment.

How about the welfare implications of an increased indebtedness for the economy with hyperbolic discounting consumers? The middle and bottom panels of Table 4 summarize the welfare effects in hyperbolic discounting models with varying degrees of the strength of hyperbolic discounting. Figure 7 compares the heterogeneity of welfare effects across consumers with different

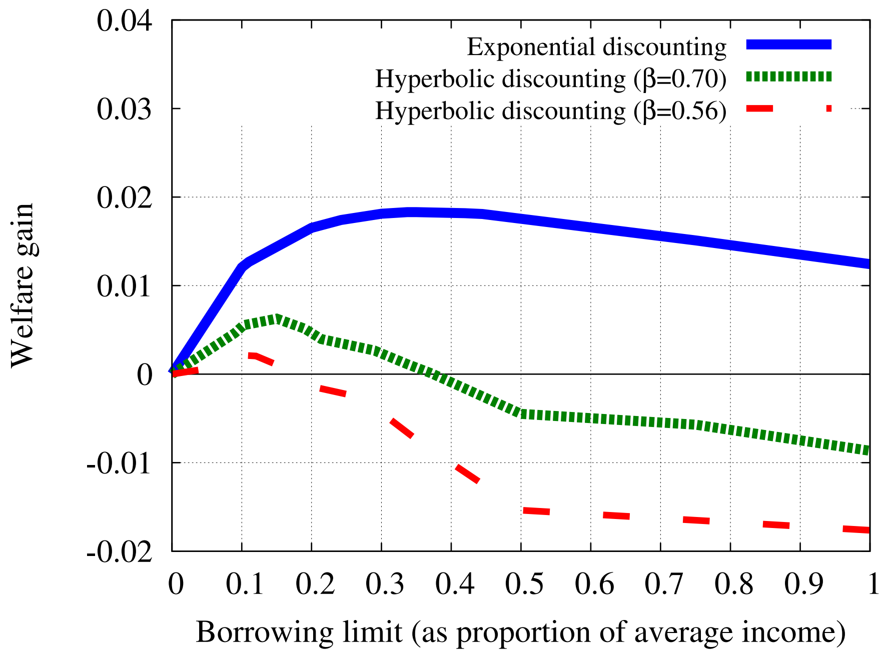


Figure 8: Level of borrowing limit and social welfare

initial productivity in the models with exponential and hyperbolic discounting. Regarding social welfare defined as the ex-ante expected life-time utility, although the welfare effect is small but positive (0.5% increase in per-period consumption), if the 1970 economy and the 1980 economy with the baseline ($\beta = 0.70$) hyperbolic discounting are compared, there is a social welfare loss equivalent to a 0.2% decrease in consumption between the 1980 and 2000 economies. In the case of stronger ($\beta = 0.56$) hyperbolic discounting, the welfare loss between the 1980 and 2000 economies is as large as 0.4% decrease in consumption. Cross-sectionally, although the U-shape is still observed in Figure 7, there are significant differences. The difference is especially striking for consumers with low initial productivity. Their gain from having a relaxed borrowing limit is significantly smaller in the case of the baseline hyperbolic discounting and negative in the case of strong hyperbolic discounting. The key reason is the negative welfare effect of over-borrowing. Those who are close to the borrowing limit benefit from having a less strict borrowing limit, which facilitates consumption smoothing across age and states, but suffer from borrowing more than the level associated with the highest welfare.

The discussion in this section implies that the *optimal* level of the borrowing limit differs, potentially substantially, across models with different preference assumptions. Here I define *optimal* as the level of the uniform borrowing limit that is associated with the highest social welfare defined as the ex-ante expected life-time utility. Figure 8 exhibits social welfare under different levels of the borrowing limit in the models with exponential and hyperbolic discounting. Three things are worth pointing out. First, the solid line, which is for the model with exponential discounting consumers, is located above the other lines, which are associated with the hyperbolic discounting models; the welfare gain is always higher in the exponential discounting model, conditional on the same level of borrowing limit. Second, all lines are hump shaped, because the equilibrium effect from a lower capital stock dominates at some point for all economies. Third, the optimal level of the borrowing limit is decreasing in the strength of the hyperbolic discounting. This is mainly because a hyperbolic discounting preference implies a smaller (or negative) welfare gain from the relaxed borrowing limit for low and medium productivity consumers.

