

# ADVERSE SELECTION IN THE ANNUITY MARKET AND THE ROLE FOR SOCIAL SECURITY\*

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## **Abstract**

This paper studies the role of social security in providing annuity insurance when there is adverse selection in the annuity market. I calculate the welfare gain from mandatory annuitization in the current U.S. social security system using a life cycle model in which individuals have private information about their mortality. I calibrate the model to the current U.S. social security replacement ratio, fraction of annuitized wealth and mortality heterogeneity in the Health and Retirement Study. The main findings of the paper are the following: 1) the overall welfare gain from having mandatory annuitization through the current U.S. social security system is 0.27 percent of consumption; 2) social security has a large effect on price of annuities because it crowds out the demand for annuities by people who have low survival expectations. This price effect has negative welfare impact of 0.29 percent of consumption. On one hand, individuals with high mortality (who will die soon and do not have demand for longevity insurance) incur large welfare losses from mandatory participation. On the other hand the effect on prices limits the benefit to the low mortality individuals. These two effects results to the overall small ex ante welfare gain.

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# 1 Introduction

“... the existence of asymmetric information *may* justify a social insurance program (a government annuity in this case) but does not necessarily do so. The case for a mandatory annuity program depends on calculations that could be done but that have not yet been done.”(Feldstein (2005), page 4)

Mandatory annuitization is a key feature of the current U.S. social security system. Its value is derived from its ability to overcome potential inefficiencies due to adverse selection in the annuity market.<sup>1</sup> The purpose of this paper is to quantify the value of mandatory annuitization in the current U.S. social security system using a quantitative framework in which informational frictions in the annuity market are explicitly modeled. I will bias this exercise toward finding an upper bound for the value of mandatory annuitization, and I conclude that this value is small.

To do this, I develop a dynamic life cycle model in which individuals have private information about their mortality. Uncertainty about time of death generates demand for longevity insurance. In this environment individuals can purchase annuity contracts at linear prices. I assume that contracts are non-exclusive and insurers cannot observe individuals' trades. The lack of observability implies that insurers cannot classify individuals by their risk type. As a result, the unit price of insurance coverage is identical for all agents. Individuals with higher mortality (who on average die earlier) demand little insurance (or nothing at all). This makes lower mortality types (types with higher risk of survival) more represented in the market. This, in turn, leads the equilibrium price of annuities to be higher than the overall actuarially fair value of their payment.

In this environment, I define and characterize the set of ex ante efficient allocations. I show that these allocations are independent of individuals' mortality risk type and are only contingent on survival, which is publicly observed. This feature implies that ex ante efficient allocations can be implemented by a system of mandatory annuitization in which every individual is taxed, lump sum, before the retirement and receives a benefit contingent on survival after retirement.<sup>2</sup> The ex ante efficient allocation will be the benchmark for the best outcome that any social security system can achieve.

The environment I study has three important features. First, it abstracts from any heterogeneity other than mortality types (e.g., in tastes, bequest motives, abilities, income

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<sup>1</sup>Existence of adverse selection is well-documented by Finkelstein and Poterba (2002, 2004, 2006) and by Friedman and Warshawsky (1990) and McCarthy and Mitchell (2003) among others.

<sup>2</sup>The ex ante efficient allocation is achieved by forcing individuals with higher mortality (who on average die earlier) to pool with those of higher mortality (who on average die later). In a decentralized environment in which the choice of participation in the insurance pool is not mandatory, the participation of higher mortality types is always less than efficient.

shocks, etc.). It also abstracts from any distortionary effects of policy on labor supply and retirement decisions. This enables me to focus only on the inefficiencies caused by adverse selection. Furthermore, it implies that optimal policies are uniform across individuals. This gives a uniform mandatory annuitization policy the best chance to produce large welfare gains. Second, I assume individuals know all the information about their mortality risk type at the beginning of time. This assumes away any possibility of insuring against the realization of risk type in the market, exaggerates the effect of adverse selection, and hence provides an upper bound of the usefulness of policy. Finally, studying the annuitization over the life cycle, as opposed to a decision at retirement, enables me to highlight how individuals make decisions over their life and prepare for retirement based on their private expectation about the time of death.

The quantitative exercise of this paper consists of welfare comparisons between three economies: 1) an economy with no social security in which individuals share their longevity risk only through the annuity and life insurance markets, 2) the same economy with the addition of a social security system that is calibrated to the current U.S. system, and 3) an economy in which ex ante efficient allocations are implemented.

The key quantitative object in the model is the distribution of mortality risk types. This distribution determines the extent of private information in the economy. Following the demography literature, I model heterogeneity in mortality risk as a *frailty* parameter that shifts the force of mortality.<sup>3</sup> This parameter, once realized at birth, stays constant throughout one's lifetime. Individuals with a higher frailty parameter are more likely to die at any given age. I parametrize the initial distribution of mortality types (frailty) and use the data on subjective survival probabilities in the Health and Retirement Study (HRS) to estimate those parameters.<sup>4</sup> I calibrate the model to the current U.S. social security replacement ratio and choose the preference parameters to match the fraction of annuitized wealth through social security and pension at retirement in the HRS.

The three main findings of the paper are as follows: 1) the overall welfare gain from having mandatory annuitization through the current U.S. social security system is 0.27 percent of consumption; 2) social security has large effect on price of annuities because it crowds out the demand for annuities by people who have low survival expectation. This price effect has negative welfare impact of 0.29 percent of consumption. In other words in the absence of this price effect, the welfare gains from social security would have been as large as 0.56 percent; 3) the overall welfare gain from implementing ex ante efficient allocation over the

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<sup>3</sup>See, for example, Butt and Haberman (2004), Vaupel et al. (1979), and Manton et al. (1981).

<sup>4</sup>Hurd and McGarry (1995, 2002) and Smith et al. (2001) document that these probabilities are consistent with life tables and ex post mortality experience. They argue that they are good predictors of individuals' mortality.

market equilibrium without social security is 0.91 percent.

To understand the intuition for these results, I look at the effect of the presence of social security on individuals' participation in the annuity market and on equilibrium prices. In the presence of social security, 40 percent of the population (those with higher than average mortality) are not active in the annuity market.<sup>5</sup> These individuals get more annuitization than they need from social security. On the other hand, individuals with lower than average mortality, expecting longer life spans, accumulate more assets and have higher demand for annuitized wealth. These individuals purchase annuities in the market. However, since higher mortality types (good risk types) are not in the market, the equilibrium price of annuities are about 9 percent higher than they would have otherwise been in the absence of social security.<sup>6</sup>

In addition to the main results, I perform several sensitivity checks. I find that, contrary to common wisdom, increasing the degree of risk aversion in preferences does not lead to higher welfare gains from mandatory annuitization. At higher risk aversions, an individual of high mortality demands more insurance and at any given price is willing to participate more in the market. On the other hand, lower mortality types (who are generally over-insured) have a stronger preference for a smooth path of consumption. They increase their consumption in earlier periods by reducing their demand for annuities. This results in a flatter profile of the annuity purchase and a lower equilibrium price. Therefore, it is true that at higher levels of risk aversion the social value of insurance is higher, but at the same time there is better insurance available in the market, and the distance between equilibrium allocations and ex ante efficient allocations is reduced. Consequently, even when I repeat the welfare comparison with assuming a high degree of risk aversion, the welfare gains from mandatory annuitization are not large.

## 1.1 Related literature

This paper is related to three strands of literature. The first strand of the related literature focuses on the potential welfare-improving role for mandatory insurance in an environment with adverse selection. This was pioneered by [Akerlof \(1970\)](#), [Rothschild and Stiglitz \(1976\)](#) and [Wilson \(1977\)](#) in their seminal contribution that started the literature.<sup>7</sup> The role of

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<sup>5</sup>The model slightly under-estimates the the lack of annuitization in the data. [Johnson et al. \(2004\)](#) find that 43 percent of all adults (52 percent for males) in Health and Retirement Study hold defined benefit pensions or annuity in their own names.

<sup>6</sup>The effect of social security on annuity prices was first studied by [Abel \(1986\)](#). [Walliser \(2000\)](#) has investigated this issue quantitatively and found that removing social security can reduce the price of annuities up to 3 percent. However, the type of annuity contracts that they consider is different than the ones considered in this paper.

<sup>7</sup>See [Dionne et al. \(2000\)](#) for an excellent survey on theories of insurance markets with adverse selection.

mandatory annuitization in the annuity market with adverse selection was first studied by [Eckstein et al. \(1985\)](#) and [Eichenbaum and Peled \(1987\)](#). The contribution of this paper is the quantitative assessment of the welfare gains due to mandatory annuitization.

Second, this paper is related to the large literature measuring the insurance value of annuitization for representative life cycle consumers (e.g., [Kotlikoff and Spivak \(1981\)](#), [Mitchell et al. \(1999\)](#), [Brown \(2001\)](#), [Brown et al. \(2005\)](#), [Lockwood \(2009\)](#) ).<sup>8</sup> The exercise in these articles is to determine how much incremental, nonannuitized wealth would be equivalent to providing access to actuarially fair annuity markets.<sup>9</sup> A robust finding of this approach is that a 65-year-old adult with population average mortality gains up to 30 to 50 percent of his retirement wealth from access to actuarially fair insurance (with the exception of [Lockwood \(2009\)](#)).<sup>10</sup> A key feature of all these studies is the static comparison between full insurance and no insurance at all.<sup>11</sup> In contrast, in the current paper I allow for the annuitization through private annuity markets over the life cycle.<sup>12</sup> This allows me to distinguish between risk sharing that is provided by the market and self insurance and to study how it changes in response to changes in publicly provided insurance.

Welfare gains from provision of annuity insurance in social security is also studied by [Hubbard and Judd \(1987\)](#), [İmrohoroğlu et al. \(1995\)](#) and [Hong and Ríos-Rull \(2007\)](#). None of these papers study an environment with adverse selection, which is the friction that makes mandatory annuitization valuable in my framework. [Einav et al. \(2010\)](#) study the welfare gains from mandatory annuitization in the U.K. annuity market. In their environment individuals are heterogeneous both in their survival probabilities and their preferences. The preference heterogeneity implies that a uniform policy of mandatory annuitization cannot be optimal. In contrast, in this paper, the only source of heterogeneity is in survival probabilities and uniform mandatory annuitization is unambiguously welfare improving, ex ante. Therefore, I consider an environment in which social security has the best chance of producing a

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<sup>8</sup>[Brown \(2003\)](#) and [Gong and Webb \(2008\)](#) consider observable heterogeneous types and study the redistribution impact of mandatory annuitization.

<sup>9</sup>[Lockwood \(2009\)](#) is an exception in that he considers the comparison between no annuity and annuity that is available at actuarially unfair market rates.

<sup>10</sup>[Gong and Webb \(2008\)](#) did this exercise allowing for pre-annuitized wealth (e.g., through defined benefit pensions) and still found a big welfare gain (9 percent).

<sup>11</sup>A large part of this welfare gain comes from the fact that in the absence of any longevity insurance, individuals should rely on their savings in liquid assets. Therefore, with positive probability each individual dies with positive assets. Upon their death their assets evaporate from the economy. On the other hand, under full insurance these assets are annuitized. Upon individual's death, his/her assets are transferred to those who survived (the assets do not leave the economy).

<sup>12</sup>[Butrica and Mermin \(2006\)](#), [Dushi and Webb \(2004\)](#), [Johnson et al. \(2004\)](#), [Moore and Mitchell \(1999\)](#), and [Poterba et al. \(2007\)](#) document that a significant fraction of the wealth of retired adults is annuitized. In particular, [Butrica and Mermin \(2006\)](#) find that 10 percent of the wealth at retirement is annuitized through private annuities and pensions and about 45 percent through social security.

potentially large welfare gain. Despite that, I find that welfare gains are small.

Social security is a large program with many purposes.<sup>13</sup> On the normative side, its role is broadly categorized by [Diamond \(1977\)](#) into income redistribution, provision of insurance (when there is market failure) and paternalism toward irrational savings by individuals. These aspects of social security have been studied extensively in the literature.<sup>14</sup> For tractability reasons, in this paper I abstract from many features of social security and focus only on one of the many possible benefits, i.e., mandatory annuitization. This is an obvious advantage that a government system has (in imposing participation by everyone) that no private market system can mimic. I study an environment in which this is the only role for social security.

The paper is organized as follows. [Section 2](#) describes the environment, defines and characterizes efficient allocations, and introduces the equilibrium notion. [Section 3](#) contains a two-period example that highlights some features of the environment. [Section 4](#) and [\(5\)](#) contain the parametric specification and calibrations. [Section 6](#) reports the findings, and [Section 7](#) concludes.

## 2 Model

In this section I first describe the environment. Then, I describe the ex ante efficient allocation in which a social planner chooses the allocation to maximize ex ante welfare subject to informational and feasibility constraints. This will be a benchmark for what is the best possible outcome in this environment. I show that this allocation is independent of individual's private type and therefore can be implemented with a type-independent social security tax and transfer. I also, describe a decentralized market arrangement in which individuals can trade annuities and can hold non-contingent savings. There is also a type-independent social security tax and transfer which I later calibrate to the U.S. economy. The goal of the paper is to compare welfare between various equilibrium allocations (for various level of tax and transfer) and the ex ante efficient allocation.

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<sup>13</sup>See [Mulligan and Sala-i-Martin \(1999a,b\)](#) for an extensive survey on normative and positive theories of social security.

<sup>14</sup>For example, [Golosov and Tsyvinski \(2006\)](#) study the disability insurance aspect of social security and [Gottardi and Kubler \(2006\)](#) evaluate the role of social security in improving inter-generational risk sharing. Finally, [Emre \(2006\)](#) points out to the positive role of mandatory savings in social security when there is lack of commitment by the government.

## 2.1 Information

Consider a continuum economy with atom-less measure space of agents' labels  $(I, \mathcal{I}, \iota)$ . The economy starts at date zero and ends at date  $\infty > T \geq 1$ . Individuals are born at the beginning of period zero and face an uncertain life span. An individual who survives to age  $t$  faces the uncertainty of surviving to age  $t + 1$  or dying at the end of age  $t$ . Anyone who survives to age  $T$  will die at the end of that age. I index the survival state at date  $t$  by  $s_t \in S = \{0, 1\}$ , in which 1 means the individual has survived to age  $t$  and 0 means the individual has died before age  $t$ . Agents' survival is an ex post state of the world in the sense that it is realized after all trade decisions are made. There is also a set of possible individual types or characteristics,  $\Theta$ . Individuals' type,  $\theta \in \Theta$ , determines their likelihood of survival in each period. I assume that  $\Theta = [\underline{\theta}, \bar{\theta}] \subseteq \mathbb{R}_+$  and  $\theta \in \Theta$  is the index of frailty. Individuals with lower  $\theta$  have a higher probability of survival (and a longer expected lifetime). I assume  $\theta$  is private information and known only by the individual. Furthermore, I assume  $\theta \in \Theta$  is an ex ante state of the world and is realized before any transaction takes place.

To sum up, there are three sets of possible economic agents. First, there is a set  $I$  of labels (without loss of generality, we can assume this is a unit interval). Second, there is the set  $I \times \Theta$  of possible type-contingent agents, indexed with their label and their ex ante (private) types. And finally, there is the set  $I \times \Theta \times S^T$  of possible types and survival contingent individuals indexed by their label  $i \in I$ , ex ante (private) types  $\theta \in \Theta$  and ex post survival state  $s_t \in S^t$ . Note that if  $s_t = 0$ , then  $s_{t'} = 0$  for all  $t' > t$ .

Suppose there is a well-defined distribution  $G_0 \in \Delta(\Theta)$  with full support. Suppose each type realization  $\theta \in \Theta$  determines the conditional probability of survival to date  $t$  in period zero (i.e., conditional probability that  $s_t = 1$ ). I denote this conditional probability by  $P_t(s_t = 1|\theta)$ , or  $P_t(\theta)$  for short. Therefore, the joint probability that an individual's type is in the set  $Z \subseteq \Theta$  and survives to period  $t$  is  $\mu_t(Z, s_t = 1) = \int_{\theta \in Z} P_t(\theta) dG_0(\theta)$ .

Type realization and survival (conditional on types) are i.i.d. and there is no aggregate uncertainty about distribution of types and actual survival for the agents in any subset of  $I$ .<sup>15</sup> In other words, let  $A \subseteq I$  be any non-zero measure subset of  $I$ . Then exactly  $\iota(A)$  fraction of agents have labels in  $A$ . Furthermore, fraction  $G_0(Z)$  of agents with label in  $A$  have type  $\theta \in Z \subseteq \Theta$ , and out of this fraction exactly  $\mu_t(Z, s = 1)$  will survive through period  $t$  (conditional on being alive in period zero).

