Finite Horizon Life-cycle Horizon Learning

Erin Cottle Hunt

Department of Economics
Lafayette College

Sept 21, 2019
What I do

Develop a new model of bounded rationality
Develop a new model of bounded rationality

- Finite Horizon Learning, *within* a Life-cycle model
What I do

Develop a new model of bounded rationality

- Finite Horizon Learning, *within* a Life-cycle model
- Simulate social security policy changes and recessions
Why it matters

• Extend adaptive learning literature into a new class of models
Why it matters

• Extend adaptive learning literature into a new class of models

• Show rational expectations equilibrium is stable under learning
Why it matters

- Extend adaptive learning literature into a new class of models
- Show rational expectations equilibrium is stable under learning
- Develop new framework for modeling announced/surprise changes
Why it matters

• Extend adaptive learning literature into a new class of models

• Show rational expectations equilibrium is stable under learning

• Develop new framework for modeling announced/surprise changes

• Learning dynamics propagate recession shock; introduce overshooting for announced policy changes
Outline

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions
Expectations

Two main approaches to modeling expectations
Two main approaches to modeling expectations

- Rational Expectations
Two main approaches to modeling expectations

- Rational Expectations
- Adaptive Learning
  - Sargent (1993), Evans and Honkapohja (2001)
Adaptive Learning Overview

Adaptive Learning

- Reduced form adaptive learning
  - Evans and Honkapohja (2001) and Bullard and Mitra (2002)
Adaptive Learning

- Reduced form adaptive learning
  - Evans and Honkapohja (2001) and Bullard and Mitra (2002)

- Micro-foundations
Adaptive Learning

- Reduced form adaptive learning
  - Evans and Honkapohja (2001) and Bullard and Mitra (2002)

- Micro-foundations
  - Euler-equation learning (Honkapohja, Mitra, and Evans (2002), Evans and Honkapohja (2006))
  - Infinite Horizon Learning (Marcet and Sargent (1989), Preston (2005), Bullard and Russell (1999))
Adaptive Learning Overview

Adaptive Learning

- Reduced form adaptive learning
  - Evans and Honkapohja (2001) and Bullard and Mitra (2002)

- Micro-foundations
  - Euler-equation learning (Honkapohja, Mitra, and Evans (2002), Evans and Honkapohja (2006))
  - Infinite Horizon Learning (Marcet and Sargent (1989), Preston (2005), Bullard and Russell (1999))
  - Finite Horizon Learning (Branch, Evans, and McGough (2013))
Finite Horizon Learning appealing assumption

- Real life forecasts are over a finite horizon
Finite Horizon Learning appealing assumption

- Real life forecasts are over a finite horizon
- Allows agents to respond to announced policy (Evans et al. (2009), Mitra and Evans (2013), Gasteiger and Zhang (2014), Caprioli (2015))
Finite Horizon Learning appealing assumption

- Real life forecasts are over a finite horizon
- Allows agents to respond to announced policy (Evans et al. (2009), Mitra and Evans (2013), Gasteiger and Zhang (2014), Caprioli (2015))
- Somewhat similar in spirt to short-planning horizon literature
<table>
<thead>
<tr>
<th>Outline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive Learning Overview</td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Expectations</td>
</tr>
<tr>
<td>Examples</td>
</tr>
<tr>
<td>Conclusion and Extensions</td>
</tr>
</tbody>
</table>
Model Summary

- Households
- Government
- Firms
- Competitive Markets
Model Summary

- **Households**
  - Work and pay taxes; retire and receive social security
  - Choose savings and consumption to maximize utility
- **Government**
- **Firms**
- **Competitive Markets**
Model Summary

- **Households**
  - Work and pay taxes; retire and receive social security
  - Choose savings and consumption to maximize utility

- **Government**
  - Taxes workers, pays retirement benefits, issues bonds

- **Firms**

- **Competitive Markets**
Model Summary

- **Households**
  - Work and pay taxes; retire and receive social security
  - Choose savings and consumption to maximize utility
- **Government**
  - Taxes workers, pays retirement benefits, issues bonds
- **Firms**
  - Turn labor and capital into output
- **Competitive Markets**
Model Summary