For the exponential discounting model, the level of the uniform borrowing limit that maximizes social welfare is 37% of average income. Interestingly, this level turns out to be very close to the level in the 2000 economy (34%), which generates a 7% debt-to-output ratio in the steady state. In other words, the model with exponential discounting consumers implies that the 2000 U.S. economy is close to the optimal level in terms of the tightness of the borrowing limit. On the other hand, in the baseline ($\beta = 0.70$) hyperbolic discounting model, the optimal level is 15% of average income. This optimal level is substantially lower than in the hyperbolic discounting model, because of the welfare loss from over-borrowing. Furthermore, the optimal level is substantially lower than 29%, which is the borrowing limit of the 2000 economy. In other words, according to the baseline hyperbolic discounting model, the 2000 U.S. economy features an excessively relaxed borrowing limit. In the case of stronger hyperbolic discounting ($\beta = 0.56$), the optimal borrowing limit declines further to 11% of the average income.

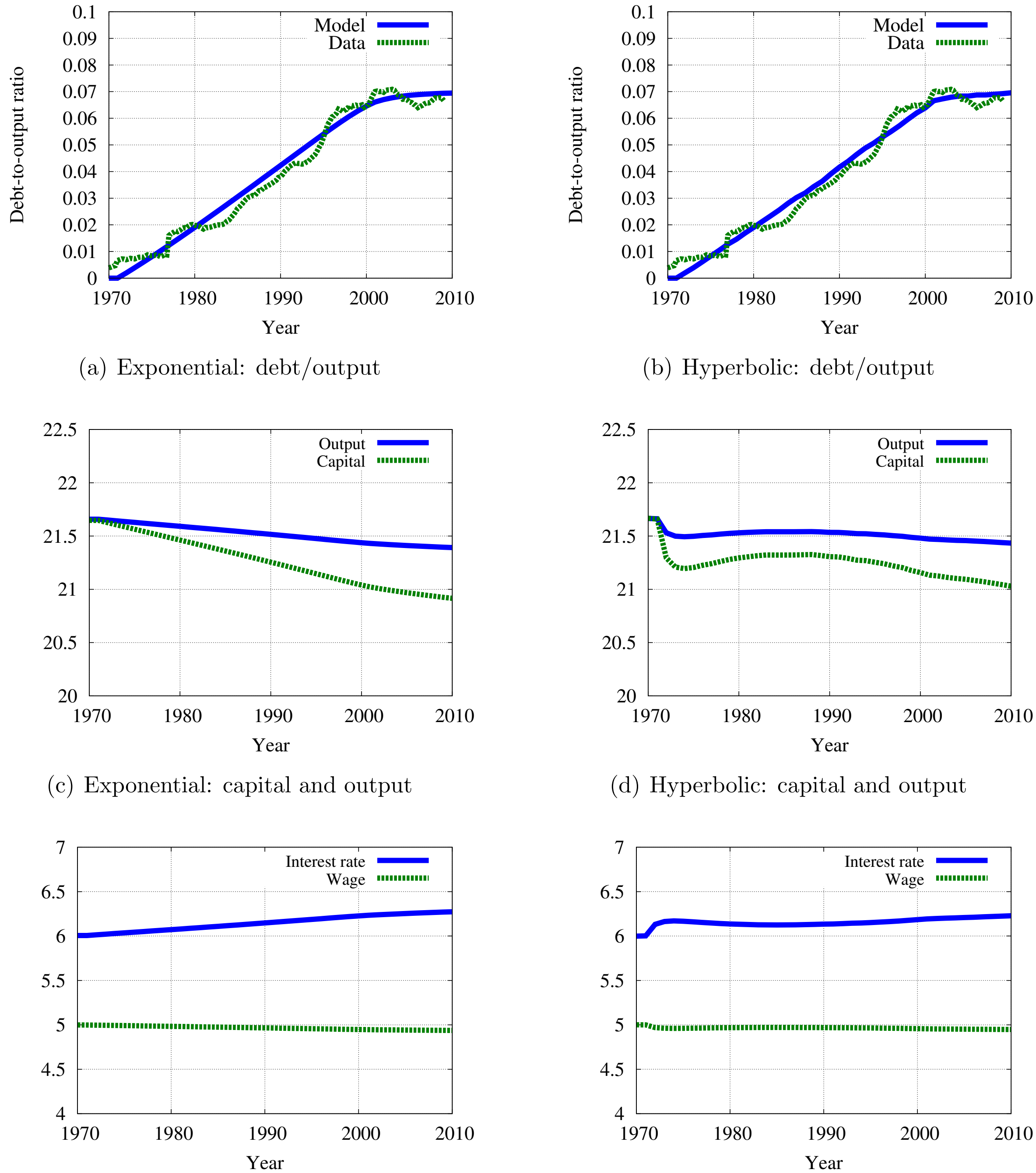
Just as commitment by using an illiquid asset is valued in Laibson (1997) and forced saving might be welfare-improving in Malin (2008), the existence of a tight borrowing constraint prevents consumers from over-consuming and thus potentially has a welfare-improving role. If the borrowing limit is tightened to the optimal level of 15% in the baseline hyperbolic discounting model, there is a sizable welfare gain (about 0.4% increase in per-period consumption).