Individuals who die exit the economy. Therefore, in each period the distribution of types (conditional on survival) becomes more skewed toward the higher survival (lower  $\theta$ ) types. Let  $G_t$  be the distribution of types conditional on survival to date  $t$ ; then, the fraction of

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<sup>15</sup>Subject to the usual caveat on continuum of i.i.d. random variables. See Judd (1985) and Uhlig (1996).



people with type in any set  $Z \subseteq \Theta$  is

$$G_t(Z) = \frac{\int_{z \in Z} P_t(z) dG_0(z)}{\int_{\theta \in \Theta} P_t(\theta) dG_0(\theta)} \quad \forall Z \subseteq \Theta. \quad (1)$$

## 2.2 Preferences

Individuals have time separable utility over consumption,  $u(\cdot)$ , as long as they live. They also enjoy utility from leaving a bequest at the time of death,  $v(\cdot)$ . These functions are assumed to be twice continuously differentiable with  $u', v' > 0$  and  $u'', v'' < 0$  and satisfy the usual INADA conditions. Let  $x_t(\theta) = \frac{P_{t+1}(\theta)}{P_t(\theta)}$  be one-period conditional survival probability for type  $\theta$  (probability of surviving to age  $t + 1$  condition on being alive at  $t$ ). Then type  $\theta$ 's utility out of a given sequence of consumption,  $c_t$ , and bequest,  $b_t$ , is

$$\sum_{t=0}^T P_t(\theta) \beta^t [u(c_t) + (1 - x_{t+1}(\theta)) \beta v(b_t)], \quad 0 < \beta \leq 1.$$

Preference for bequest can be motivated and interpreted in several ways. I follow [Abel and Warshawsky \(1988\)](#) and interpret it as altruism towards future generation (or surviving spouse) whereby,  $v(b)$  stands for reduced form lifetime value function of a child that is born after the individual's death and receives bequest  $b$  (or simply the surviving spouse that lives for fix number of periods after the agent dies).<sup>16</sup>

Each individual is endowed with a unit of labor endowment which is inelastically supplied for constant wage  $w$  in every period  $t \leq J < T$  (after period  $J$  the individual cannot work).<sup>17</sup> There is also a saving technology with gross rate  $R \geq \frac{1}{\beta}$ .

An *allocation* is a map from agents' label, type, and survival state to positive real line, i.e.,

$$c_t : I \times \Theta \times S^t \rightarrow \mathbb{R}_+ \quad 0 \leq t \leq T$$

$$b_t : I \times \Theta \times S^{t+1} \rightarrow \mathbb{R}_+ \quad 0 \leq t \leq T.$$

I will focus on symmetric allocations that depend only on type  $\theta$  and survival and not an individual's label. Furthermore, since the agents do not care about consumption in the state in which they are dead (and about bequests in the state in which they are alive), I will drop the realization of the survival state from the argument of the allocation function. Therefore, it is understood that  $c_t(\theta)$  is the consumption of all  $\theta$  type individuals condition on their survival at age  $t$  (and similarly  $b_t(\theta)$  is the bequest that type  $\theta$  leaves if he dies at

<sup>16</sup>Look also at [Braun and Muermann \(2004\)](#) for an alternative interpretation based on *regret* motive.

<sup>17</sup>Allowing for age varying wage profile does not affect the results.



the end of age  $t$ ). An allocation is *feasible* if

$$\int \sum_{t=0}^T \frac{P_t(\theta)}{R^t} \left[ c_t(\theta) + \frac{(1 - x_{t+1}(\theta))}{R} b_t(\theta) \right] dG_0(\theta) = w \int \sum_{t=0}^J \frac{P_t(\theta)}{R^t} dG_0(\theta). \quad (2)$$

In the environment described above, the agents face the risk of outliving their assets. Also, from the ex ante point of view (before birth), agents face the risk of their type realization. Individuals whose type  $\theta$  imply a higher survival probability need more resources to finance consumption through their lifetime relative to those types who have lower survival. Therefore, there is a need for insurance against these risks. Next, we study the ex ante efficient allocations as a benchmark that provides perfect insurance against both types of risks.

### 2.3 Ex ante efficient allocations

Consider the problem of a social planner who maximizes the expected discounted utility of agents behind the veil of ignorance, i.e., before agents are born.

$$\max_{c_t(\theta), b_t(\theta) \geq 0} \int \sum_{t=0}^T P_t(\theta) \beta^t [u(c_t(\theta)) + (1 - x_{t+1}(\theta)) \beta v(b_t(\theta))] dgG_0(\theta)$$

subject to (2)

It is straightforward to verify that the allocations that solve the above problem must satisfy

$$\begin{aligned} c_t(\theta) &= c_t(\theta') = c_t \text{ for all } \theta, \theta' \in \Theta, \forall t \\ b_t(\theta) &= b_t(\theta') = b_t \text{ for all } \theta, \theta' \in \Theta, \forall t \end{aligned}$$

and

$$u'(c_t) = \beta R u'(c_{t+1}) = \beta R v'(b_t).$$

As is evident from the above equations, the allocations do not depend on individuals' type  $\theta$ . The intuition for this result is the following. In this environment, individuals are heterogeneous ex ante (differ in the risk of survival) but identical ex post. There is no difference among dead individuals. There is also no difference among people who survive. Therefore, there is no reason that the planner should discriminate between them ex post.

The fact that allocations are independent of heterogeneous risk type means that a "one size fits all" identical allocation not only is ex ante efficient under full information, but also is incentive compatible and hence implementable even if risk type  $\theta$  is private information.

This means that the efficient allocation can be implemented by lump-sum tax and transfer.<sup>18</sup>

Two key assumptions drive this result. One is that the planner (as well as individuals) is expected utility maximizer. Removing this assumption leads to efficient allocations that are type specific. The other assumption is that mortality risk is the only heterogeneity in this environment. If individuals are heterogeneous in other characteristics (such as ability or taste), then the efficient allocations are type specific and therefore incentive compatibility constraints are trivially satisfied. There are numerous studies that document a negative correlation between socio-economic status (such as education, income, etc.) and mortality (see, for example, Deaton and Paxson (2001)). An example of preference heterogeneity that is correlated with mortality is Einav et al. (2010). They consider a model in which there is heterogeneity in mortality as well as preference for bequest. They estimate the joint distribution of mortality index and bequest parameter and find that they are positively correlated (people with higher mortality also have stronger taste for bequest). In their environment a uniform policy of mandatory annuitization is not optimal precisely because of preference heterogeneity. Abstracting from these other sources of heterogeneity in this paper gives a uniform social security policy the highest chance to produce a large welfare gain.

In the next section I describe a decentralized environment in which individuals can share the risk of their longevity in private annuity market and possibly through a uniform (across mortality type) social security system.

## 2.4 Competitive equilibrium with asymmetric information

### 2.4.1 Survival contingent contracts

Individuals can purchase annuity contracts during the last period of work (model age  $J$ ). One unit of annuity contract pays one unit of consumption good contingent on survival for as long as the agent survives starting at age  $J + 1$ . Contracts are assumed to be non-exclusive and cannot be contingent on the agent's past trades or volume of the transaction. Contracts are linear in the sense that to purchase  $a$  unit of annuity coverage, the individual pays  $qa$ .

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<sup>18</sup>An example of implementation is discussed in section 6.5

<sup>19</sup>Allowing individuals to purchase annuity at other ages does not affect the results. Individuals choose to purchase annuity at only one age. However freedom to choose that age gives rise to a multiplicity problem. In the interest of avoiding this problem I restrict the trade to happen only at the time of retirement. Choosing a different age for trade alters the calibration but does not affect the quantitative findings of the paper.

### 2.4.2 Consumer problem

Let  $k_t$  be the amount of non-contingent saving by the individual and  $b_t$  be the bequest they leave if they die at the end age  $t$ . The optimization problem faced by this individual is

$$\max_{c_t, b_t, k_{t+1}, a \geq 0} \sum_{t=0}^T P_t(\theta) \beta^t [u(c_t) + (1 - x_{t+1}(\theta)) \beta v(b_t)]$$

subject to

$$c_t + k_{t+1} = Rk_t + (1 - \tau)w \quad \text{for } t < J \quad (3)$$

$$c_J + k_{J+1} + qa = Rk_J + (1 - \tau)w \quad (4)$$

$$c_t + k_{t+1} = Rk_t + a + z \quad \text{for } t > J \quad (5)$$

$$b_t = Rk_{t+1} \quad (6)$$

$$k_0 \quad \text{is given} \quad (7)$$

in which  $a$  denotes annuity coverage purchased,  $\tau$  is social security tax rate and  $z$  is social security benefit. Note that the individual faces short sale constraints on annuity as well as saving. Note also that  $x_{T+1}(\theta) = 0$  for all  $\theta$ . Given price  $q$  the type  $\theta$  individual's demand for annuity is  $a(\theta; q)$  and aggregate demand for annuity is  $y(q) = \int a(\theta; q) dG_J$ .

### 2.4.3 Social security

There is a fully funded social security system that taxes individuals at ages 0 to  $J$  at rate  $\tau$  (since labor is inelastically supplied, this is in fact a lump-sum tax) and transfers constant social security benefit  $z$  to everyone at ages  $t > J$  for as long as they are alive. The social security, therefore, is in fact a mandatory annuity insurance.<sup>20</sup>

Let  $SSA_t$  denote the social security assets at date  $t$ . During periods  $0 \leq t \leq J$ , the mandatory contributions are collected from whoever is alive, and social security assets accumulate

$$SSA_{t+1} = R(SSA_t + \tau e \int P_t(\theta) dG_0(\theta)) \quad t \leq J \quad (8)$$

$$SSA_{t+1} = R(SSA_t - z \int P_t(\theta) dG_0(\theta)) \quad t > J,$$

in which  $SSA_0 = SSA_{T+1} = 0$ . Note that  $\int P_t(\theta) dG_0(\theta)$  is the total fraction of people (of

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<sup>20</sup>In reality social security system in the U.S. is a much more complicated policy and has many other feature embedded to it (progressivity, survival benefit, etc.). It is also set up as pay as you go system and is not fully funded. I abstract from all these aspects and focus only on one feature of the system: mandatory annuitization.

all types) that survive to age  $t$ .

#### 2.4.4 Annuity Insurers

There are large number of insurers who sell life annuity contracts to individuals of age  $J$ . Faced with the aggregate demand for annuity  $y(\cdot)$  and the anticipated distribution of payouts  $F(\cdot)$ , they choose annuity price  $q$ ,

$$\max_{q \geq 0} qy(q) - \int \sum_{t=J+1}^T \frac{y(q)}{R^{t-J}} \frac{P_t(\theta)}{P_J(\theta)} dF(\theta; q) \quad (9)$$

$F(\theta; q)$  determines what fraction of each unit of total annuity obligations by insurer is to be paid to type  $\theta$ . It determines the risk is the annuity insurer's pool. In the equilibrium –which I will define shortly–  $F(\theta; q)$  is required to be consistent with individuals' demand for annuity. I assume that annuity insurers engage in Bertrand competition and therefore, they make non-positive profit.

#### 2.4.5 Competitive equilibrium

Competitive equilibrium is defined as follows.

**Definition 1** *A competitive equilibrium with asymmetric information is the sequence of consumers' allocations,  $(c_t^*(\theta), b_t^*(\theta), a^*(\theta), k_{t+1}^*(\theta))_{\theta \in \Theta}$ , annuity insurer decisions, annuity price ( $q^*$ ), anticipated distribution of payouts by insurers, ( $F^*$ ), and social security policy  $(\tau, z, SSA_{t+1})$  such that:*

1.  $(c_t^*(\theta), a^*(\theta), k_{t+1}^*(\theta))_{\theta \in \Theta}$  solves consumer's problem for all  $\theta \in \Theta$  given annuity price  $q^*$ .
2.  $q^*$  is the lowest price such that

$$q^* = \int \sum_{t=J+1}^T \frac{P_t(\theta)}{R^{t-J} P_J(\theta)} dF(\theta; q^*)$$

if  $\int a(\theta; q^*) dG_J > 0$ . Otherwise

$$q^* = \sup_{\theta} \sum_{t=J+1}^T \frac{P_t(\theta)}{P_J(\theta) R^{t-J}}$$

3. Allocations are feasible

$$\int \sum_{t=0}^T \frac{P_t(\theta)}{R^t} \left[ c_t^*(\theta) + \frac{(1 - x_{t+1}(\theta))}{R} b_t^*(\theta) \right] dG_0(\theta) = w \int \sum_{t=0}^J \frac{P_t(\theta)}{R^t} dG_0(\theta). \quad (10)$$

4.  $F^*$  is consistent with consumers' choices, i.e., for any price  $q$ , the fraction of total annuity coverage bought by individuals with type in  $Z \subseteq \Theta$  is

$$F^*(Z; q) = \frac{\int_{\theta \in Z} a^*(\theta; q) dG_J(\theta)}{\int_{\theta \in \Theta} a^*(\theta; q) dG_J(\theta)} \quad \text{and with positive mass only on } \underline{\theta} \text{ if } a^*(\theta) = 0 \forall \theta.$$

in which  $G_J(\cdot)$  is defined in equation (1).

5. Social security budget balances (equation (8)).

The equilibrium notion is similar to Bisin and Gottardi (1999, 2003) and also Dubey and Geanakoplos (2001). Bisin et al. (1998) prove that almost linear contracts emerge as the result of non-exclusivity in a moral hazard environment, and they conjecture that the same result will hold in an environment with adverse selection.<sup>21</sup> The idea behind the non-exclusivity is that the insurers cannot observe and monitor individuals' trades. People may buy multiple insurance contracts from multiple insurers. Empirical evidence suggests that in the annuity market, insurers do not attempt to use menus of prices to classify individuals based on risk characteristics, even when they can condition prices on observable characteristics that correlate with mortality (see Finkelstein and Poterba (2006) for more details).

Using zero profit condition and consistency conditions (condition 4 in equilibrium definition), we can get the equation for equilibrium price

$$q^* \int a(\theta; q^*) dG_J(\theta) = \int a(\theta; q^*) \sum_{t=J+1}^T \left( \frac{P_t(\theta)}{P_J(\theta)} \frac{1}{R^{t-J}} \right) dG_J(\theta) \quad (11)$$

### 3 Two-period Example

Deriving qualitative results in the general case is difficult. To gain insights about some properties of equilibrium prices and allocations, I study a two-period example.

The economy lasts for two periods. All individuals live through the first period. They are alive in the second period with probability  $P(\theta)$ .  $\theta$  is non-negative number, has distribution

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<sup>21</sup>Ales and Maziero (2009) consider a static Rothschild-Stiglitz environment and show that linear prices emerge as a result of non-exclusivity.

$G(\cdot)$  (with density  $g(\cdot)$ ), and indexes individuals' frailty.  $P(\cdot)$  is a decreasing function of  $\theta$ . Individuals enjoy consumption while they are alive and leave bequests when they die. Assume that there is no discounting and return on saving is one.

The timing is the following: 1) At the beginning of period 0 before any decision is made, individuals learn their  $\theta$  (therefore, they know the probability that they will be alive in the second period,  $P(\theta)$ ); 2) They make decisions about consumption, saving (which they leave as bequest if they die) and annuity. The consumer problem is

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta) (u(c_1) + v(k_2))$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

$$c_1 + k_2 \leq k_1 + a + z.$$

The goal is to establish the following results: 1) Household decision over purchase of annuity is monotone in their type. Individuals with higher  $P(\theta)$  (higher probability of survival) purchase more annuity. 2) Equilibrium prices are unfair (they are above average actuarially fair prices). The first proposition establishes these results. After these results are established, I show that increasing the social security tax increases equilibrium price of annuity.

**Proposition 1** *Annuity purchase for each mortality type,  $a(\theta; q)$ , is a monotone decreasing function of  $\theta$  (index of mortality). Furthermore, equilibrium price,  $q^*$  is higher than the average survival risk in the economy,  $q^* > \int P(\theta)dG(\theta)$ .*

**Proof.** See Appendix A. ■

This proposition highlights the effect of adverse selection in increasing the price of insurance above the actuarially fair price in this environment. Individuals with a higher probability of survival demand more annuity insurance at any price. They also survive to the second period with higher probability and therefore are more likely to claim the insurance they have purchased. Any unit of coverage that is sold to these individuals is more risky from the point of view insurers. On the other hand, individuals with a lower probability of survival are less risky for insurers since they are less likely to survive and claim the insurance coverage. However, since they are less likely to survive, they purchase less insurance (relative to high survival types). As a result, the insurers are left with a pool of claims that are more likely to be materialized than the average probability of survival in the population. The risk in each insurers pool is higher than what is implied by average risk of survival by individual agents

in the economy. Therefore, the equilibrium price of annuity is higher than the actuarially fair value of its payout. This is the essence of adverse selection in this environment.

Next, I show that increasing social security taxes leads to an increase in the price of annuity.