• Households
  • Work and pay taxes; retire and receive social security
  • Choose savings and consumption to maximize utility
• Government
  • Taxes workers, pays retirement benefits, issues bonds
• Firms
  • Turn labor and capital into output
• Competitive Markets
  • Determine prices of labor, capital, bonds, and output
Outline

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions
Expectations: Adapative Learning

New Model: Finite Horizon Life-cycle Learning

- Agents combine limited structural knowledge of macroeconomy with full knowledge of government policy
- as in Evans, Honkapohja, and Mitra (2009, 2013)
Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning
Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length $H$
Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length $H$
- Agents forecast prices using adaptive expectations
Expectations: Adaptive Learning

Finite Horizon Life-cycle Learning

- Agents look forward over a planning horizon of length $H$
- Agents forecast prices using adaptive expectations
- Decisions are optimal, conditional on expected future savings
Finite Horizon Life-cycle Learning

Agents forecast wages, \( w \), the gross interest rate \( R \) and government bonds \( b \) adaptively:

\[
\begin{align*}
  w_{t+1} &= \gamma w_t + (1 - \gamma) w_t \\
  R_{t+1} &= \gamma R_t + (1 - \gamma) R_t \\
  b_{t+1} &= \gamma b_t + (1 - \gamma) b_t
\end{align*}
\]
Finite Horizon Life-cycle Learning

Agents forecast wages, \( w \), the gross interest rate \( R \) and government bonds \( b \) adaptively:

\[
  w_{t+1}^e = \gamma w_t + (1 - \gamma)w_t^e
\]

with a gain parameter \( \gamma \in (0, 1) \).

similar equations with same gain for interest rate and bonds
Finite Horizon Life-cycle Learning

also forecast a terminal asset holding

\[ a_{t,\text{terminal}}^j = \gamma a_{t-1}^j + (1 - \gamma) a_{t-1,\text{terminal}}^j \]
for \( j = 1, \cdots, J - 1 \)

\( a_{t,\text{terminal}}^j \) is amount of assets an agent expects to hold at the end of age \( j \).

\( a_6 = 0 \); agents deplete their savings account at the end of the lifecycle
Finite Horizon Life-cycle Learning

Suppose planning horizon $H = 2$
Finite Horizon Life-cycle Learning

Suppose planning horizon $H = 2$

- Young agent chooses consumption and savings ($c^1$ and $a^1$) and plans for the next period ($c^2$ and $a^2$) according to:

$$u'(c^1_t) = \beta R^e_{t,t+1} u'(c^2_{t+1})$$

$$u'(c^2_{t+1}) = \beta R^e_{t,t+2} u'(R^e_{t,t+2} a^2_{t+1} + y^e_{t+2} - a^3_{t,\text{terminal}})$$

where $y^e_{t+2}$ is the time $t$ expectation of age $t + 2$ income, and $a^3_{t,\text{terminal}}$ is the terminal condition
Finite Horizon Life-cycle Learning

Suppose planning horizon $H = 2$

- Young agent chooses consumption and savings ($c^1$ and $a^1$) and plans for the next period ($c^2$ and $a^2$) according to:

$$u'(c^1_t) = \beta R_{t,t+1}^e u'(c^2_{t,t+1})$$

$$u'(c^2_{t,t+1}) = \beta R_{t,t+2}^e u'(R_{t,t+2}^e a_{t,t+2} + y_{t,t+2}^e - a_{t,terminal}^3)$$

- Older agents are following similar process choosing consumption and savings according to planning horizon and forecasts

where $y_{t,t+2}^e$ is the time $t$ expectation of age $t + 2$ income, and $a_{t,terminal}^3$ is the terminal condition.
Finite Horizon Life-cycle Learning

For a planning horizon of length $H$, and $J$ cohorts, there will be $J - H$ terminal conditions and $H(J - H) + \frac{H(H-1)}{2}$ household first order equations.