7 Results: Transition Analysis

This section presents the results of the analysis with the equilibrium transition path. In constructing the transition path between the initial steady state and the final one, I assume that the initial steady state is characterized by no borrowing ($\underline{a} = 0$). The initial steady state corresponds to the 1970 economy in Section 6. The final steady state is characterized by the borrowing limit associated with a 7% debt-to-output ratio. This state corresponds to the 2000 economy. Notice that the borrowing limit in the final steady state is different depending on the model, but all models generate the observed amount of debt in 2000. I assume that the borrowing limit relaxes linearly between period 0 (corresponds to year 1970) and period 30 (corresponds to 2000). After period 30, the borrowing limit stays at the level in the 2000 economy. An alternative assumption is to set the borrowing limit to the level in the 2000 economy from the beginning of the transition (1971), but it turns out that this generates a counterfactual transition path of the debt-to-output ratio: the debt increases immediately in the 1970s, while the debt-to-output ratio gradually increases in the U.S. economy (Figure 1). In what follows, I first present the transition path of macroeconomic aggregates generated by the models (Section 7.1). The welfare analysis that explicitly takes into account the transition to the new steady state follows (Section 7.2). Appendix A.2 describes the computational algorithm.

7.1 Macroeconomic Aggregates

Figure 9 compares the macroeconomic aggregates between 1970 and 2010, for the model with the standard exponential discounting and the one with hyperbolic discounting ($\beta = 0.70$). The