**Theorem 1** *Suppose  $v(\cdot) = \xi u(\cdot)$  for some constant  $\xi > 0$  and  $u(\cdot)$  is homothetic. Then, if aggregate demand for annuity is positive, equilibrium price in the annuity market is an increasing function of social security tax,  $\tau$ .*

**Proof.** See Appendix B. ■

Social security is a substitute for annuity that is purchased in the market. An increase in social security tax causes everyone to reduce their demand for annuity in the market. However, it has a larger effect on the demand for annuity by lower survival types. The reason is that increasing tax (and benefits) of social security has two effects. On one hand it substitutes annuities and therefore reduces demand for them in the market. This effect is the same for all types. On the other hand it provides annuities at cheaper rates (than it is available in the market). This generates an income effect which increases demand. But the magnitude of this income effect depends in probability of survival and it is larger for high survival types. The reason is that these types survival with higher probability and are more likely to collect the social security benefit. Therefore, the overall reduction in annuity demand is larger for low survival types than the high survival types and the profile of annuity purchase becomes more skewed towards the high survival types. As a result, increasing social security increases the risk in the annuity pool in the market. This in turn leads to a higher equilibrium price.

Although increasing social security taxes increases the price in the annuity market, its effect on welfare is not negative. In fact, as we see in Figure 1, increasing social security taxes improves ex ante welfare while increasing the equilibrium price of annuity. Higher social security tax forces more of the lower survival types to join the pool of mandatory annuity insurance and provides better insurance for higher survival types. Next, I investigate these welfare effects quantitatively in a dynamic model.

## 4 Parametric specifications

This section contains the parametric specifications of the quantitative model, as well as a description of the data and calibration procedure.



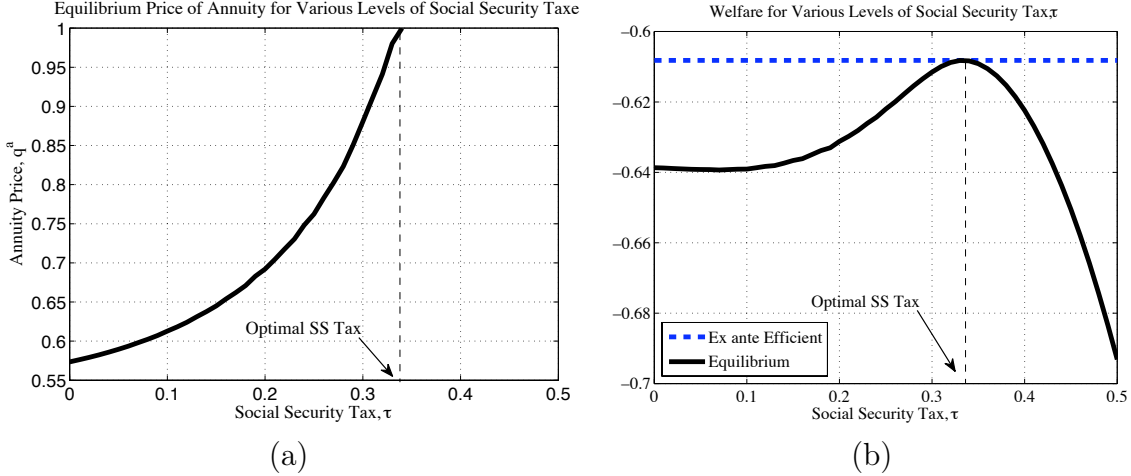


FIGURE 1: Two Period Example: panel(a) shows the equilibrium price in the annuity market for various level of social security tax, panel (b) shows individual's ex ante welfare for various level of social security tax.

**Preferences.** Individuals have CRRA utility function with coefficient of risk aversion  $\gamma$  over consumption and bequest:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \text{ and } v(b) = \xi \frac{b^{1-\gamma}}{1-\gamma}.$$

$\xi > 0$  is the weight on bequest in the utility function and is identical for every individual.<sup>22</sup> The higher  $\xi$  is, the higher is the value of bequest for individuals. I choose the parameters  $\gamma$  and  $\xi$  so that the fraction of annuitized wealth through social security and annuity purchase matches with the ones in the HRS data. More details on data and calibration are laid out in the next section.<sup>23</sup>

**Demographics.** In what follows, I model aging as a continuous time process, and later I derive the age-specific probabilities.

Individuals are indexed by their *frailty* type,  $\theta \in \mathbb{R}_+$ . Let  $h_t(\theta)$  be the force of mortality of an individual at age  $t$  with a *frailty* of  $\theta$ . The *frailty* can be modeled in many ways. Here I

<sup>22</sup>See Abel and Warshawsky (1988) for relation between this *joy of giving* parameter and altruism. This exact parametric form arises if we assume child (or spouse) has the same CRRA utility function and lives for fixed number of periods after the agents death. In this paper I make no attempt to model the exact details of inter-family/inter-generational link and leave it for future research. However, the quantitative results regarding welfare gains are robust to a wide range of values for  $\xi$ .

<sup>23</sup>In a recent paper, Lockwood (2009) has studied demand for annuities for a general class of bequest functions. He finds that personal gains from annuitization are small regardless of what functional form is assumed for bequest. Using the utility functions in his paper will alter my calibration results, but does not affect the magnitude of welfare gains by much.

follow Vaupel et al. (1979) and Manton et al. (1981) and assume the following:

$$\frac{h_t(\theta)}{h_t(\theta')} = \frac{\theta}{\theta'} \quad (12)$$

or alternatively

$$h_t(\theta) = \theta h_t.$$

An individual with frailty of 1 might be called a *standard* individual. I denote the force of mortality of standard individual by  $h_t$  (note that this is, in general, different from the average population force of mortality). The frailty index shifts the force of mortality. Furthermore, an individual's frailty does not depend on age. Therefore,  $\theta > \theta'$  means that an individual with frailty  $\theta$  has a higher likelihood of death at any age  $t$  than an individual with frailty  $\theta'$  condition that they are both alive at age  $t$ . Let  $H_t(\theta)$  be the cumulative mortality hazard; that is,

$$H_t(\theta) = \int_0^t h_s(\theta) ds = \theta \int_0^t h_s ds = \theta H_t. \quad (13)$$

Once again,  $H_t$  is the cumulative mortality hazard for standard individual. Finally, the probability that an individual of type  $\theta$  survives to age  $t$  is

$$P_t(\theta) = \exp(-H_t(\theta)) = \exp(-\theta H_t). \quad (14)$$

Therefore, if an individual of  $\theta$  has a 50 percent chance of survival to age  $t$ , an individual of type  $2\theta$  has a 25 percent chance of survival to the same age.<sup>24</sup>

Let  $g_0(\theta)$  be the density of frailty at birth; that is, at age  $t = 0$ . Also let  $\bar{P}_t$  be the overall survival probability in the population.  $\bar{P}_t$  corresponds to the data that can be calculated using a life table, and it is the fraction of all individuals (across all  $\theta$  types) who survive to age  $t$ . Therefore, the relationship between  $\bar{P}_t$  and  $P_t(\theta)$  is the following:

$$\bar{P}_t = \int_0^\infty P_t(\theta) g_0(\theta) d\theta. \quad (15)$$

Note that individuals with higher values of frailty  $\theta$  will have a higher probability of dying and are more likely to die earlier. This leads to a selection effect that changes the distribution of frailty types who are alive at each age  $t$ . The conditional density of type  $\theta$  who survive

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<sup>24</sup> $\theta$  encompasses all of the factors affecting human mortality other than age. Needless to say, it is also possible to model the heterogeneity as factors that directly scale the probability of survival. However, given the fact that survival and death probabilities are naturally bounded above, the model becomes complicated. Modeling frailty as it is done here is convenient because it allows more flexibility in choosing a parametric class of distributions for heterogeneity.

to age  $t$  can be found by applying *Bayes' rule*:

$$g_t(\theta) = \frac{P_t(\theta)g_0(\theta)}{\int_0^\infty P_t(\theta)g_0(\theta)d\theta} = \frac{P_t(\theta)g_0(\theta)}{\bar{P}_t}. \quad (16)$$

As the population ages, the distribution of frailty types who survive tilts toward the lower value of  $\theta$ . This implies that the overall average mortality hazard in the population does not correspond to individuals' mortality hazard. The relationship between average population mortality hazard,  $\bar{h}_t$ , and individual mortality hazard,  $h_t(\theta)$ , can be established by the following equation:

$$\bar{h}_t = \int_0^\infty \theta h_t g_t(\theta) d\theta = h_t \int_0^\infty \theta g_t(\theta) d\theta = h_t E[\theta|t] \quad (17)$$

in which  $E[\theta|t]$  is the mean frailty among survivors to age  $t$ . Note that since individuals with higher frailty die earlier and the distribution of types becomes skewed toward lower values of  $\theta$  as the population ages, the mean frailty in the population decreases, i.e.,  $E[\theta|t]$  is a decreasing function of  $t$ . This implies that overall, the population at each age  $t$  dies at a slower rate than individuals (unless  $g_0$  is degenerate). Consequently, knowing the overall mortality rate,  $\bar{h}_t$ , which can be computed from life tables, is not enough to find individuals' mortality hazard rate. To uncover the individuals' mortality hazard rates, we need to make further assumptions on the shape of distribution  $g_0$ .

Following Vaupel et al. (1979), I assume the initial distribution of individual frailty,  $\theta$ , is the gamma distribution with unit mean and variance  $\sigma_\theta^2 = \frac{1}{k}$ .<sup>25</sup>

$$g_0(\theta) \sim \mathcal{G}\left(\frac{1}{k}, k\right) = k^k \theta^{k-1} \frac{\exp(-k\theta)}{\Gamma(k)}.$$

Aside from its flexible shape, a useful feature of gamma distribution is that the frailty among survivors at any age  $t$  is itself a gamma distribution. This keeps the evolution of type distribution across ages analytically tractable and convenient for computation. To see

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<sup>25</sup>The general formula for gamma distribution is

$$\mathcal{G}(m, k) = \theta^{k-1} \frac{\exp(-\theta/m)}{m^k \Gamma(k)}$$

in which  $m$  and  $k$  are the scale and shape parameters. The mean and variance of this distribution are

$$\begin{aligned} \mu_\theta &= km \\ \sigma_\theta^2 &= km^2. \end{aligned}$$

Normalizing the mean to one implies that  $m = 1/k$  and  $k = \sigma_\theta^{-2}$ .

this, first replace for  $P_t(\theta)$  from equation (14) into equation (15) and the formula for gamma distribution to simplify the equation. We get the following relation between  $H_t$  and  $\bar{H}_t$

$$H_t = \frac{\exp(\sigma_\theta^2 \bar{H}_t) - 1}{\sigma_\theta^2}. \quad (18)$$

We can use this relation in the equation (16) and derive the formula for  $g_t(\theta)$ , which is itself a gamma distribution.

$$g_t(\theta) \sim \mathcal{G}\left(\frac{1}{k + H_t}, k\right) = (k + H_t)^k \theta^{k-1} \frac{\exp(-(k + H_t)\theta)}{\Gamma(k)} \quad (19)$$

Therefore, not only do we know the shape of distribution of frailty types at each age, we also know how the cumulative mortality hazard for standard type,  $H_t$ , is related to the population cumulative mortality hazard,  $\bar{H}_t = -\log(\bar{P}_t)$ .

The values for  $\bar{P}_t$  at each age can be calculated from cohort life tables. In the model I assume that a period is 5 years, that individuals enter the economy at the age of 30, and that everyone dies at or before age 100. Given the variance of the initial distribution of frailty at birth,  $\sigma_\theta^2$ , equation (18), together with our assumption about the frailty (equations (12) and (14)), can be used to uncover individuals' survival probabilities at each age  $t$ . These survival probabilities are, by construction, consistent with the life table data. That means, for any variance of initial distribution,  $\sigma_\theta^2$ , overall population survival in the model is exactly equal to survival probabilities calculated from the life table. However, we need an extra source of information to estimate the variance of initial distribution. I use the data on subjective survival probabilities in the Health and Retirement Study (HRS) to estimate  $\sigma_\theta^2$ . I describe the estimation procedure in the next section and relegate the details to the Appendix D.

An alternative approach taken by [Butt and Haberman \(2004\)](#) and [Einav et al. \(2010\)](#) is to make a parametric assumption on  $H_t$  as well as the initial distribution of  $\theta$  and estimate these parameters using only the life table data. The parametric assumption puts restriction on the variance of  $\theta$ . This can be seen by looking at equation (18). If a value for  $H_t$  is assumed, then parameter  $\sigma_\theta^2$  can be backed out (since  $\bar{H}_t$  is known from the data). Consequently, the extent of heterogeneity will depend on the details of the parametric form assumed for  $H_t$ . A novelty of the approach taken in this paper is that instead of identifying the extent of heterogeneity by functional forms, I let individuals' assessment about their mortality guide me in choosing the degree of heterogeneity. Next section describes the data and how I use this data to estimate the degree of heterogeneity in frailty.

## 5 Data and Calibration

In this section I describe the data and the calibration procedure. In short, I first choose the demographic parameters (initial distribution of frailty and time path of morality hazard) using Health and Retirement Study survey on self assessed probability of survival. I also choose social security tax and transfer to match replacement ratio in the current U.S. system. I then feed these values to the model and choose preference parameters to match the fraction of annuitized wealth in the Health and Retirement Study.

### 5.1 Individual survival probabilities

In order to estimate the parameters of the initial distribution of frailty, I use individual subjective survival probabilities from the Health and Retirement Study (HRS). The HRS is a biennial panel survey of individuals born in the years 1931-1941, along with their spouses. In 1992, when the first round was conducted, the sample was representative of the community-based U.S. population aged 51 to 61. The baseline sample contains 12,652 observations. The survey has been conducted every two years since. The HRS collects extensive information about health, cognition, economic status, work, and family relationships, as well as data on wealth and income. The particular observation on survival probabilities that I am going to use comes from the following survey question:

Using any number from 0 to 10 where 0 equals *absolutely no chance* and 10 equals *absolutely certain*, what do you think are the chances you will live to be 75 and more?<sup>26</sup>

Hurd and McGarry (1995, 2002) analyzed HRS data on subjective survival probabilities and found that the responses aggregated quite closely to the predictions of life tables and varied appropriately with known risk factors and determinants of mortality. Also, Smith et al. (2001) found that subjective survival probabilities are good predictors of actual survival and death.

Although the above-mentioned studies point to the potential usefulness of these responses as probabilities, there is a drawback. Gan et al. (2005) noticed the existence of focal points (0 or 1) in responses.<sup>27</sup> They propose a Bayesian updating procedure for recovering subjective survival probabilities (they study older respondents who were born before 1924). They assumed that individuals' true beliefs regarding their survival probability are unknown to

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<sup>26</sup>The question was repeated with the target age of 85, too. From wave 2 onward the respondents were asked to report a number between 0 to 100.

<sup>27</sup>They report that 30 percent of responses in wave 1 and 19 percent of responses in wave 2 are 0's or 1's.

the econometrician. However, the distribution of beliefs is known (which is taken as Bayesian prior). The individual reports a survival probability based on his true beliefs. The difference between his or her true beliefs and reported probabilities is modeled as measurement a error. Gan et al. (2005) use the self-reported probabilities to update the prior distribution and to obtain posterior distribution. They then apply the posterior distribution of survival probabilities to observed mortality among panel to estimate parameter values that best characterize each individual’s belief about his survival probabilities. They also provide the estimate for variance of the hazard scaling parameter. This parameter in their model corresponds to  $\theta$  in this paper (they call this parameter ‘the optimistic index’). I use Gan et al. (2005)’s procedure to estimate the subjective survival probabilities for male respondents in the first wave of HRS. The details of estimation procedure is laid out in the Appendix D.

This estimation procedure identifies the variance of the initial distribution of frailty types. Once the variance of initial distribution is known, equation (18) can be used to back out the baseline cumulative mortality hazard,  $H_t$  (this is mortality hazard of type  $\theta = 1$ ). In equation (18),  $\bar{H}_t = -\log(\bar{P}_t)$  and  $\bar{P}_t$  is the average survival probability from *Cohort Life Tables for the Social Security Area by Year of Birth and Sex* for males of 1930 birth cohort.<sup>28</sup>

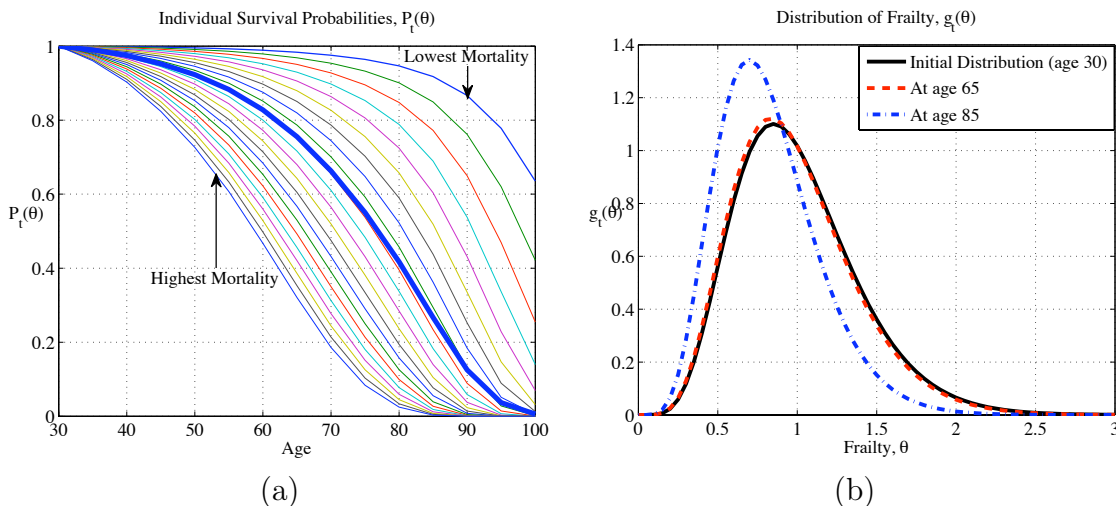


FIGURE 2: Panel (a) shows the calculated survival probabilities for each type. The thick line is the overall population survival probabilities (the life table data). Panel (b) shows the evolution of type distribution as the population ages. As argued in the text, since individuals with higher frailty ( $\theta$ ) have a higher likelihood of death at any age, the distribution of types who survive to each age  $t$  becomes skewed towards the lower value of  $\theta$  as the population ages.