Together,

- the decisions of households of all ages
- asset market and bond clearing
- expectation equations

create a recursive system that governs the dynamics of the economy.

RE model is stable under Finite Horizon Life-cycle Learning
life-cycle modeled as six decade-long periods

gain parameter $\gamma = 0.93$ set to minimize welfare cost of learning relative to RE
Outline

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions
## Examples

- Social security reform
- Recession
- conclusion
Social Security Reform

- Demographic change beginning in 1980
- Social security tax increase in 2030
Social Security Reform

- **capital** $k$
- **bonds** $b$

**Rational expectations**
Social Security Reform

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions

Rational Expectations

planning horizon 5

planning horizon 4

planning horizon 3

planning horizon 2

planning horizon 1
Social Security Reform

![Graph showing Social Security Reform](image)

- **Rational Expectations**
- **Planning Horizon 5**
- **Planning Horizon 4**
- **Planning Horizon 3**
- **Planning Horizon 2**
- **Planning Horizon 1**

Other examples
Recession

Surprise, one-period recession, modeled as TPF reduction
Recession: Savings

The graph illustrates the effects of different planning horizons on savings over time for different age groups. The graph shows how savings adjust over time under various planning horizons, with the lines representing different ages and planning horizons.

- **Rational Expectations**: The line representing rational expectations is shown as a solid blue line.
- **Planning Horizons**: The planning horizons are represented by different lines for each age group:
  - Planning horizon 1 (orange)
  - Planning horizon 2 (green)
  - Planning horizon 3 (red)
  - Planning horizon 4 (light blue)
  - Planning horizon 5 (dark blue)

The graph shows how savings adjust over time under different planning horizons, with the lines representing different ages and planning horizons.
Recession: Savings

- **Age 1:**
  - Assets: $a^1$
  - Values: 2.8, 3.0, 3.2, 3.4, 3.6

- **Age 2:**
  - Assets: $a^2$
  - Values: 6.0, 6.2, 6.4, 6.6, 6.8

- **Age 3:**
  - Assets: $a^3$
  - Values: 9.6, 9.4, 9.2, 9.0, 8.8

- **Age 4:**
  - Assets: $a^4$
  - Values: 11.4, 11.2, 11.0, 10.8, 10.6

- **Age 5:**
  - Assets: $a^5$
  - Values: 6.0, 6.2, 6.4, 6.6, 6.8

Legend:
- Blue: Rational expectations
- Orange: Planning horizon 5
- Green: Planning horizon 4
- Red: Planning horizon 3
- Blue: Planning horizon 2
- Brown: Planning horizon 1
Recession: Consumption

- Age 1 consumption ($c^1$)
- Age 2 consumption ($c^2$)
- Age 3 consumption ($c^3$)
- Age 4 consumption ($c^4$)
- Age 5 consumption ($c^5$)
- Age 6 consumption ($c^6$)

Legend:
- **blue**: rational expectations
- **orange**: planning horizon 5
- **green**: planning horizon 4
- **red**: planning horizon 3
- **brown**: planning horizon 2
- **purple**: planning horizon 1
Welfare Comparison

compares the life-time utility initial steady state with life-time utility in any other period

\[ \sum_{j=1}^{J} \beta^{j-1} u(c_{ss}^j (1 + \Delta)) = \sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^j) \]

\( \Delta \) consumption equivalent variation (CEV)

\( c_{ss}^j \) is the consumption in the initial steady state

\( c_{t+j-1}^j \) is the consumption of an agent age \( j \) in time period \( t + j - 1 \)
Recession: CEV

![Graph showing CEV over cohort birth year with different planning horizons.]

- **CEV**: Change in Expectations over Various Cohorts
- **Cohort Birth Year**: Year of birth for each cohort
- **Rational Expectations**: Baseline performance
- **Planning Horizon**:
  - 1
  - 2
  - 3
  - 4
  - 5

The graph illustrates how different planning horizons affect CEV across various birth years.
Recession: CEV different gain parameters

![Graph showing CEV for different gain parameters and planning horizons.]