(a) Exponential: debt/output

(b) Hyperbolic: debt/output

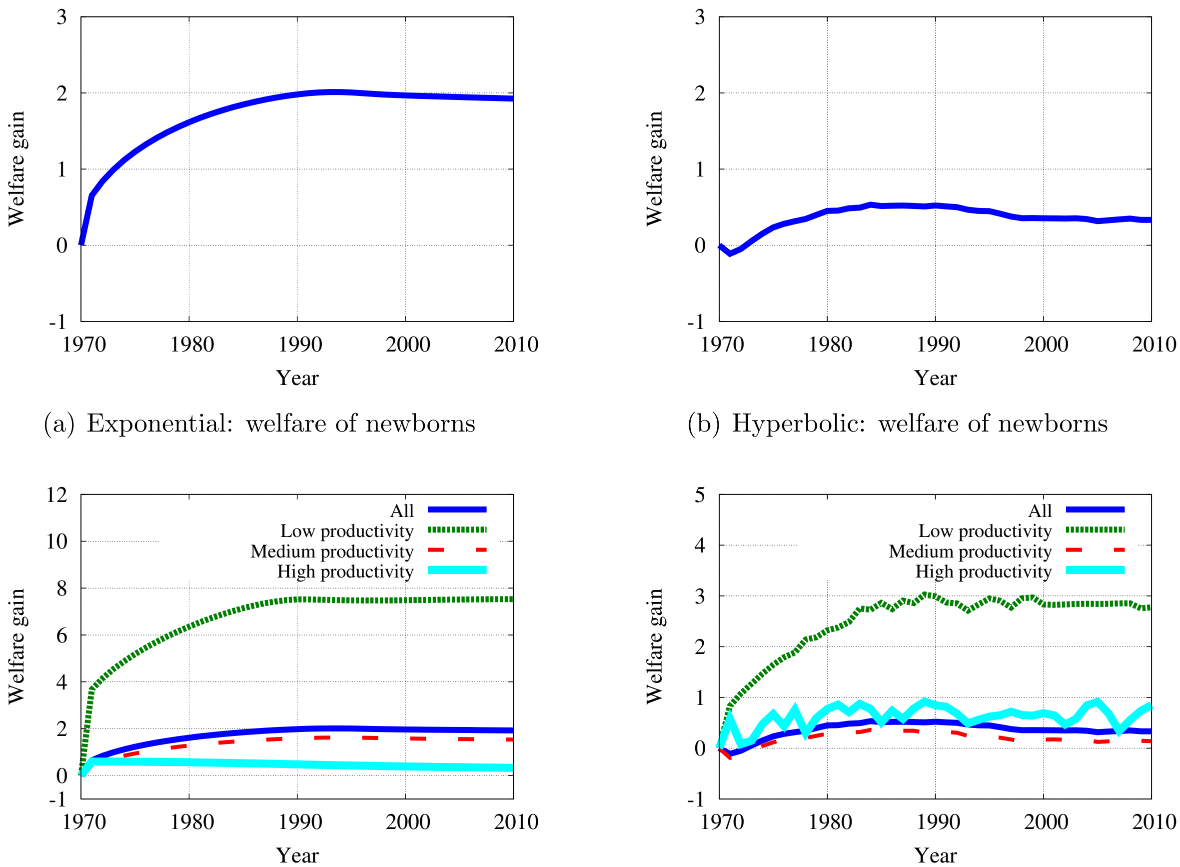
(c) Exponential: capital and output

(d) Hyperbolic: capital and output

(e) Exponential: prices

(f) Hyperbolic: prices

Figure 9: Comparison of macroeconomic aggregates along the transition path



(a) Exponential: welfare of newborns

(b) Hyperbolic: welfare of newborns

(c) Exponential: heterogeneity of welfare

(d) Hyperbolic: heterogeneity of welfare

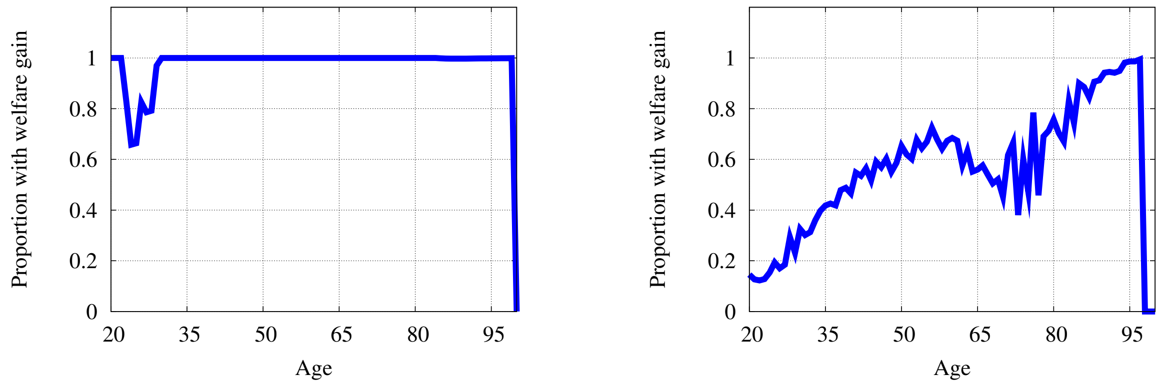
Figure 10: Comparison of welfare effects along the transition path

results with the exponential discounting model are on the left side, while those of the hyperbolic discounting model are on the right side of the figure. Panels (a) and (b) compare the path of the debt-to-output ratio of the models and of the data (same as in Figure 1). It is clear that both models capture the dynamics of the debt-to-output ratio in the data quite well. In both models, the debt-to-output ratio gradually increases from the initial level of zero in 1970 and reaches about 7% around 2000. Panels (c) and (d) compare the transition path of the capital stock and output. Although there are some non-monotonic dynamics in the model with hyperbolic discounting, the long-run trend is the decline in the capital stock over time, as consumers borrow more and more over time. As a result, output also continues to decrease over time. This is the source of the negative general equilibrium effect on welfare. Since labor is supplied inelastically, a declining capital stock yields a declining trend of wage and the increasing trend of the interest rate in the economy. These trends are present in both models, as shown in panels (e) and (f).