Once  $H_t$  is known, equation (14) can be used to compute individuals’ survival probabilities  $P(\theta)$ . Computed survival probabilities are plotted in Figure (2). Panel (a) shows the path of survival probability to each age for various frailty type,  $\theta$ . As I argued above, the distribution

<sup>28</sup>Table 7 in Bell and Miller (2005).

of frailty types who are alive at each age evolves as individuals age. Panel (b) in Figure 2 shows the evolution of this distribution. Since frailty is not observable, interpreting the degree of heterogeneity from variance of initial distribution for frailty is not straight-forward. However, the heterogeneity in frailty implies heterogeneity in life expectancy at each age. The estimation implies a standard deviation of 4 years for life expectancy at age 30 which indicates a large degree of heterogeneity.<sup>29</sup>

In what follows, I assume that these subjective probabilities are true probabilities and represent the true risk of survival for each frailty type. I then, feed these probabilities to the model and compute equilibrium allocations. I choose the preference parameters so that these computed allocations match some moments in the HRS data on annuitized wealth. Next, I describe this data.

## 5.2 Fraction of annuitized wealth

Butrica and Mermin (2006) use the HRS data on household wealth and income to construct measures of fraction of annuitized wealth at old ages. In their measure of wealth they include financial assets, housing equity, and other assets. Financial assets include IRA balances; stock and mutual fund values; bond funds; checking, savings, money market, and certificates of deposit account balances; and trusts, less unsecured debt. Housing equity is the value of home less mortgages and home loans. Other assets include the net value of other estates; vehicles; and businesses. To construct total retirement wealth they add the present discounted value of expected future stream of payment from social security, defined benefit pensions and annuities. The results of their calculation are presented in Table 1.

The upper section of Table 1 shows the result of the calculations for married couples who have median expenditure.<sup>30</sup> The lower section contains the same information for unmarried adults. The table shows that typical adults older than 65 have a significant percentage of their retirement wealth annuitized. For unmarried adults between 70 to 79, 45 percent of the wealth is annuitized through social security. These individuals, on average, choose to hold an extra 11 percent of their wealth in the form of an annuitized payment stream through either defined benefit pension plans or annuities purchased in the private market. In calibration I treat these defined benefit pension holdings as annuity that is purchased from the employer. This is, admittedly, a strong assumption. In reality individuals have less control over a) how much they contribute towards their defined benefit coverage and, b) whether or not they can work for an employer who does or does not offer defined benefit plan. In contrast, in

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<sup>29</sup>Gong and Webb (2008) also use Gan et al. (2005)'s procedure to estimate the degree of heterogeneity in subjective survival probabilities. My results are very similar to these other papers. .

<sup>30</sup>These are averages over households with expenditure between the 45th and 55th percentile.



TABLE 1: Wealth Among Adults Ages 65+ with Median Expenditure

	Financial Assets	Housing/ Other	Social Security(1)	Pensions/ Annuities(2)	SSI (3)	Annuitized Wealth (1)+(2)+(3)
<b>Married</b>	19%	26%	41%	14%	0%	55%
<b>By age</b>						
65-69	16	23	43	17	0	60
70-79	17	28	41	14	0	55
>=80	30	33	30	7	0	37
<b>Unmarried</b>	14%	26%	48%	10%	1%	59%
<b>By age</b>						
65-69	4	26	59	11	0	70
70-79	16	29	45	10	0	55
>=80	26	25	40	7	2	49

**Note:** These are adults who collected either social security or social security disability insurance in 2002/2004 HRS. All percentages are computed as the mean ratio.

**Source:** Tables 2 and 3 in [Butrica and Mermin \(2006\)](#).

my model they are implicitly free to choose the employer with less or more generous defined benefit pension plan and these employers offer a range of options to them. Because of this flexibility the extent of adverse selection is more severe in my model. In reality, many low survival types do not have the flexibility to opt out of a defined benefit pension plan (or have various other incentives for not doing so). Therefore, they keep the annuity risk in the pool of a defined benefit pension plan low. Hence, the adverse selection, ex ante, is not as bad as it is in my model. Therefore, in my welfare calculation, I am providing a generous upper bound for the welfare gains of mandatory annuitization.

TABLE 2: Calibration Summary

Parameter	Value	Note
$\gamma$ (risk aversion)	1.47	Match fraction of social security wealth at 70
$\xi$ (weight on bequest)	0.8	Match fraction of pension wealth at 70
$\beta$ (discount factor)	$0.97^5$	
$R$ (return on savings)	$1.03^5$	
$\tau$ (social security tax rate)	0.08	Match U.S. replacement ratio (45 percent)
$\sigma_\theta^2$ (variance of $G_0(\theta)$ )	0.12	Estimated using HRS survey on subjective survival probabilities (see Appendix D)
$P_t(\theta)$ (survival probabilities)		Chosen to match life table for the 1930 birth cohort

The calibration procedure is the following. Each period is 5 years. I assume 3 percent annual real rate of return on liquid assets ( $R = 1.03^5$ ) and assume no growth so that  $\beta = \frac{1}{R}$ . I choose social security taxes  $\tau$  of 8 percent to match the average U.S. social security

replacement ratio of 45 percent using equation (8).<sup>31</sup> I then choose the coefficient of risk aversion,  $\gamma$ , and weight on bequest,  $\xi$ , in such a way that the percentage of annuitized wealth through social security and annuity matches the corresponding numbers in the lower section of Table 1 for 70-year-old unmarried adults. The summary of calibration results is presented in Table 2. Individuals' income is constant and normalized to 1 for the pre-retirement period (age 30 to 65), and afterward it is zero. Figure 3 shows the fit of the model in matching the average fraction of annuitized wealth at retirement.

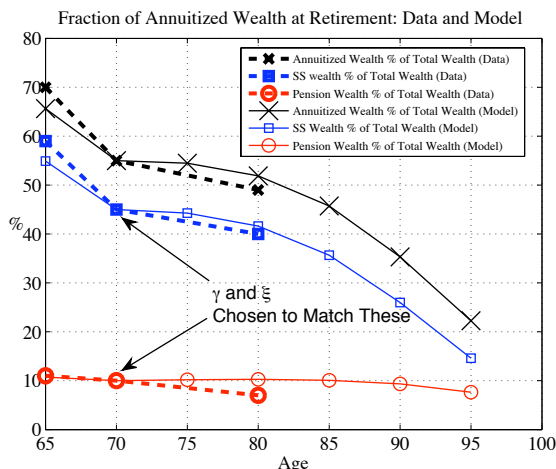


FIGURE 3: Thick dashed lines are data from the lower section of Table 1. Thin lines are data generated by model.

## 6 Findings

In this section I report the findings. First I present and describe the life cycle profile of allocations for the calibrated economy with the current U.S. social security replacement ratio. Next, I present and describe the allocations under counterfactual scenario in which there is no social security. I compare the annuitization decision and welfare in two economies and also report the welfare gains from implementing the ex ante efficient allocations. Finally, I report this welfare for various values of key parameters to check for robustness.

<sup>31</sup>The value of social security tax rate is smaller than what is usually found in the life cycle literature. The reason is that social security tax is usually calibrated to balance a pay as go budget in which the implicit return on social security is tied to the population growth and the life table. In contrast, in order to abstract from efficiency losses due to low rate of returns on social security assets, I assume returns on social security is the same as market returns (3 percent annually). This implies that lower tax rate is required to balance the budget. Given the goal of this exercise is to measure the potential benefits of social security in mitigating adverse selection, I am abstracting from other frictions and inefficiencies that may arise from particular financing scheme.

## 6.1 Model with current U.S. social security

Figure 4 shows the profile of consumption and holdings of liquid assets for different mortality types. Individuals whose frailty type is in the bottom 5 percent of initial type distribution (lowest 5 percent of mortality) start the life cycle with a low level of consumption (dashed line with  $\times$ ). These individuals expect to survive to a very old age and therefore accumulate assets when they are young up to last period before retirement, when they spend a large fraction of their accumulated wealth to purchase annuity (Figure 5). After they purchase annuity, they increase their consumption. On average, these individuals outlive the participants in the annuity market. The market price that they pay for annuity insurance is much lower than the value they get from the stream of payment (since they evaluate the payment stream by their own high probability of survival). Therefore, once they purchase annuity, the future consumption becomes cheap for them and they increase their consumption.

On the other hand, individuals with frailty type in the top 5 percent of initial type distribution expect shorter life spans and start the life cycle with higher consumption level (dashed line with  $+$ ). These individuals accumulate liquid assets at a slower rate (panel (b) in Figure 4). They also do not spend those assets on annuity purchase. Since they have a high likelihood of dying at each age, the expected value of leaving a bequest is higher for them than the expected value of future consumption. Consequently, they accumulate assets up to retirement and run down their assets afterward until they die.

The average profile of consumption (thick line in Figure 4) is the average over consumption of the individuals who are alive in each age. The average profile is almost constant up to retirement, at which point it starts to go down up to age 85. The graph shows a large increase in the profile of consumption at older ages. The reason lies in the fact that as the population gets older, high frailty types (who have low consumption) die and exit the economy. Therefore, the majority of individuals who survive to very old ages are those who have low frailty ( $\theta$ ). These individuals also have a high level of consumption at old ages. This leads to a rise in consumption profile at old ages.<sup>32</sup> Figure 5 shows the profile of annuity purchase. Individuals can only purchase annuity during the last period before retirement when they are between 60 and 65 years old.

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<sup>32</sup>This implication of the model can potentially be checked by investigating the Consumer Expenditure Survey (CEX) data across different education levels. However, one difficulty is poor data availability at the old ages.

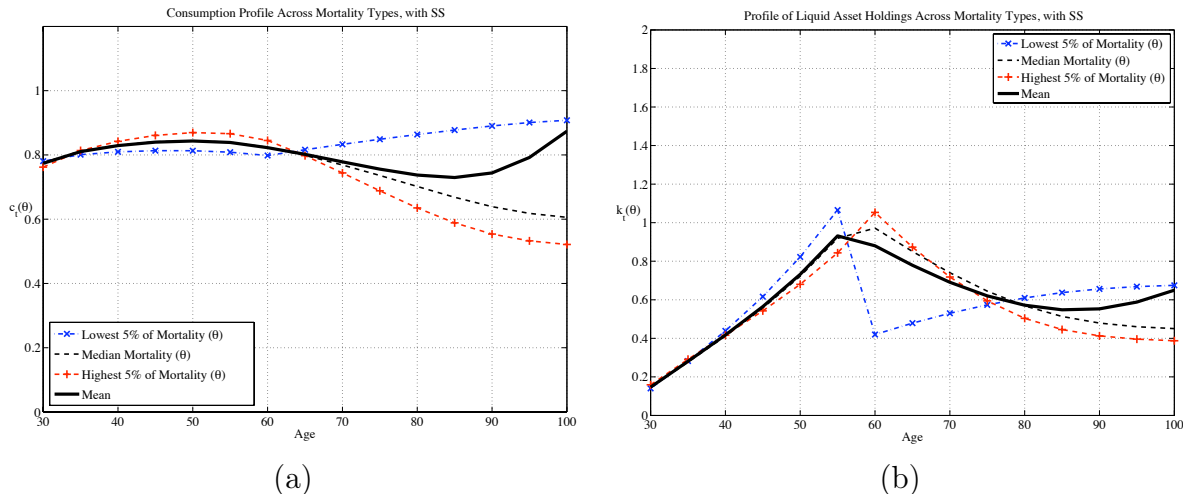


FIGURE 4: Life cycle profile of consumption (panel (a)) and holdings of liquid assets (panel (b)) across mortality types with U.S. social security replacement ratio.

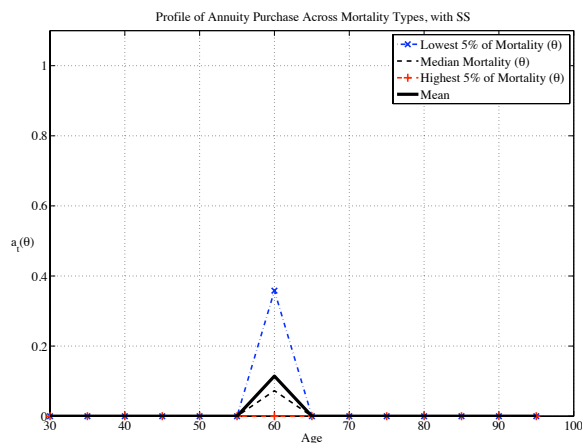


FIGURE 5: Profile of annuity purchases over life cycle across mortality types when there is social security with current US replacement ratio.

## 6.2 Model without social security

Figure 6 shows the allocations over the life cycle in the same economy but with no social security. In this economy individuals purchase all the annuity insurance they need from the market. Panel (a) shows that there is little change in the life cycle profile of low mortality types. However, high mortality types significantly reduce their consumption at old ages. They also consume slightly more in pre-retirement periods (relative to the economy with social security). The reason that this increase is not large is that although they don't have to pay social security taxes and have more income to spend, they also have to save more

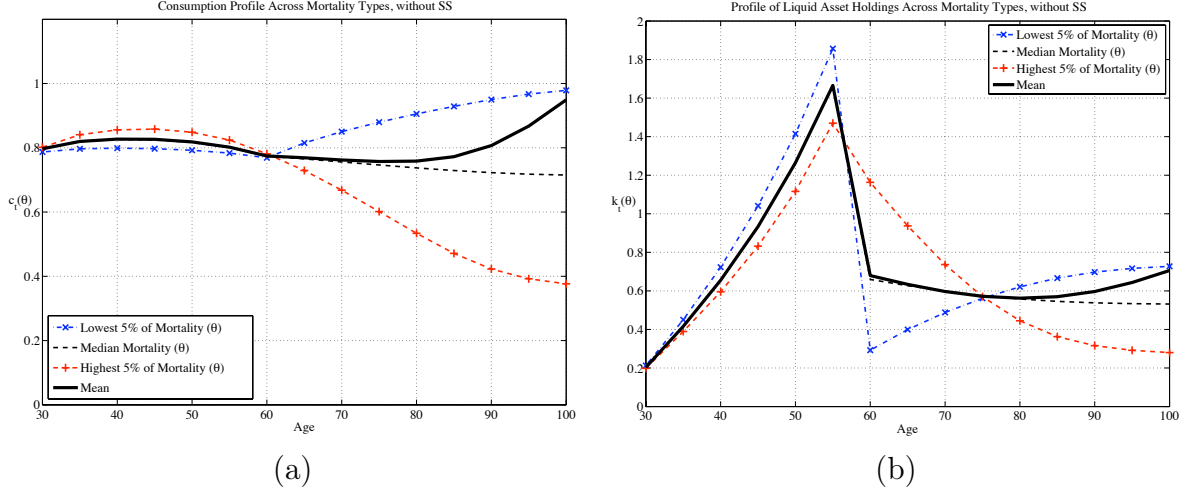


FIGURE 6: Life cycle profile of consumption (panel (a)) and holdings of liquid assets (panel (b)) across mortality types without social security

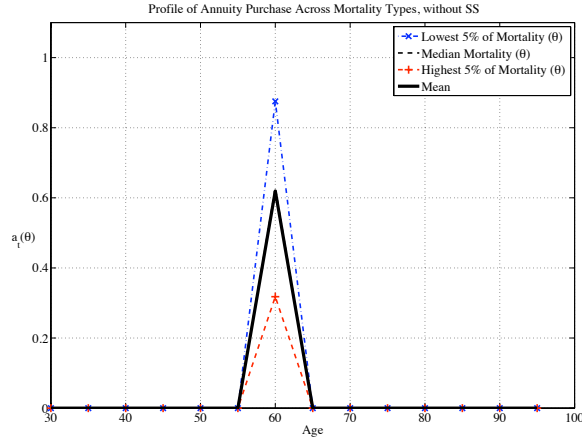


FIGURE 7: Profile of annuity purchases over the life cycle across mortality types without social security.

(relative to the case in which there is social security) to finance retirement.

Figure 7 shows that in the absence of social security, the purchase of annuity is significantly increased. Almost everyone purchases annuity between ages 60 and 65. The average annuity purchase goes up by an order of 6.

### 6.3 Comparing two environments

In this section I compare the annuitization decision of individuals in two environments. Panel (a) in Figure 8 shows the profile of annuity purchase across different types. About 60 percent of the population with lower mortality (higher survival risk) purchases annuity. This is larger than the findings of Johnson et al. (2004) who report that only 43 percent of all adults in

HRS hold defined benefit pensions or annuity in their own names (52 percent for males). Although, the model overestimates the number of annuitants, it is still doing a good job in accounting for large fraction of non-annuitization in the population.<sup>33</sup>

Because the higher mortality types do not participate in the market, the quality of risk sharing in the annuity market is low. The model predicts that the price of annuity in the market is 13 percent higher than the overall actuarially fair value of the payments (evaluated using average survival probabilities in the population from the cohort life table). This is consistent with the evidence on annuity prices in the U.S. market (see for example [Mitchell et al. \(1999\)](#)). In the absence of social security almost everyone purchases annuity in the market, even the individuals with the highest mortality types. As a result, the annuities are cheaper by about 9 percent relative to the economy with social security.<sup>34</sup>

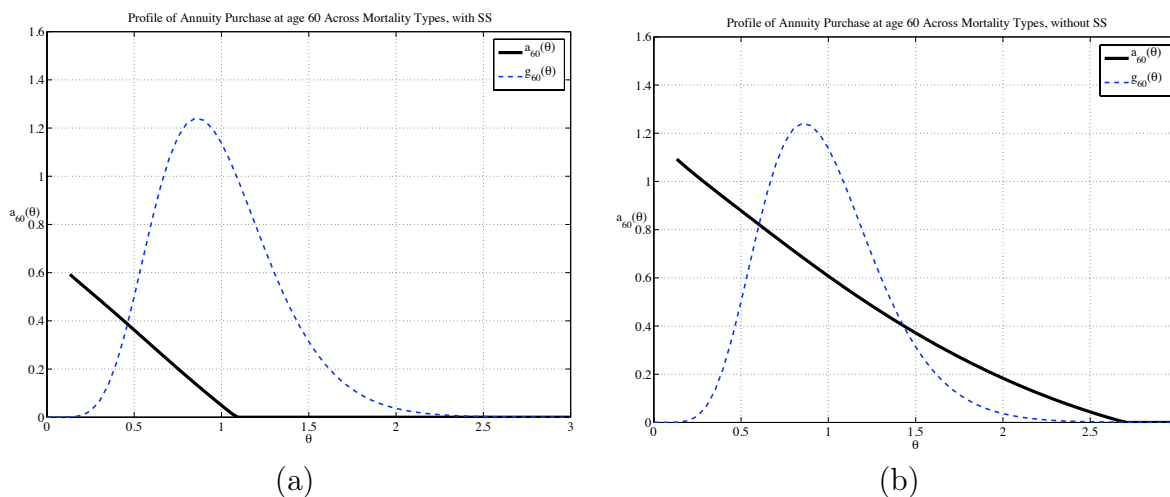


FIGURE 8: Profile of annuity purchase at age 60 across mortality types. Panel (a) shows the amount of annuity insurance individuals buy when there is social security with the current U.S. system's replacement ratio. Half of the population choose to buy zero. Panel (b) shows the annuity purchases in the same economy but with no social security.

In Figure 9, I present the same fact using the fraction of wealth annuitized at age 70. Panel (a) shows the fraction of wealth annuitized in the economy with social security. The plain dashed line is the fraction of total retirement annuitized through social security. Notice that this fraction is increasing in the frailty type ( $\theta$ ). In this economy every individual receives

<sup>33</sup>There are many explanation for why few people buy annuities in the market, including: pre-annuitized wealth through social security and defined benefit pension; adverse selection; bequest motives; precautionary saving; and restrictions on type of contracts that are offered in market. In a recent paper [Pashchenko \(2010\)](#) shows that in order to fully account for non-annuitization one needs a model that includes all these features and frictions.

<sup>34</sup>[Walliser \(2000\)](#) also finds that in the absence of social security annuity prices would be lower. Although he reports much smaller numbers (2-3 percent). This is partly due the fact that the annuity contracts that he considers are not life annuities. Instead they are one period survival contingent bonds.

the same payment from social security. However, individuals with lower mortality types expecting longer life spans accumulate more assets. As a result, social security is a smaller fraction of their total wealth at retirement, and they turn to the annuity market to purchase more annuity insurance (the dot-dashed line). On the other hand, high mortality individuals expect a shorter life, and so they accumulate fewer assets. Social security constitutes a large fraction of their retirement wealth. They have too much wealth annuitized. This further suppresses their demand for annuitized assets, and they choose to buy zero annuity in the market.

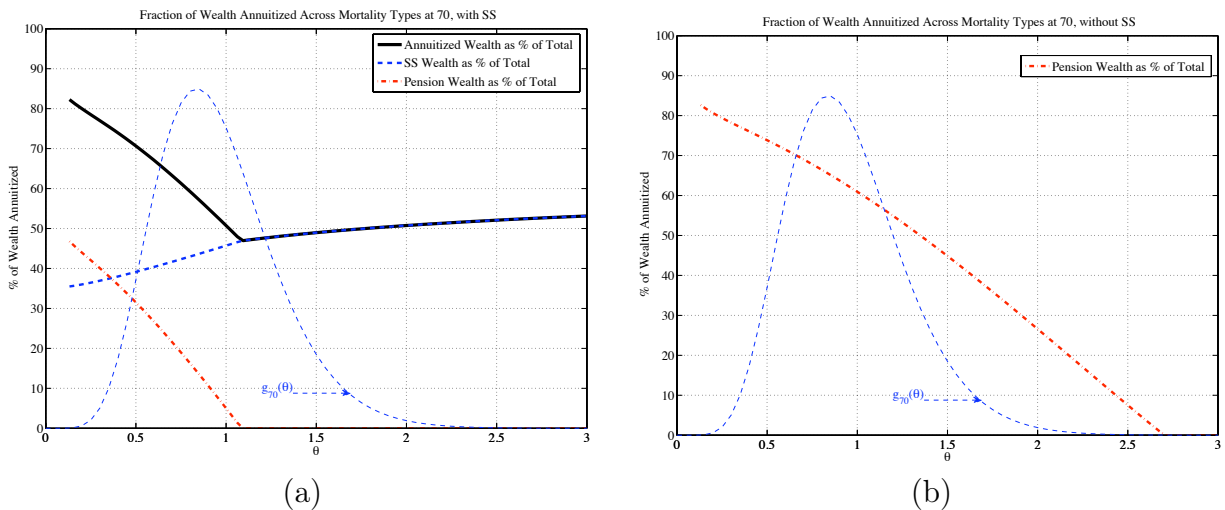


FIGURE 9: Fraction of total wealth that is annuitized. Panel (a) shows the result when there is social security with the current U.S. system replacement ratio. Panel (b) shows the result in the same economy, but without social security. The dashed line is the type distribution for those who are alive at 70. The numbers are in percentages.

Panel (b) in Figure 9 shows the fraction of wealth annuitized when there is no social security. Low mortality types hold almost the same level of annuitized wealth compared to the economy with social security. On the other hand, although higher mortality types purchase a positive amount of annuity insurance, they choose a significantly lower level of annuity coverage. When they are given the choice, they choose to annuitize a much smaller fraction of their income (compared to the fraction that is annuitized through the current U.S. social security system).

## 6.4 Welfare

The central finding of this paper are presented in Figure 10. The solid line presents the welfare gain of the current U.S. social security replacement ratio over an economy with no social security. In other words this is the percentage increase in consumption at each age that is



required to make an individual of each type  $\theta$  indifferent between living in an economy without social security and living in an economy with the current US social security replacement ratio. It is clear that individuals with high mortality suffer a loss from presence of social security since they expect to live a shorter life and do not expect to gain from mandatory annuitization. On the other hand individuals with low mortality expect to survival to older ages with higher probability. Mandatory annuitization in effect transfers resources from high mortality types to them. Therefore, they gain from current US replacement ratio. If the welfare is evaluated ex ante, all individuals prefer an economy with current US replacement ratio over an economy with no mandatory annuitization. This ex ante welfare gain is about 0.27 percent of lifetime consumption.

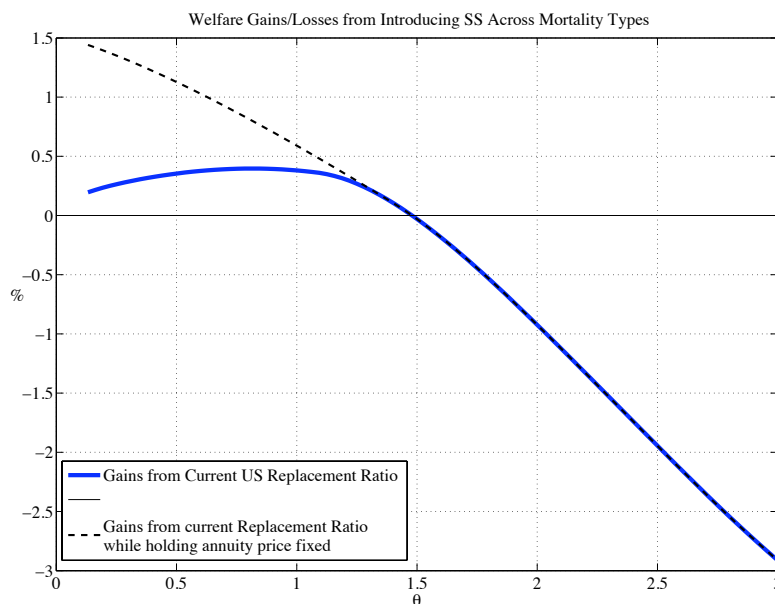


FIGURE 10: Welfare gain by individual mortality types. The thick line is the welfare gain/losses for each mortality type from living in an economy with current US replacement ratio over an economy with no social security. The dashed line is welfare gain from counterfactual experiment of introducing current US social security replacement ratio but holding the price of annuity fixed at the equilibrium it would have been in the absence of social security.

The mandatory annuitization in the social security has two effects. On one hand it transfers resources from high mortality individuals to low mortality individuals. Therefore, it is ex ante beneficial to the extent that ex post benefits to the low mortality types exceed ex post costs to high mortality types. On the other hand, mandatory annuitization in the current US social security system crowds out activities in the private annuity market by driving good risk types (high mortality types) out of the market. This causes the annuity price to be about 9 percent higher than it would have been otherwise (in the absence of social security) and has a negative impact on the welfare gain enjoyed by the low mortality

types (since they are the ones who purchase additional annuity in the market).

To see how large the welfare impact of this price effect is, I conduct the following counter-factual experiment. I introduce US social security replacement ratio, but hold the price of annuity fixed at the equilibrium level it would have been without social security. The dashed line in Figure 10 shows the welfare gains/losses from this counter-factual experiment. The gains for low mortality types are substantially reduced due to this price effect (it would have been substantially higher without it). Ex ante welfare gain from this counter-factual experiment is about 0.56 percent of life time consumption. In other words the effect of social security on annuity prices eliminates about 0.29 percent of potential ex ante welfare gains.

## 6.5 Implementing ex ante efficient allocation

In this section I describe a simple policy to implement ex ante efficient allocations in this model and study the welfare gains from implementing ex ante efficient allocation.

### Implementation

Recall from section 2.3 that ex ante efficient allocations are independent of mortality types. Everyone receives same allocations independent of their mortality types. Also, the allocations must satisfy

$$u'(c_t) = \beta R u'(c_{t+1}) = \beta R v'(b_t).$$

In which  $b_t$  is the bequest that is left if the individual dies at the end of age  $t$ . I will maintain the assumption that  $\beta R = 1$ , hence allocations are constant over time. Let  $(c^*, b^*)$  be ex ante efficient level of consumption (contingent on survival) and bequest (contingent on death). I propose the a system of social security tax rate,  $\tau^*$ , social security retirement benefit,  $z^*$  and a sequence of survival benefit  $(l_0^*, l_1^*, \dots, l_T^*)$  that pays  $l_t^*$  if the individual dies at the end of age  $t$ . Consider consumer problem of section 2.4.2 with the proposed social security policy

$$\max_{c_t, b_t, k_{t+1}, a \geq 0} \sum_{t=0}^T P_t(\theta) \beta^t [u(c_t(\theta)) + (1 - x_{t+1}(\theta)) \beta v(Rk_{t+1}(\theta) + l_t^*)]$$

subject to

$$\begin{aligned} c_t + k_{t+1} &= Rk_t + (1 - \tau^*)w & \text{for } t < J \\ c_J + k_{J+1} + qa &= Rk_J + (1 - \tau^*)w \\ c_t + k_{t+1} &= Rk_t + a + z^* & \text{for } t > J \\ k_0 &= 0 \end{aligned}$$

**Proposition 2** Suppose  $\beta R = 1$  and let  $(c^*, b^*)$  be the ex ante efficient level of consumption and bequest. A social security policy  $(\tau^*, z^*, l_t^*)$  such that

$$z^* = c^* + \left(\frac{1}{R} - 1\right) b^*$$

$$l_t^* = \begin{cases} (w(1 - \tau^*) - c^*) \sum_{j=t}^{J-1} \frac{1}{R^{j-t+1}} + \left(1 - \frac{1}{R^{J-t}}\right) b^* & \text{for } t < J \\ 0 & \text{for } t \geq J \end{cases}$$

and

$$\int \left[ \sum_{t=0}^J \frac{P_t(\theta)}{R^t} \frac{(1 - x_{t+1}(\theta))}{R} l_t^* + \sum_{t=J+1}^T \frac{P_t(\theta)}{R^t} z^* \right] dG_0(\theta) = (1 - \tau^*) w \int \sum_{t=0}^J \frac{P_t(\theta)}{R^t} dG_0(\theta)$$

implements  $(c^*, b^*)$ .

**Proof.**

The goal is to show that facing the policy  $(\tau^*, z^*, l_t^*)$ , individual will choose  $(c^*, b^*)$ . I first show that, for any type  $\theta$  individuals, if they purchase zero annuity they will choose allocation  $(c^*, b^*)$ . Then I show that given these choices, purchasing zero annuity is optimal. Consider individual's first order condition

$$P_t(\theta)u'(c_t) = P_{t+1}(\theta)\beta R u'(c_{t+1}) + P_t(\theta)(1 - x_{t+1}(\theta))\beta R v'(Rk_{t+1}(\theta) + l_t^*)$$

Notice that if individual chooses  $c_t(\theta) = c^*$  and  $k_{t+1}(\theta) = \frac{b^* - l_t^*}{R}$ , the first order condition is satisfied since a property of allocation  $(c^*, b^*)$  is that  $u'(c^*) = v'(b^*)$ . Also, it is straight forward to check that these choices satisfy household's budget constraints and government's budget constraint by construction.

Now consider the following annuity price

$$q = \sup_{\theta} \sum_{t=J+1}^T \frac{P_t(\theta)}{P_J(\theta)R^{t-J}}$$

at this prices and with  $c_t(\theta) = c^*$  we have

$$qu'(c_J(\theta)) \geq \sum_{t=J+1}^T \frac{P_t(\theta)}{P_J(\theta)} \beta^{t-J} u'(c_t(\theta))$$

and hence no one chooses to purchase annuity. ■

## Welfare gain from implementing ex ante efficient allocation

Figure 11 shows the ex post welfare gains/losses from implementing ex ante efficient allocation relative to an economy with no social security. The overall ex ante welfare gain is as large as 0.91 percent of lifetime consumption. Important to note is that the social security tax rate required to implement this allocation is 14 percent. This is much higher than calibrated tax rate of 8 percent that is required to implement the current U.S. social security replacement ratio of 0.45. One reason for this large welfare gains is that in this environment taxes do not have distortionary effect on labor supply and retirement. One way to interpret these results is that I am only reporting the benefits associated with these policies and completely abstract from the efficiency losses due to distortionary taxation. This is in particular important in this case, since implementing ex ante efficient allocation requires the tax rates to be almost doubled. In a model with distortionary taxes, this can potentially lead to a large dead weight loss.

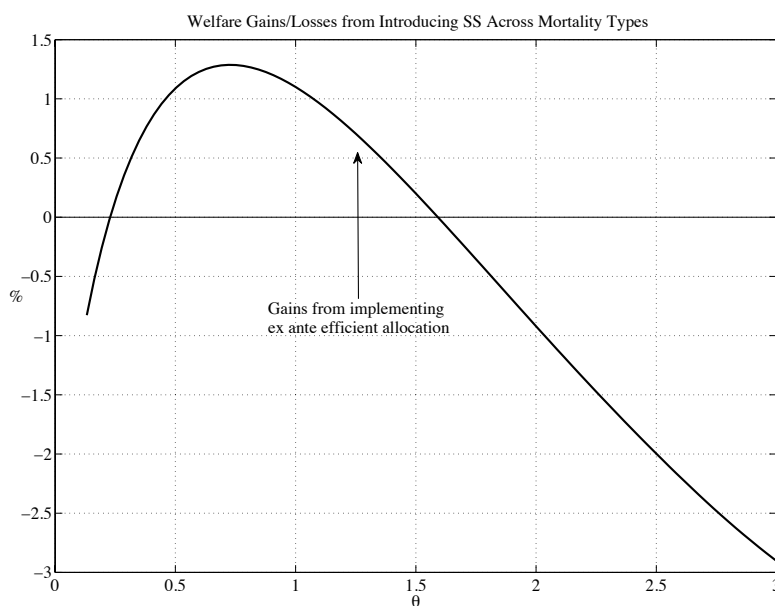


FIGURE 11: Welfare gains/losses from implementing ex ante efficient allocation for individual mortality types.