7.2 Welfare Implications

Although the dynamics of macroeconomic aggregates are observationally similar between the models with exponential discounting and with hyperbolic discounting, welfare implications are different, as shown using the steady-state comparison in Section 6. Figure 10 compares the welfare implications along the transition path in two models, again with figures for the exponential discounting model on the left and those for the hyperbolic discounting model on the right. First, panels (a) and (b) compare the ex-ante expected life-time utility of newborns (age-20 consumers) in different years along the transition path, in two model economies. Welfare is measured as the uniform increase in per-period consumption. The welfare in the initial steady state is used as the basis of comparison. For example, in panel (a), the welfare gain is approximately 2% in 2000. This means that an age-20 consumer in 2000 along the transition path is better off than if he had been born in 1970 (the initial steady state), by an increase in per-period consumption equivalent to 2%. Two things can be learned from comparing panels (a) and (b). First, the welfare gain from a relaxed borrowing limit is substantially higher in the model with exponential discounting, throughout the transition path. Second, as shown in Section 6, consumers born in 2000 are slightly worse off than those born in 1980 in the hyperbolic discounting model, while consumers born in 2000 are substantially better off then those born in 1980 in the exponential discounting model. As emphasized in Obiols-Homs (forthcoming), there is a negative general equilibrium effect on welfare (notice a small drop in welfare at the end of the transition path in panel (a)), but it is quantitatively small compared with the large gain during the transition between 1970 and 2000 in the exponential discounting model. Those born in 2000 in the hyperbolic discounting model are worse off compared with those born in 1980 by 0.05% of flow consumption, while those born in 2000 in the exponential discounting model are better off than those born in 1980 by 0.4% of flow consumption. Panels (c) and (d) exhibit the heterogeneity of the welfare effect on newborns (age-20 consumers) on the transition path in the exponential and hyperbolic discounting models. Intuitively, in both models, those who benefit most from having a relaxed borrowing limit are the ones with initially low productivity, although the size of the welfare gain is substantially smaller in the hyperbolic discounting model because the welfare loss from over-borrowing offsets some of the gains from improved insurance. What is also interesting is that the medium productivity consumers gain more than the high productivity consumers in the exponential discounting model, while the order is the reverse in the hyperbolic discounting model: medium productivity consumers in the hyperbolic discounting model gain less than those with high productivity.

Since I explicitly solve for the equilibrium transition path, I can measure the welfare gain of the consumers that are present in 1970 as the economy switches from the initial steady state to the transition path. Figure 11 shows the proportion of consumers who gain from the transition among each age group, in 1970, for both the exponential discounting model (panel (a)) and the hyperbolic discounting model (panel (b)). We can see that a very large proportion of consumers in 1970 gain from the switch to the transition path in the exponential discounting model. In total, 97% of consumers in 1970 (initial steady state) gain from the switch. On the other hand, in the hyperbolic discounting model, many consumers, especially the young ones, suffer from the transition. In total, only 47% of consumers in 1970 gain from switching to the transition path. In sum, the transition analysis confirms the findings of the steady-state comparison: although the macroeconomic implications are similar between the models with different preference assumption, welfare implications are strikingly different, if the welfare loss from over-borrowing is taken into account in the model with hyperbolic discounting. The ex-ante expected life-time utility of



(a) Exponential: proportion with welfare gain

(b) Hyperbolic: proportion with welfare gain

Figure 11: Comparison of welfare effects on the initial consumers

newborns declines from 1980 and 2000 along the transition path, as in the steady-state analysis, but the size of the welfare loss is found to be smaller (0.05% of flow consumption compared with 0.2%).

8 Discussion

In this section, I will discuss some of the assumptions in the model and their implications for the main results of the paper.[12] First, I assume that all consumers have the same preference: in the hyperbolic discounting models, all consumers share the same short-term and long-term discount factors. Although it is feasible to introduce heterogeneity with respect to discount factors across consumers, it is very difficult to separately identify heterogeneous lower short-term discount factors and lower long-term discount factors. Moreover, the homogeneous discount factor is a common assumption in the literature. If heterogeneous discounting factors are introduced, the welfare results would depend on what kind of consumers are more likely to be borrowing constrained. If those with a lower short-term discount factor (stronger hyperbolic discounting) are borrowing constrained, the main results of the paper are likely to be carried over. On the other hand, if consumers with a higher short-term discount factor are borrowing constrained, the results with respect to social welfare would be similar to those of the exponential discounting model: consumers do not suffer severely from over-consumption. Considering that the hyperbolic discounting preference is often associated with consumers' behavior regarding borrowing and defaulting, it is safe to think that the former is more likely.

Second, as Laibson (1997) showed, hyperbolic discounting consumers would optimally try to use commitment devices, if available, to restrain themselves from over-consuming in the future. Examples are durable goods (such as housing) or retirement saving instruments (such as individual retirement accounts (IRAs)). Although it is likely that if such commitment devices are available, consumers in the model will use them more extensively if the borrowing constraint is relaxed, notice that I calibrate the borrowing constraint to match the observed level of indebt-

[12]This section benefits from insightful comments by Christopher Sleet.

edness. If these commitment devices are available for consumers in the model, the degree of relaxation of the borrowing constraint will be calibrated to be even larger. Therefore, introducing commitment devices that allow consumers to restrain themselves from over-consuming and over-borrowing will not necessarily weaken the main results of the paper. Exactly how the main results of the paper will be affected by the introduction of the commitment devices is left for future research.

Third, although the paper focuses on unsecured borrowing, there was a parallel substantial increase in the indebtedness regarding secured borrowing (mainly mortgage loans). Moreover, as I argued above, housing can be considered as a commitment device against future over-consumption or over-borrowing. Regarding the role of housing as a commitment device, since the 1980s, there has been a development in the mortgage markets that makes it easer and cheaper to borrow against home equity. In other words, it is likely that the role of housing as a commitment device for saving might have weakened. The relaxed borrowing constraint considered in the paper can be interpreted as partly capturing such a development. The analysis with housing and mortgage loans, especially the link between the development in the mortgage markets and the saving behavior of hyperbolic discounting consumers, is an interesting future topic.

9 Conclusion

This paper investigates the macroeconomic and welfare implications of rising indebtedness in the U.S. using the model with hyperbolic discounting consumers. There are five main findings. First, when calibrated to the same set of targets, models with exponential and hyperbolic discounting exhibit similar life-cycle profiles. Even if the short-term discount factor of the hyperbolic discounting models induces consumers to borrow more and consume more in the current period than in the exponential discounting model, the long-term discount rate must be calibrated much higher than the exponential discounting model to match the observed aggregate capital stock. Second, there are some differences cross-sectionally: basically, hyperbolic discounting models expand the wealth distribution and push more consumers to borrowing or zero saving. Third, not only are the two models observationally similar in terms of aggregates, but they also have similar predictions in terms of macroeconomic changes in response to a relaxed borrowing limit. Fourth, although the macroeconomic implications of the relaxed borrowing limit are similar between the two models, the welfare implications are very different: hyperbolic discounting models imply significantly lower or even negative welfare effects associated with rising indebtedness. In particular, I find that when debt increases to the same extent as in the period between 1980 and 2000, there is a loss of social welfare as large as a 0.2% decrease in per-period consumption in the economy populated with hyperbolic discounting consumers. The difference in the welfare implications is very striking between the hyperbolic discounting model and the standard exponential discounting model, since the standard model with exponential discounting consumers implies a welfare gain (0.6% increase in per-period consumption) during the period between 1980 and 2000. Finally, the optimal borrowing limit is substantially lower in the hyperbolic discounting model. One could interpret that the severe restriction on borrowing could be welfare-improving according to the hyperbolic discounting model.

There are two ways to interpret the set of findings in the current paper. First, even though the models with exponential and hyperbolic discounting are observationally similar along many dimensions, they have very different welfare implications. Therefore, from the normative perspective, even though there is little need to use the non-standard hyperbolic preference to study macroeconomic implications, it is important to find other and better ways to distinguish between the two models. Second, it might be possible to combine the model implications and survey evidence to distinguish between the two models. If we know how much consumers suffer from over-borrowing and over-consuming, we could map the results into the strength of hyperbolic discounting, in particular, the level of the short-term discount factor.

One interesting and important extension from the current paper is associated with consumer bankruptcy. The increase in consumer debt has been accompanied by a substantial increase in consumer bankruptcy filings. Recently, the consumer bankruptcy law was reformed to make bankruptcy more costly and not available to consumers with relatively high income in order to discourage abuse of the law.[13] The standard equilibrium models of consumer bankruptcy[14] imply that a tougher bankruptcy law might benefit consumers by allowing stronger commitment to repay. But it is not clear if the intuition holds when consumers could suffer from *over-consuming*, as some argue. Nakajima (2009) investigates this issue.

[13]See White (2007) for details.

[14]See Livshits et al. (2007) and Chatterjee et al. (2007).