## 6.6 Comparison to autarky

In all the welfare calculations presented so far it is assumed that an annuity market exist. As another benchmark we could also repeat these calculations assuming that individuals have no access to annuity and have to rely on their non-contingent liquid asset to finance their consumption (and bequest) after retirement. The dashed line in figure 12 shows the

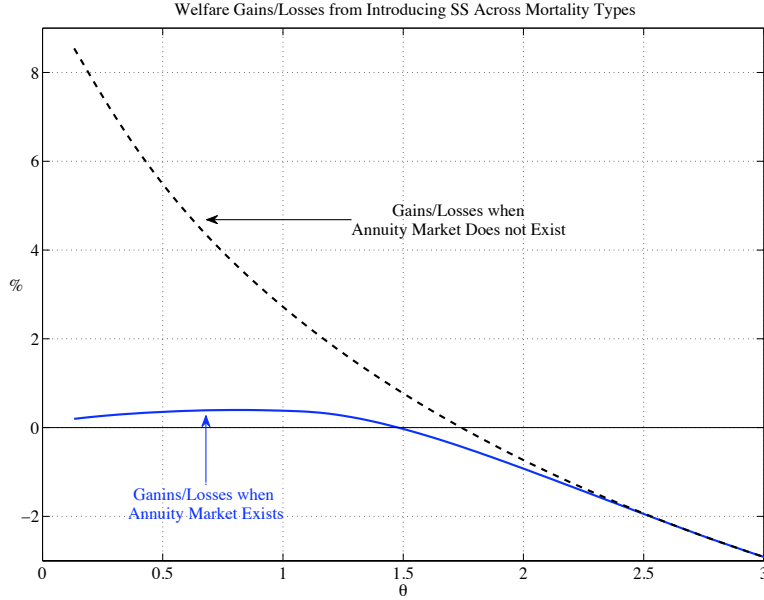


FIGURE 12: Welfare gains by individual mortality types in two environment. Dashed line shows the welfare gains and losses in an economy in which there is no annuity market. The solid line shows the welfare gains and losses in an economy in which annuity contracts are traded.

welfare gains and loses under this assumption. We see that in the absence of annuity market low mortality types make huge gains from current U.S. replacement ratio. On the other hand high mortality types losses are not that different (relative to the case in which annuity markets are present). This suggest that if we are in a world that annuity market do not exists altogether, then the welfare gains from current U.S. replacement ratios could be very large. The calculated welfare gains from the current U.S. replacement ratio in an autarky environment is about 2.84 percent in consumption which is more than ten times larger than the 0.27 percent computed under the assumption that annuity market exists. Also, the gains from implementing ex ante efficient (over autarky) is about 3.71 percent which is four times larger than 0.91 percent which was computed for the case where annuity contracts are present.

These results suggest that when evaluating welfare benefit of mandatory annuitization in social security it is important to implicitly model the existence of annuity market and consider the response of the prices in that market to different policies.

## 6.7 Sensitivity

In order to check how robust the calculation is with respect to changes in parameters, I calculate welfare gains from implementing current US social security replacement ratio over the economy with no social security for different levels of parameters. The results are

summarized in Figure 13. Panel (a) is the result of welfare calculation for various levels of risk aversion while holding the bequest parameter,  $\xi$ , at its benchmark value ( $\xi = 0.8$ ). As I argue in the appendix C, using the two-period example, increasing the risk aversion coefficient does not lead to high welfare gains. The reason is that for high levels of risk aversions, high mortality types have a greater demand for insurance. Therefore, they buy more at any price. On the other hand, low mortality types (who have an upward-sloping consumption profile) prefer a smoother and flatter profile of consumption. Therefore, they demand less annuity and increase their consumption at younger ages. These two effects result in a flatter profile of annuity purchase. This, in turn, implies that the price of annuity in the market is closer to the overall actuarially fair price. Therefore, for high levels of risk aversion, the quality of risk sharing in the market is better and there are smaller gains from implementing the ex ante efficient allocation, even though the social value of insurance is increased.

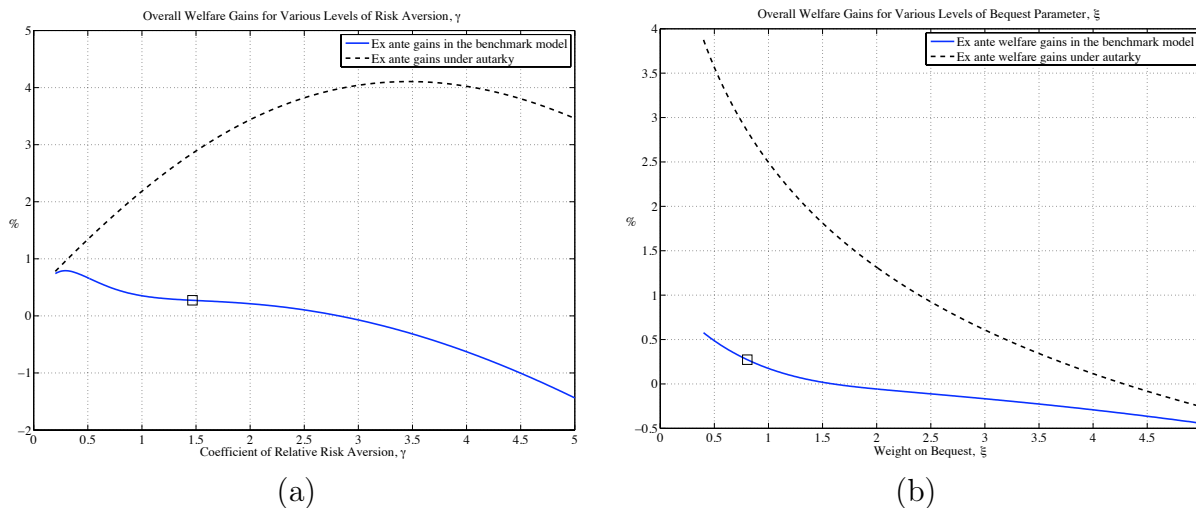


FIGURE 13: Panel (a) shows the overall welfare gain from current U.S. replacement ratio over an economy with no social security for various levels of risk aversion. Panel (b) shows the same calculation for various levels of bequest parameter. The square indicates value of ex ante welfare gain at the benchmark calibration. Dashed line show the same calculations for autarky economy.

Panel (b) in Figure 13 shows the calculated welfare gain for various levels of bequest parameter, holding the coefficient of risk aversion at its benchmark level ( $\gamma = 1.47$ ). The results of comparative static for the two-period example suggest that when starting from low levels of  $\xi$  and increasing it, the price in the annuity will rise.<sup>35</sup> The reason is that increasing weight on bequest reduces the value of annuitization (since it increases the value of death at the margin). This affects the higher mortality types more, i.e., everything else equal, their demand is reduced more relative to low mortality types for higher levels of  $\xi$ . As a result,

<sup>35</sup>Appendix C

the profile of annuity purchase across types becomes steeper and the price of annuity rises further away from the overall actuarially fair price. However, the price of annuity in the market is bounded above (by the fair price of the lowest mortality types). As the bequest parameter increases, eventually the price hits the upper bound and does not increase further. However, the value of annuity insurance in the economy reduces as we increase the weight on bequest. Therefore, even though the market outcome does not provide good risk sharing, the welfare gain eventually decreases (and becomes negative) for the high bequest parameter because the annuity insurance has little value.

Figure 13 also indicates welfare gains at various parameter values for autarky economy in which annuity markets do not exist and individuals rely on self insurance to finance retirement. Not only the welfare gains are significantly higher at all parameter values, the direction of comparative static also is reverse. Social security is more beneficial for higher risk aversion. It is also more beneficial for higher value of bequest parameter. This, once again, highlights the importance of an explicit model of annuity market in studying the welfare benefit of social security.

## 7 Conclusion

Friedman (1962) argues that governments ought to give individuals the freedom of choice to “purchase their annuities from private concerns.” However, if individuals are heterogeneous in their mortality and have private information about it, freedom of choice does not lead to the best outcome ex ante. Given choices, individuals with high mortality (who have lower risk of survival to old ages) do not participate in the annuity market and limit the risk sharing available to individuals with low mortality (who have a higher risk of survival). Mandatory annuitization can improve welfare by forcing everyone to pool their longevity risk and therefore can improve ex ante welfare. This mechanism has been proposed as one normative rationale to have a social security system.

In this paper I investigated the quantitative importance of this welfare improvement and found that it is small. The welfare cost to high mortality types from losing their freedom of choice over the desired level of annuitization almost offsets the improved longevity insurance to low mortality types in a mandatory annuitization scheme. One important reason that contributes to small welfare gains by low mortality types is the effect social security has on annuity prices.

The environment studied in this paper abstracts from any heterogeneity other than mortality types (e.g., in tastes, bequest motives, abilities, income shocks, etc.). It also abstracts



from any distortionary effects of policy on labor supply and retirement decisions. An obvious first step to extend the results is to include these features in the environment. In particular, the negative correlation between mortality and measures of individual ability (e.g., education) is well documented.<sup>36</sup> However, the implication of these empirical results for the design of an optimal retirement plan is unknown. A challenge in incorporating these heterogeneities in the model is that the ex ante efficient allocation no longer has the simple type-independent form. In fact, the two-dimensionality of private information makes the characterization of the constraint efficient allocation very difficult. In such an environment mandatory annuitization, not only has small value but may also have negative welfare effects.<sup>37</sup>

Another extension would be to include endogenous labor supply and/or the retirement decisions. The effect of mortality heterogeneity on labor supply and retirement decision is unknown. Mandatory annuitization policy may potentially have an effect on retirement decision. These effects are ignored in this paper.

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<sup>36</sup>See, for example [Deaton and Paxson \(2001\)](#).

<sup>37</sup>[Einav et al. \(2010\)](#) provide an instructive example of the negative welfare effects of mandatory annuitization when individuals are heterogeneous in mortality risk as well as their preference for bequest.

# Appendix

## A Proof of Proposition 1

**Proof.** Let's re-write the individual problem

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta)(u(c_1) + v(k_2))$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

$$c_1 + k_2 \leq k_1 + a + z.$$

To simplify the problem define

$$U(x) = \max_{c_1, k_2} u(c_1) + v(k_2)$$

subject to

$$c_1 + k_2 \leq x$$

Now the consumer problem can be written as

$$\max u(c_0) + (1 - P(\theta))v(k_1) + P(\theta)U(k_1 + a + z)$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

Let  $c_0(\theta)$ ,  $k_1(\theta)$  and  $a(\theta)$  be the solution for type  $\theta$ .<sup>38</sup> The first order conditions are

$$u'(c_0(\theta)) = P(\theta)U'(z + k_1(\theta) + a(\theta)) + (1 - P(\theta))v'(k_1(\theta)) \quad (20)$$

and

$$qu'(c_0(\theta)) \geq P(\theta)U'(z + k_1(\theta) + a(\theta)) \text{ with "=" if } a > 0. \quad (21)$$

**Claim:** Suppose  $\tilde{\theta} < \theta$ ,  $a(\theta) > 0$  and  $a(\tilde{\theta}) > 0$ , then  $a(\tilde{\theta}) > a(\theta)$ .

Note that in order for annuity demand to be positive for at least some  $\theta$  types, we must have  $q < 1$ . Also, note that if  $\tilde{\theta} < \theta$ , then  $P(\tilde{\theta}) > P(\theta)$  ( $\theta$  is an index of mortality). Replace for  $c_0 = w(1 - \tau) - k_1 - qa$  in (21)

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<sup>38</sup>Allocations depend on  $q$ , too. I ignore this for now to simplify notation.

$$\frac{qu'(w(1-\tau) - k_1(\theta) - qa(\theta))}{U'(z + k_1(\theta) + a(\theta))} = P(\theta) < P(\tilde{\theta}) = \frac{qu'(w(1-\tau) - k_1(\tilde{\theta}) - qa(\tilde{\theta}))}{U'(z + k_1(\tilde{\theta}) + a(\tilde{\theta}))} \quad (22)$$

Now add (21) and (20)

$$(1-q)u'(c_0(\theta)) = (1-P(\theta))v'(k_1(\theta))$$

and replace for  $c_0$  again

$$\frac{(1-q)u'(w(1-\tau) - k_1(\theta) - qa(\theta))}{v'(k_1(\theta))} = 1-P(\theta) > 1-P(\tilde{\theta}) = \frac{(1-q)u'(w(1-\tau) - k_1(\tilde{\theta}) - qa(\tilde{\theta}))}{v'(k_1(\tilde{\theta}))} \quad (23)$$

Suppose  $k_1(\theta) + qa(\theta) \geq k_1(\tilde{\theta}) + qa(\tilde{\theta})$ . Then in order for inequality (22) to hold it must be true that  $k_1(\theta) + a(\theta) < k_1(\tilde{\theta}) + a(\tilde{\theta})$ . This implies  $a(\tilde{\theta}) > a(\theta)$ . On the other hand if  $k_1(\theta) + qa(\theta) < k_1(\tilde{\theta}) + qa(\tilde{\theta})$ , then inequality (23) implies that  $k_1(\theta) > k_1(\tilde{\theta})$  which in turn implies that  $a(\tilde{\theta}) > a(\theta)$ .

This implies that there exists a cut off  $\theta_c$  such that  $a(\theta) > 0$  for all  $\theta < \theta_c$  and  $a(\theta) = 0$  for all  $\theta > \theta_c$ .

Next we show that equilibrium price is higher than average risk in the economy. Note that if no individual buys annuity, then by equilibrium definition insurers have the most pessimistic belief about about buyers and equilibrium price is

$$q^* = \max_{\theta} P(\theta) > \int P(\theta)dG(\theta).$$

Assume that a positive mass of purchase annuity so that aggregate annuity purchase is  $\int a(\theta)dG(\theta) > 0$ . Then zero profit condition implies that

$$q^* \int_{\underline{\theta}}^{\bar{\theta}} a(\theta)dG(\theta) - P(\theta) \int_{\underline{\theta}}^{\bar{\theta}} a(\theta)dG(\theta) = 0$$

If  $q^* > P(\theta)$  for all  $\theta \leq \theta_c$ , the claim of the proposition is established. Also,  $q^* < P(\theta)$  for all  $\theta \leq \theta_c$  cannot be an equilibrium since at such a price there demand for annuity but not insurer will supply annuity. Therefore, there must exist a type  $\theta^* < \theta_c$  such that  $q < P(\theta)$  for  $\theta < \theta^*$  and  $q^* > P(\theta)$  for  $\theta > \theta^*$ .

We can write the insurer's profit as

$$\begin{aligned}
0 &= \int_{\underline{\theta}}^{\bar{\theta}} (q^* - P(\theta))a(\theta)dG(\theta) \\
&= \int_{\underline{\theta}}^{\theta^*} (q^* - P(\theta))a(\theta)dG(\theta) + \int_{\theta^*}^{\bar{\theta}} (q^* - P(\theta))a(\theta)dG(\theta) \\
&< a(\theta^*) \int_{\underline{\theta}}^{\theta^*} (q^* - P(\theta))dG(\theta) + a(\theta^*) \int_{\theta^*}^{\bar{\theta}} (q^* - P(\theta))dG(\theta) \\
&= a(\theta^*) \int_{\underline{\theta}}^{\bar{\theta}} (q^* - P(\theta))dG(\theta)
\end{aligned}$$

since  $\theta^* < \theta_c$  we know that  $a(\theta^*) > 0$  and therefore

$$q^* > \int_{\underline{\theta}}^{\bar{\theta}} P(\theta)dG(\theta)$$

■

## B Proof of Theorem 1

The problem of the consumer is

$$\max u(c_0) + (1 - P(\theta))\xi u(k_1) + P(\theta)U(k_1 + a + z)$$

subject to

$$c_0 + k_1 + qa \leq w(1 - \tau)$$

Given that  $u(\cdot)$  is homothetic and given how  $U(\cdot)$  is defined,  $U(\cdot)$  is also homothetic. Make the following change of variable  $x = k + a + z$  and rewrite the consumer problem

$$\max u(c_0) + (1 - P(\theta))\xi u(k_1) + P(\theta)U(x)$$

subject to

$$c_0 + (1 - q)k_1 + qx \leq w(1 - \tau) + qz.$$

By homotheticity of the objective function we know that (if solution is interior)

$$\begin{aligned} c_0(\theta; q, \tau) &= \phi_c(\theta, q)(w(1 - \tau) + qz) \\ k_1(\theta; q, \tau) &= \phi_k(\theta, q)(w(1 - \tau) + qz) \\ x(\theta; q, \tau) &= \phi_x(\theta, q)(w(1 - \tau) + qz) \end{aligned}$$

for some functions  $\phi_c(\theta, q)$ ,  $\phi_k(\theta, q)$  and  $\phi_x(\theta, q)$ . Then

$$a(\theta; q, \tau) = \max[\phi_a(\theta, q)(w(1 - \tau) + qz) - z, 0]$$

for some function  $\phi_a(\theta, q) = \phi_x(\theta, q) - \phi_k(\theta, q)$ . Note that since  $a(\theta; q, \tau)$  is a monotone decreasing function of  $\theta$  we know that  $\phi_a(\theta, q)$  is positive and monotone decreasing in  $\theta$ .