Appendix A

A.1 Calibration Appendix

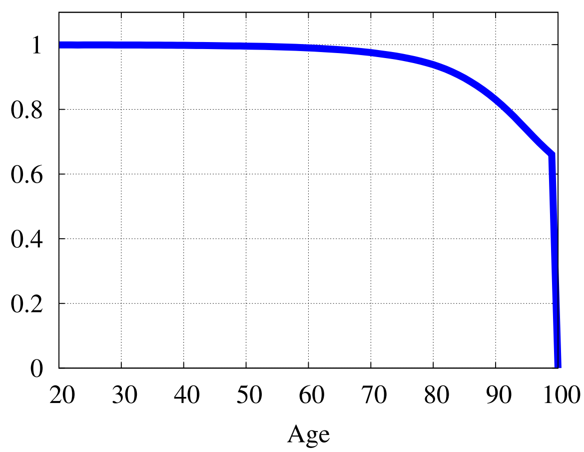


Figure 12: Conditional survival probabilities

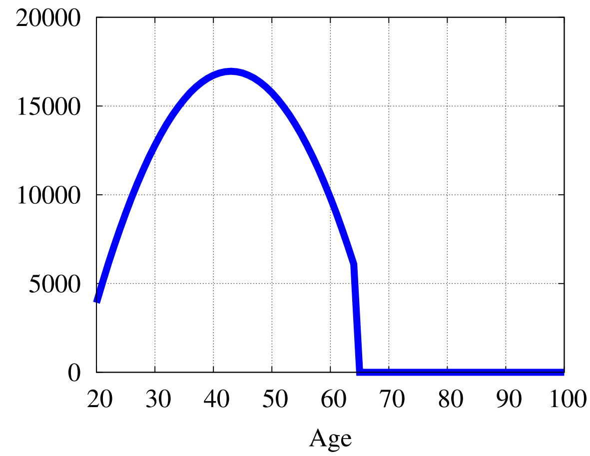


Figure 13: Average life-cycle profile of labor productivity

A.2 Computational Appendix

I will first describe below the computational algorithm to solve the steady-state equilibrium of the model. Since the focus is the steady-state equilibrium, I drop the time script in the algorithm. I will focus on the model with hyperbolic discounting consumers. The solution method for the exponential discounting model is straightforward.

Algorithm 1 (Computation algorithm for solving steady-state equilibrium)

1. *Set the initial guess of the aggregate capital K^0 and per-consumer transfer d^0. Notice that since there is no labor-leisure decision, aggregate labor supply $\overline{L}$ can be computed independently from the model.*

2. *Given K^0 and $\overline{L}$, compute the interest rate r and the wage w. The transfer used in the iteration is equal to the guess, i.e., $d = d^0$. The Social Security benefit $\bar{b}$ can be computed given w and the age distribution, which is exogenous to the model. Once $\bar{b}$ is obtained, $\{b_i\}_{i=1}^{I}$ can be set as $b_i = \bar{b}$ for $i \geq I_R$ and $b_i = 0$ for $i < I_R$.*

3. *Given $\{r, w, d, b_i\}$, solve the consumer's optimization problem using backward induction.*

   (a) *Set $V(I+1, p, a) = 0$ for all p and a.*

   (b) *Solve the problem of the consumer of age I, using the Bellman equation (3) for all p and a. Future value function is interpolated using piece-wise linear functions. The optimal savings level is found using a golden section search. Notice that the Bellman equation is NOT used for updating the value function. This step yields the optimal decision rule $g^a(I, p, a)$.*

(c) *Use the optimal decision rule $g^a(I,p,a)$ just obtained and (6) to update the value and obtain $V(I,p,a)$ $\forall p,a$.*

(d) *With $V(I,p,a)$ at hand, we can solve the problem of age $I-1$ consumers. Keep going back in the same way until the value function and the optimal decision rule for age 1 (initial age) consumers are obtained.*

4. *Using the obtained optimal decision rule $g^a(i,p,a)$, simulate the model.*

   (a) *Set the type distribution for the newborns, which is exogenously given. In particular, all newborns have $i=1$ and $a=0$. The initial distribution of p is subject to $\{\pi_p^0\}$.*

   (b) *Update the type distribution using the stochastic process for p and the optimal decision rule $g^a(I,p,a)$. The optimal decision rule is interpolated using piece-wise linear approximation.*

   (c) *Keep updating until age I (last age).*

5. *Compute the aggregate capital stock K^1 and total amount of accidental bequests implied by the simulated distribution. Notice that consumers survive according to the survival probability, and there is population growth, which makes the size of the younger population larger. Make these adjustments when computing the aggregate capital stock and accidental bequests. Specifically, when the measure of age 1 consumers is normalized to one, measures of age i consumers, $\widetilde{\mu}_i$ can be represented as follows:*

$$\widetilde{\mu}_i = \frac{1}{(1+\nu)^{i-1}} \prod_{j=0}^{i-1} s_j \tag{21}$$

   where $s_0 = 1$. Once the aggregate amount of accidental bequests is computed, we can compute the per-consumer lump-sum transfer d^1.

6. *Compare $\{K^0, d^0\}$ and $\{K^1, d^1\}$. If they are closer than a predetermined tolerance level, we can assume that $\{K^0, d^0\}$ was the consistent guess and stop. Otherwise, update $\{K^0, d^0\}$ and go back to step 2.*

Next, I will describe the solution algorithm of an equilibrium that features the deterministic transition between two steady states. Denote the initial and the new steady state by $t=0$ and $t=\infty$, respectively. In particular, let μ_0 be the type distribution of consumers in the initial steady state, which is the initial distribution along the transition path, and $V_\infty(i,p,a)$ as the value function in the new steady state. The only difference between the two steady states is the borrowing limit $\underline{a}$; total factor productivity Z is assumed to be constant over time. I also assume that the transition is complete after T periods, meaning that the economy is assumed to have reached the new steady state in period T. Since the model economy is likely to converge to the new steady state only asymptotically, a large T is desirable for a good approximation. Now, in period 0 the economy is in the initial steady state, but in period 1, the transition is revealed. In particular, the sequence of the borrowing limit $\{\underline{a}_t\}_{t=0}^{T}$ is revealed. Let $\underline{a}_1 = \underline{a}_0 = 0$ and $\underline{a}_t = \underline{a}_\infty$

for $t = \widetilde{T}, \widetilde{T}+1, \widetilde{T}+2, ..., T$. and $\underline{a}_t$ gradually increases between period 1 and period $\widetilde{T} < T$. I set $\widetilde{T} = 30$: when $t = 0$ corresponds to year 1970 (initial steady state without borrowing), and one period is a year, $t = \widetilde{T} = 30$ corresponds to 2000. After 2000, the borrowing limit remains at the level as in 2000.

Algorithm 2 (Computation algorithm for solving equilibrium transition path)

1. *Set the guess of sequences* $\{K_t^0, d_t^0\}_{t=0}^T$. *Notice that since there is no labor-leisure decision, aggregate labor supply* $\{\overline{L}_t\}_{t=0}^T$ *can be computed independently from the model.*

2. *Given* $\{K_t^0, d_t^0, \overline{L}_t\}_{t=0}^T$*, compute the sequence* $\{r_t, w_t, d_t, b_{t,i}\}_{t=0}^T$.

3. *Given* $\{r_t, w_t, d_t, b_{t,i}\}_{t=0}^T$*, solve the consumer's optimization problem using backward induction.*

   (a) *Start from period* T*. Notice that we know the value function* $V_{T+1}(i,p,a) = V_\infty(i,p,a)$ *for* $\forall(i,p,a)$ *since the economy is assumed to have converged to the new steady state in period* T.

   (b) *Solve the consumer's problem for* $\forall(i,p,a)$ *in period* T*, given* $V_{T+1}(i,p,a)$*. The solution method for the model with hyperbolic discounting consumers is the same as in the steady-state equilibrium described in Algorithm 1. The optimal decision rule* $g_T^a(i,p,a)$ *and the value function* $V_T(i,p,a)$ *are obtained. Notice that since the value function for the next period is given, there is no need to go back from age* I *as in Algorithm 1.*

   (c) *Keep going back until* $t = 0$.

4. *Using the obtained optimal decision rule* $g_t^a(i,p,a)$*, simulate the model.*

   (a) *The type distribution in period* 0 *is given by* μ_0.

   (b) *Update the type distribution using the stochastic process for* p *and the optimal decision rule* $g_t^a(i,p,a)$*. The optimal decision rule is interpolated using piece-wise linear approximation. Make sure to normalize the population size each period.*

   (c) *Keep updating until period* T *(last period).*

5. *Compute* $\{K_t^1, d_t^1\}_{t=0}^T$ *using the sequence of distribution* $\{\mu_t\}_{t=0}^T$ *generated in the last step.*

6. *Compare* $\{K_t^0, d_t^0\}_{t=0}^T$ *and* $\{K_t^1, d_t^1\}_{t=0}^T$*. If they are closer than a predetermined tolerance level, we can assume that* $\{K_t^0, d_t^0\}_{t=0}^T$ *constitutes an equilibrium. Otherwise, update* $\{K_t^0, d_t^0\}_{t=0}^T$ *and go back to step 2.*

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