Replace for  $z = \frac{\tau w}{\bar{P}}$  where  $\bar{P} = \int P(\theta)dG(\theta)$

$$a(\theta; q, \tau) = \max[\phi_a(\theta, q)(w(1 - \tau) + \frac{q}{\bar{P}}w\tau) - \frac{w\tau}{\bar{P}}, 0].$$

Note that  $a(\theta; q, \tau)$  is differentiable in  $\tau$ .

**Lemma 1** *Whenever  $a(\theta; q, \tau) > 0$ ,  $\frac{\partial a(\theta; q, \tau)}{\partial \tau} < 0$  and  $\frac{\partial a(\tilde{\theta}; q, \tau)}{\partial \tau} > \frac{\partial a(\theta; q, \tau)}{\partial \tau}$  for any  $\tilde{\theta} < \theta$ .*

**Proof.** First we show that  $q\phi(\theta, q) < 1$ . Note that from the consumer budget constraint we know

$$\phi_c(\theta, q) + (1 - q)\phi_k(\theta, q) + q\phi_x(\theta, q) = 1$$

then

$$\phi_c(\theta, q) + 1\phi_k(\theta, q) + q(\phi_x(\theta, q) - \phi_k(\theta, q)) = 1$$

and therefore

$$q\phi_a(\theta, q) = q(\phi_x(\theta, q) - \phi_k(\theta, q)) < 1.$$

If  $a(\theta; q, \tau) > 0$ ,

$$a(\theta; q, \tau) = \phi_a(\theta, q)w + \phi_a(\theta, q)(\frac{q}{\bar{P}} - 1)w\tau - \frac{w\tau}{\bar{P}}$$

and

$$\frac{\partial a(\theta; q, \tau)}{\partial \tau} = \frac{q\phi_a(\theta, q)}{\bar{P}}w - \frac{1}{\bar{P}}w - \phi_a(\theta, q)w < 0.$$

Now suppose  $\tilde{\theta} < \theta$  and suppose  $a(\theta; q, \tau) > 0$ . Then  $a(\tilde{\theta}; q, \tau) > a(\theta; q, \tau)$  implies that

$\phi_a(\tilde{\theta}, q) > \phi_a(\theta, q)$ . Therefore,

$$\frac{\partial a(\tilde{\theta}; q, \tau)}{\partial \tau} - \frac{\partial a(\theta; q, \tau)}{\partial \tau} = (\phi_a(\tilde{\theta}, q) - \phi_a(\theta, q))\left(\frac{q}{\bar{P}} - 1\right)w\tau > 0$$

■

Next, I proceed to prove the theorem.

Define function  $H(q; \tau)$  as

$$H(q; \tau) = \frac{\int P(\theta)a(\theta; q, \tau)dG(\theta)}{\int a(\theta; q, \tau)dG(\theta)}.$$

Equilibrium price is the lowest fixed point of this function.

$$q^* = \inf_q \{q | H(q; \tau) \leq q\}.$$

I will show that increase in social security tax,  $\tau$ , shifts this function upward and therefore increases the equilibrium price.

First note that for any price  $q$  we have

$$\begin{aligned} H(q; \tau) - \int P(\theta)dG(\theta) &= \frac{\int P(\theta)a(\theta; q, \tau)dG(\theta) - \int a(\theta; q, \tau)dG(\theta) \int P(\theta)dG(\theta)}{\int a(\theta; q, \tau)dG(\theta)} \\ &= \frac{Cov(P(\theta), a(\theta; q, \tau))}{\int a(\theta; q, \tau)dG(\theta)} > 0. \end{aligned}$$

in which the last inequality is true because both  $P(\theta)$  and  $a(\theta; q, \tau)$  are monotone decreasing in  $\theta$ .

Since  $a(\theta; q, \tau)$  is differentiable in  $\tau$ ,  $H(q; \tau)$  is also differentiable in  $\tau$ . For any price  $q$

$$\begin{aligned} \frac{\partial H(q; \tau)}{\partial \tau} &= \frac{\left(\int P(\theta) \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta)\right) \left(\int a(\theta; q, \tau) dG(\theta)\right) - \left(\int P(\theta) a(\theta; q, \tau) dG(\theta)\right) \left(\int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta)\right)}{\left(\int a(\theta; q, \tau) dG(\theta)\right)^2} \\ &= \frac{\int P(\theta) \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta) - H(q; \tau) \int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta)}{\int a(\theta; q, \tau) dG(\theta)} \\ &= \frac{\left(\int (P(\theta) - H(q; \tau)) dG(\theta)\right) \left(\int \frac{\partial a(\theta; q, \tau)}{\partial \tau} dG(\theta)\right) + Cov(P(\theta) - H(q; \tau), \frac{\partial a(\theta; q, \tau)}{\partial \tau})}{\int a(\theta; q, \tau) dG(\theta)} \\ &> 0 \end{aligned}$$

The inequality is true because  $\frac{\partial a(\theta; q, \tau)}{\partial \tau} \leq 0$  for all  $\theta$  and  $H(q; \tau) > \int P(\theta) dG(\theta)$ . Also, as I have shown above  $\frac{\partial a(\theta; q, \tau)}{\partial \tau}$  is monotone decreasing in  $\theta$ . This proves that  $H(q; \tau)$  is always increasing increasing with  $\tau$ . Therefore, its lowest fixed point must also increase as  $\tau$  increases.<sup>39</sup> This completes the proof of the theorem.

## C Comparative Statics Examples

In this section assume (as I do in the quantitative exercises in the main text) that utility over consumption is CRRA with coefficient  $\gamma$

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

and utility over bequest is

$$v(b) = \xi \frac{b^{1-\gamma}}{1-\gamma}.$$

I will perform comparative statics over utility parameters  $\gamma$  and  $\xi$ . In particular, I will show how the equilibrium price in the annuity market changes as we change these parameters.

### Risk Aversion

Suppose there is no demand for bequest ( $\xi = 0$ ) and there is no social security. The first-order conditions for individual  $\theta$  is <sup>40</sup>

$$q^a c_0^{-\gamma} = P(\theta) a^{-\gamma}$$

and the budget constraint

$$c_0 + q^a a = w.$$

Therefore,

$$a(q; P(\theta)) = \frac{w}{q + \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}}}.$$

Fixing price  $q$  and taking a derivative with respect to  $\gamma$ ,

$$\frac{\partial a(q; P(\theta))}{\partial \gamma} = \frac{w \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}} \log\left(\frac{q}{P(\theta)}\right)}{\gamma^2 \left(q + \left(\frac{q}{P(\theta)}\right)^{\frac{1}{\gamma}}\right)^2}.$$

<sup>39</sup>See Milgrom and Roberts (1994) for detailed arguments.

<sup>40</sup>Since annuity makes survival contingent payment it dominates the liquid assets, and therefore I omit the first-order condition with respect to  $k_1$ . For more detailed analysis of this, see Brown et al. (2005).

Notice that for individuals with  $P(\theta) < q$ , increasing  $\gamma$  leads to more demand for insurance. This is because more risk-averse individuals demand more insurance. However, notice that if  $P(\theta) > q$ , then the effect is reversed. For these individuals, the elasticity of substitution effect dominates. They favor a smoother consumption path across time, and therefore their demand for annuity reduces (instead they consume more in the first period). Overall, higher demand by high mortality types ( $P(\theta) < q$ ) and lower demand by low mortality types ( $P(\theta) > q$ ) means that the overall risk in the insurers' pool is lower. That is because there is more demand by good risk types (who are less likely to claim their coverage) and less demand by high risk types (who are more likely to claim their coverage). This leads to a reduction in the equilibrium price.

**Lemma 2** *Equilibrium annuity price is a decreasing function of coefficient of risk aversion.*

**Proof.**

Consider the zero profit condition of insurers:

$$\int (q - P(\theta)a(q; P(\theta))dG(\theta) = 0.$$

Take the derivative with respect to  $\gamma$ :

$$\int \frac{\partial q}{\partial \gamma} a(q; P(\theta))dG(\theta) + \int (q - P(\theta)) \left[ \frac{\partial q}{\partial \gamma} \frac{\partial a(q; P(\theta))}{\partial q} + \frac{\partial a(q; P(\theta))}{\partial \gamma} \right] dG(\theta) = 0.$$

Collecting terms:

$$\frac{\partial q}{\partial \gamma} = \frac{- \int (q - P(\theta)) \frac{\partial a(q; \theta)}{\partial \gamma} dG(\theta)}{\int [a(q; \theta) + (q - P(\theta)) \frac{\partial a(q; \theta)}{\partial q}] dG(\theta)}.$$

The denominator is always positive since  $\frac{\partial a(q; \theta)}{\partial q}$  and  $P(\theta)$  are both decreasing functions of  $\theta$ . The numerator is always negative since  $\frac{\partial a(q; \theta)}{\partial \gamma}$  is positive whenever  $P(\theta) < q$  and is negative whenever  $P(\theta) > q$ . Therefore, it follows that

$$\frac{\partial q}{\partial \gamma} < 0.$$

■

Figure 14 is a graphical illustration of this result. Panel (a) shows the profile of annuity purchase as a function of probability of survival,  $P(\theta)$ , for a given price. We see that at a higher level of risk aversion, the profile of annuity purchase is less steep. This leads to a less skewed anticipated distribution for payouts (panel (b)) and, in turn, the prices are going to



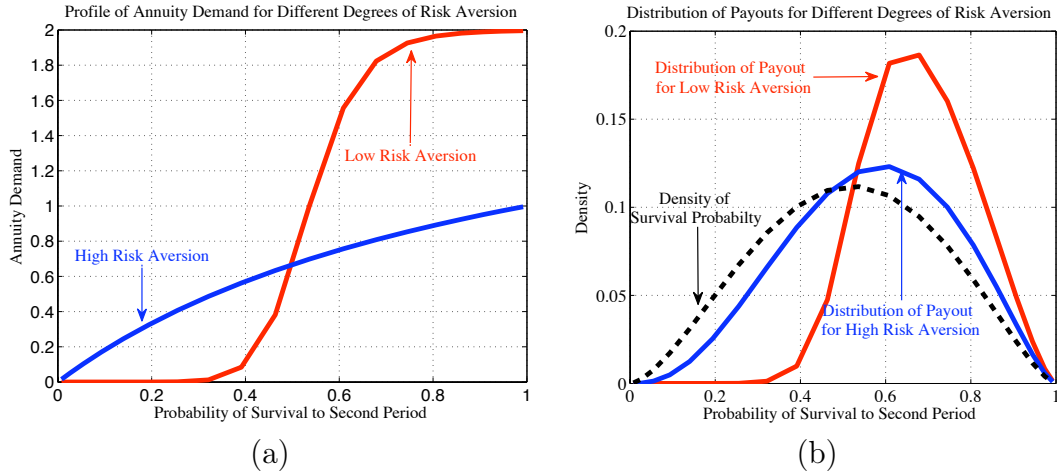


FIGURE 14: Panel (a) shows the profile of annuity purchase across survival types for a given price and for different degrees of risk aversion. Panel (b) shows the anticipated distribution of payouts for various degrees of risk aversion.

be lower. Figure 15 illustrates how the price changes with degree of risk aversion. Also, panel (b) shows how the welfare difference between equilibrium allocations and ex ante efficient allocations changes with various levels of risk aversion. At very low risk aversion, insurance is not valued and therefore the welfare difference is small. At high levels if risk aversion, the social value of insurance is higher. However, since the equilibrium price is lower, the better insurance is available in the market and the overall welfare difference is reduced.

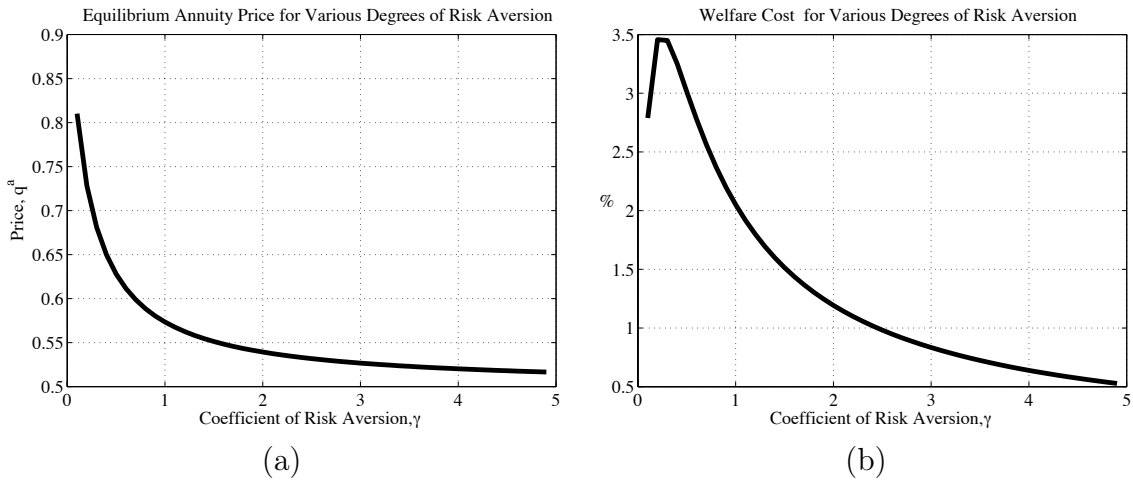


FIGURE 15: Panel (a) shows how the equilibrium price changes with the degree of risk aversion. Panel (b) shows the welfare cost as function of risk aversion.

## Bequest

Suppose that the utility function for both consumption and bequest is logarithmic ( $\gamma = 1$ ) and the weight on bequest is  $\xi$ . The demand for annuity (when it is positive) is

$$a(q; P(\theta)) = \frac{(1 + \xi) \frac{P(\theta)}{q} - \xi \frac{(1-P(\theta))}{1-q}}{1 + P(\theta) + \xi} w.$$

Once again I hold the price fixed and take the derivative with respect to  $\xi$

$$\frac{\partial a(q; P(\theta))}{\partial \xi} = \frac{wP(\theta)(P(\theta) - q)}{q(1 - q)(1 + P(\theta) + \xi)^2}.$$

At any given price, individuals with  $P(\theta) > q$  have more demand for annuity as the weight on bequest increases. On the other hand, individuals with  $P(\theta) < q$  have less demand as bequest parameters increase and their demand eventually drops to zero. Figure 16 illustrates this effect for levels of  $\xi$  at a given price.

**Lemma 3** *Equilibrium annuity price is an increasing function of bequest parameter,  $\xi$ .*

**Proof.**

Given the expression for  $\frac{\partial a(q; \theta)}{\partial \xi}$ , the proof works exactly as the proof for the previous lemma. ■

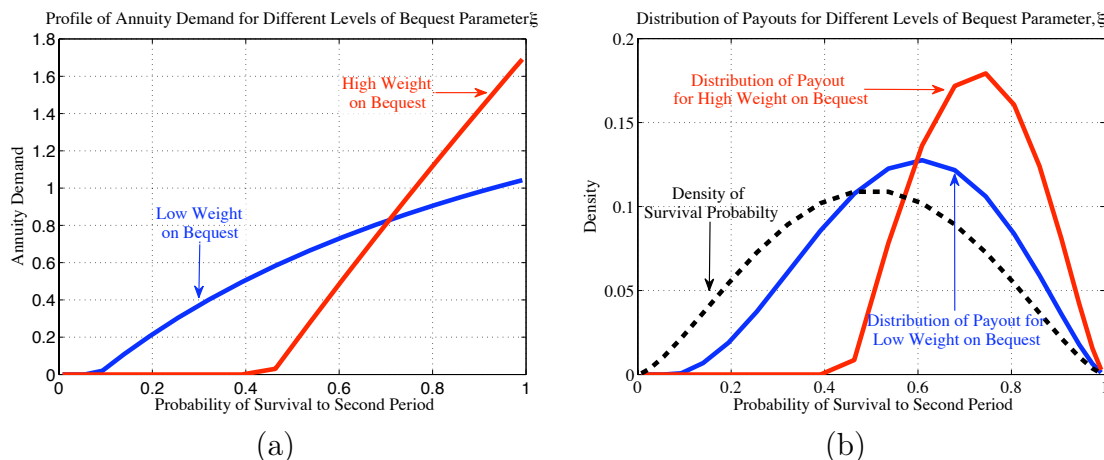


FIGURE 16: Panel (a) shows that higher weight on bequest makes the profile of annuity purchase more steep. This, in turn, leads to more skewed anticipated distribution of payouts (panel (b)) and higher annuity prices.

Although this lemma shows that increasing the weight on bequest leads to an increase in prices, the effect on welfare is unclear. Panel (b) in Figure 17 shows how the welfare changes with  $\xi$ . When  $\xi$  is very low, a small increase leads to increase in the welfare cost. This is the case when individuals of various risk types purchase positive annuity coverage. However, because the annuity profile is steeper when  $\xi$  increases, the price increases and the welfare cost goes up. But when  $\xi$  is high enough so that the price is at its highest possible value, a further increase in  $\xi$  reduces the welfare cost. It is true that at this high price, many individuals do not purchase annuity or purchase very little. But at the same time, the value of dying with positive assets is higher (because  $\xi$  is higher). Since the price cannot be above one, a further increase in  $\xi$  cannot increase the price and cannot make the market outcome worse. On the other hand, it increases the value of dying with positive liquid assets and overall welfare becomes higher. As a result, the distance between market allocation and ex ante efficient allocation goes down.

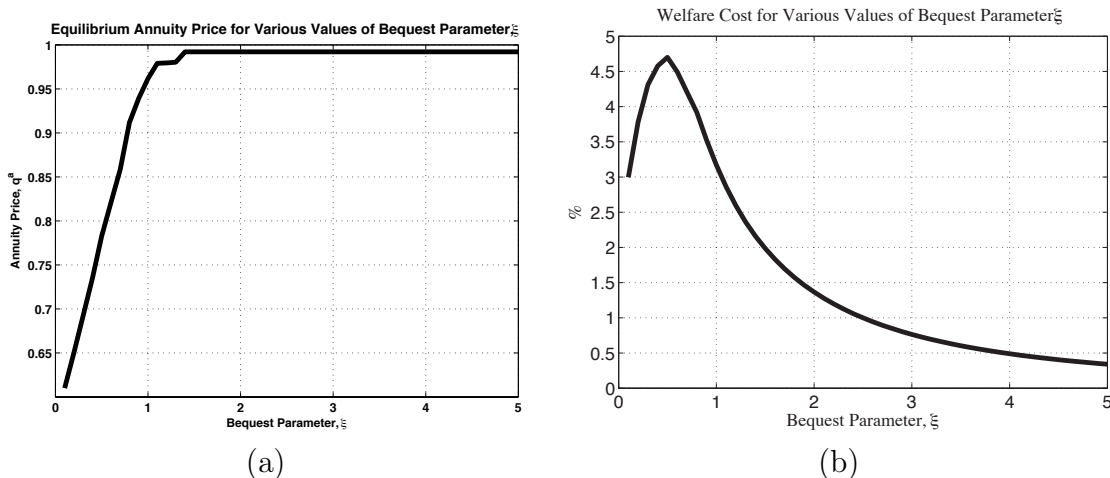


FIGURE 17: Equilibrium annuity prices (panel (a)) and welfare (panel (b)) as function of weight on bequest.

## D Estimation of Mortality Heterogeneity

In this appendix I describe the procedure for estimating the distribution of mortality heterogeneity using the HRS data on subjective survival probabilities. The data that is used is the answer to the following question:

Using any number from 0 to 10 where 0 equals *absolutely no chance* and 10 equals *absolutely certain*, what do you think are the chances you will live to be 75 and more?

It is documented by [Hurd and McGarry \(1995, 2002\)](#) that the responses aggregated quite closely to the predictions of life tables and varies appropriately with known risk factors and determinants of mortality. Therefore, these responses can potentially be used as true probabilities. However, at the individual level there are some difficulties. A large fraction of responses are either zero or one. [Gan et al. \(2005\)](#) documented that 30% of respondents in wave 1 (survey year 1992) and 19% of respondents in wave 2 (survey year 1994) responded zero or one and argue that these responses cannot represent true probabilities since the distribution of true probabilities should be continuous, and moreover, true probabilities cannot be equal zero or one. They propose a Bayesian update model to recover the true subjective survival curves for each respondent. Here I follow their procedure to estimate the heterogeneity in mortality using the data on male respondents who answered the question about survival probability in wave 1 (year 1992).

## Notations and assumptions on mortality model

Before I describe the estimation procedure, recall the notations and assumptions of the mortality model:

- $\theta$ : frailty index
- $H_t(\theta)$ : cumulative mortality hazard for individual of type  $\theta$ . I assume that

$$H_t(\theta) = \theta H_t,$$

in which  $H_t$  is independent of  $\theta$ .

- $P_t(\theta)$ : probability that an individual with type  $\theta$  survive to age  $t$  from birth. Note that by definition we have

$$P_t(\theta) = \exp(-H_t(\theta)) = \exp(-\theta H_t).$$

- $g_0(\theta)$ : density of the initial type distribution.  $g_0(\theta)$  is a gamma distribution with location parameter  $m_0$  and shape parameter  $k$ .

$$g_0(\theta) = \mathcal{G}(m_0, k) = \theta^{k-1} \frac{\exp(-\theta/m_0)}{m_0^k \Gamma(k)}.$$

Individual frailty index,  $\theta$ , shifts the mortality hazard. In what follows only the ratios of two different frailty index matters and not their levels. Therefore, without loss of generality we can normalize the mean of the initial distribution to 1. Hence,  $m_0 = 1/k$

and therefore the variance of initial distribution,  $\sigma_\theta^2$ , is  $\sigma_\theta^2 = 1/k$ . We can write the gamma distribution as

$$g_0(\theta) = \mathcal{G}\left(\frac{1}{k}, k\right) = k^k \theta^{k-1} \frac{\exp(-k\theta)}{\Gamma(k)},$$

in which  $k = 1/\sigma_\theta^2$

- $\bar{P}_t$ : average (life table) probability of survival to age  $t$  from birth.

$$\bar{P}_t = \int_0^\infty P_t(\theta) g_0(\theta) d\theta. \quad (24)$$

Also let  $\bar{H}_t = -\log(\bar{P}_t)$  denote the life table cumulative mortality hazard.

- $g_t(\theta)$ : density of types who survive to age  $t$ .

$$g_t(\theta) = \frac{g_0(\theta) P_t(\theta)}{\bar{P}_t}.$$

**Claim 2** *If initial distribution of types is gamma, then types who survive to age  $t$  also have gamma distribution.*

**Proof.**

$$\begin{aligned} g_t(\theta) &= \frac{g_0(\theta) P_t(\theta)}{\bar{P}_t} \\ &= \frac{k^k \theta^{k-1} \frac{\exp(-k\theta)}{\Gamma(k)} \exp(-\theta H_t)}{\bar{P}_t} \\ &= \frac{1}{\bar{P}_t} k^k \theta^{k-1} \frac{\exp(-\theta(H_t + k))}{\Gamma(k)} \\ &= \frac{1}{\bar{P}_t} k^k \theta^{k-1} \frac{\exp\left(-\theta / \left(\frac{1}{H_t + k}\right)\right)}{\Gamma(k)} \left(\frac{(H_t + k)}{(H_t + k)}\right)^k \\ &= \frac{1}{\bar{P}_t} \theta^{k-1} \frac{\exp\left(-\theta / \left(\frac{1}{H_t + k}\right)\right)}{\left(\frac{1}{H_t + k}\right)^k \Gamma(k)} \left(\frac{k}{H_t + k}\right)^k \end{aligned}$$

Note that

$$\int_0^\infty g_t(\theta) d\theta = 1$$

This implies

$$\bar{P}_t = \left( \frac{k}{H_t + k} \right)^k$$

or

$$-k \log((H_t + k)/k) = \log \bar{P}_t = -\bar{H}_t$$

Therefore

$$H_t = k \exp\left(\frac{\bar{H}_t}{k}\right) - 1 \quad (25)$$

and hence

$$g_t(\theta) = \theta^{k-1} \frac{\exp(-\theta/m_t)}{m_t^k \Gamma(k)},$$

in which  $m_t = \frac{1}{k} \exp(-\frac{\bar{H}_t}{k})$ . Therefore, the distribution of  $\theta$  at age  $t$  is gamma with parameters  $(m_t, k)$ .

■

Note that from equation (25) we can compute the baseline mortality hazard,  $H_t$ , if we know the variance of initial distribution,  $\sigma_\theta^2 = 1/k$ .

$$H_t = \exp(\sigma_\theta^2 \bar{H}_t) - \frac{1}{\sigma_\theta^2}.$$

Suppose the respondent  $i$  has frailty type  $\theta^i$ . Then, the true probability that this individual survives to age 75 condition on being alive at age  $t$  is

$$\frac{P_{75}(\theta^i)}{P_t(\theta^i)} = \exp(-\theta^i(H_{75} - H_t)).$$

Let  $r_t^i$  the report that this person makes about probability of survival to age 75. Following Gan et al. (2005) suppose the report is random and let  $f(r_t^i | \frac{P_{75}(\theta^i)}{P_t(\theta^i)})$  be density of distribution of reports condition on the true probability of survival being  $\frac{P_{75}(\theta^i)}{P_t(\theta^i)}$ . I follow Gan et al. (2005) and assume that conditional on  $\frac{P_{75}(\theta^i)}{P_t(\theta^i)}$ ,  $r_t^i$  has a censored normal distribution, i.e., there is a  $\mu_f^i$  and  $\sigma_f^2$  such that

$$f\left(r \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}\right) = \phi\left(\frac{r - \mu_f^i}{\sigma_f}\right) \text{ for } 0 < r < 1,$$

and

$$\begin{aligned}\Pr\left(r = 0 \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}\right) &= 1 - \Phi\left(\frac{\mu_t^i}{\sigma_f}\right), \\ \Pr\left(r = 1 \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}\right) &= 1 - \Phi\left(\frac{1 - \mu_t^i}{\sigma_f}\right),\end{aligned}$$

in which  $\phi(\cdot)$  is the standard normal p.d.f and  $\Phi(\cdot)$  is standard normal c.d.f. Furthermore, I assume that each individual makes no error on average ( $\mathbb{E}[r \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}] = \frac{P_{75}(\theta^i)}{P_t(\theta^i)}$ ). Therefore, for each  $\frac{P_{75}(\theta^i)}{P_t(\theta^i)}$  the following restriction must hold

$$\frac{P_{75}(\theta^i)}{P_t(\theta^i)} = \Pr\left(r = 1 \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}\right) + \int_0^1 r f\left(r \mid \frac{P_{75}(\theta^i)}{P_t(\theta^i)}\right) dr. \quad (26)$$

Note that this implies that the (uncensored) normal distribution has the same variance for all  $\theta$  types and all ages. However, the mean depends on type and also on the age at which the report is being made (hence  $\mu_t^i$  is indexed by both  $i$  and  $t$ ). Given the true probability of survival  $\frac{P_{75}(\theta^i)}{P_t(\theta^i)}$  and the standard deviation  $\sigma_f$ ,  $\mu_t^i$  can be computed by solving the following equation

$$\begin{aligned}\frac{P_{75}(\theta^i)}{P_t(\theta^i)} &= \left[ \Phi\left(\frac{1 - \mu}{\sigma_r}\right) + \Phi\left(\frac{\mu}{\sigma_r}\right) - 1 \right] \left( \mu_{\theta at} - \sigma_r \frac{\phi\left(\frac{1 - \mu}{\sigma_r}\right) - \phi\left(\frac{\mu}{\sigma_r}\right)}{\Phi\left(\frac{1 - \mu}{\sigma_r}\right) + \Phi\left(\frac{\mu}{\sigma_r}\right) - 1} \right) \\ &\quad + \left[ 1 - \Phi\left(\frac{1 - \mu}{\sigma_r}\right) \right].\end{aligned} \quad (27)$$

## Estimation procedure

The prior on the density of types alive at age  $t$  is given by  $g_t(\theta)$ . Once a report  $r_t^i$  is observed, we can form a posterior about the respondent  $i$ 's type, given his report. I denote this posterior density by  $\hat{g}_t(\theta \mid r_t^i)$  and

$$\hat{g}_t(\theta \mid r_t^i) = \frac{g_t(\theta) f(r_t^i \mid \frac{P_{75}(\theta)}{P_t(\theta)})}{\int_0^\infty g_t(\eta) f(r_t^i \mid \frac{P_{75}(\eta)}{P_t(\eta)}) d\eta}.$$

We can use this posterior to form expectation about respondent  $i$ 's frailty, given the report  $r_t^i$ . Let  $\hat{\theta}(r_t^i) = \int_0^\infty \theta \hat{g}_t(\theta \mid r_t^i) d\theta$  be the conditional expectation of frailty type. Using  $\hat{\theta}(r_t^i)$  and  $H_t$  we can find estimate of true probability of survival to any age  $T$  for individual

$i$  (conditional on being alive at  $t$ )

$$\hat{P}^i(t, T) = \exp(-\hat{\theta}(r_t^i)(H_T - H_t)),$$

or

$$\hat{P}^i(t, T) = \exp \left[ -\frac{\hat{\theta}(r_t^i)}{\sigma_\theta^2} \left( \exp \left( \frac{\bar{H}_T}{\sigma_\theta^2} \right) - \exp \left( \frac{\bar{H}_t}{\sigma_\theta^2} \right) \right) \right]. \quad (28)$$

HRS has observations on:

1. Self reported probability of survival to age 75 (I only use the reports in the first wave of the interview which was 1992.), which are used to construct posterior mean of frailty types.
2. Age at the time of report ( $t^i$ ).
3. Exit information and age at exit ( $T^i$ )
  - (a) For those who died we know their age at the last interview before they die.
  - (b) For those who exited the survey before dying we have age at the last interview.

Therefore, we have right censored observations and censoring points are different for each observation. HRS survey is bi-annual. If a person has died, we know that the person has died sometime between the previous wave and the current wave. So the likelihood function is

$$\mathcal{L} = \prod_{i \text{ is dead}} \left( \hat{P}^i(t^i, T^i) - \hat{P}^i(t^i, T^i + 2) \right) \prod_{i \text{ is not dead}} \hat{P}^i(t^i, T^i)$$

Then log-likelihood is

$$\log \mathcal{L} = \sum_{i \text{ is dead}} \log \left( \hat{P}^i(t^i, T^i) - \hat{P}^i(t^i, T^i + 2) \right) + \sum_{i \text{ is not dead}} \log \hat{P}^i(t^i, T^i),$$

in which  $\hat{P}^i(t^i, T^i)$  is defined by equation (28).

The parameters of  $g_0(\cdot)$  and  $f(\cdot|\cdot)$  are estimated by maximizing the above log-likelihood function. There are two parameters that we need to estimate. One is the variance of initial type distribution,  $\sigma_\theta^2$ . The other one is the variance of the (uncensored) normal distribution that is used to construct  $f(\cdot|\cdot)$ . Therefore, the likelihood function is a function of two variables,  $\sigma_\theta^2$  and  $\sigma_f^2$ . To evaluate the likelihood function we use the following procedure:

- Given a guess of  $\sigma_\theta^2$  and  $\sigma_f^2$ , and using  $\bar{H}_t$  (computed form life table) we can compute  $H_t$ .



- For each respondent  $i$  we should find the posterior density  $\hat{g}_t(\theta|r_t^i)$ . This is then used to evaluate the integral  $\int_0^\infty \theta \hat{g}_t(\theta|r_t^i) d\theta$  numerically to find estimate of frailty for respondent  $i$ , ( $\hat{\theta}(r_t^i)$ ). Therefore, we need to know the value of  $\hat{g}_t(\theta|r_t^i)$  on a finite number of points ( on a grid of  $\theta$ ). For each of these points, we can find  $\mu_t^i$  from equation (27). Once that is known,  $f(\cdot|\frac{P_{75}(\theta)}{P_t(\theta)})$  can be evaluated (for each point of the grid of  $\theta$ ). We can now compute  $\hat{\theta}(r_t^i)$ .
- Given  $\hat{\theta}(r_t^i)$  and  $H_t$  the probability of survival to any age ( $\hat{P}^i(t, T)$ ) can be computed for respondent  $i$ .
- Likelihood of the respondent  $i$ 's survival to age  $T^i$ , conditional on being alive at age  $t$ , is  $\hat{P}^i(t, T)$  . Likelihood that the respondent  $i$  dies between age  $T^i$  and  $T^i + 2$  is  $\hat{P}^i(t^i, T^i) - \hat{P}^i(t^i, T^i + 2)$ .

Once the log-likelihood is evaluated we can use standard procedure for numerical optimization to find its maximum.

TABLE 3: Estimation Results

Variance of initial frailty distribution, $\sigma_\theta^2$	Variance of reporting error, $\sigma_f^2$
0.1256 (0.07 s.e.)	0.0348 (0.0224 s.e.)

Table 3 shows the result of estimation. The number in parenthesis are standard errors. We can see from the left column that the estimated variance for frailty heterogeneity,  $\sigma_\theta^2$ , is very close to the number that Gan et al. (2005) report as “hazard scaling” in their model.

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