- **CEV**: Common Economic Variable
- **γ**: Gain Parameter
- **rational expectations**: Blue line
- **planning horizon 5**: Orange line
- **planning horizon 4**: Green line
- **planning horizon 3**: Red line
- **planning horizon 2**: Purple line
- **planning horizon 1**: Brown line

**Gain parameter selection**

**Gain parameter graph**

**Other examples**

**Conclusion**
Outline

Adaptive Learning Overview

Model

Expectations

Examples

Conclusion and Extensions
Conclusion

New modeling framework

- Embeds finite horizon learning in a lifecycle model
  - E-stability result
Conclusion

New modeling framework

- Embeds finite horizon learning in a lifecycle model
  - E-stability result
- Trade-off between planning horizon and macro cycles
Conclusion

New modeling framework
- Embeds finite horizon learning in a lifecycle model
  - E-stability result
- Trade-off between planning horizon and macro cycles
  - Longer planning horizon
    - Respond to announced policy sooner
    - Larger forecast errors $\rightarrow$ larger cycles
Conclusion

New modeling framework

• Embeds finite horizon learning in a lifecycle model
  • E-stability result

• Trade-off between planning horizon and macro cycles
  • Longer planning horizon
    • Respond to announced policy sooner
    • Larger forecast errors $\rightarrow$ larger cycles

• Trade-off in optimal gain parameter $\gamma$
Conclusion

New modeling framework

- Embeds finite horizon learning in a lifecycle model
  - E-stability result
- Trade-off between planning horizon and macro cycles
  - Longer planning horizon
    - Respond to announced policy sooner
    - Larger forecast errors $\rightarrow$ larger cycles
- Trade-off in optimal gain parameter $\gamma$
  - Small $\gamma$ optimal for temporary shocks
  - Large $\gamma$ optimal for permanent shocks
Next Steps

• Calibrate model (rather than parameterize)
• Refine examples (add others?)
• Submit paper!
Extensions

Finite Horizon Life-cycle Learning

- Great Recession and fiscal policy
- Unfunded liabilities and explosive debt
- Optimal gain parameter or planning horizon
- Euler-equation learning in life-cycle model
The end

Thank you!
Contribution

- Adaptive Learning
Contribution

• Adaptive Learning
  • New model of Finite Horizon Life-cycle Learning
Contribution

• Adaptive Learning
  • New model of Finite Horizon Life-cycle Learning
  • Effects of anticipated policy
Demographics

• Agents live for $J$ periods and work the first $T$ periods of life
• Population grows at rate $n_t$
• Demographic change modeled as a one-time reduction in $n_t$
Model details: Household Problem

Choose savings $a^j$ (consumption $c^j$) for each age $j = 1, \cdots, J$

$$
\max_{a^j_{t+j-1}} \mathbb{E}_t^* \sum_{j=1}^{J} \beta^{j-1} u(c^j_{t+j-1})
$$

$\mathbb{E}_t^*$: time $t$ expectation, $\ast$ indicates not necessarily rational. $\beta < 1$: discount factor.
Choose savings $a^j$ (consumption $c^j$) for each age $j = 1, \ldots, J$

$$
\max_{a^j_{t+j-1}} E_t^* \sum_{j=1}^{J} \beta^{j-1} u(c^j_{t+j-1})
$$

$$
c^j_{t+j-1} + a^j_{t+j-1} \leq R_{t+j-1} a^j_{t+j-2} + y^j_{t+j-1}
$$

$E_t^*$: time $t$ expectation, $*$ indicates not necessarily rational. $\beta < 1$: discount factor.

$R$ gross interest rate. $y^j$ (age specific):
gross labor income $((1 - \tau)w$, with tax rate $\tau$ and wage $w$)
or social security benefit ($z$)
Model details: Government

- Payroll tax: $\tau_t$
- Social Security Benefits: $z^j_t = \phi_t w_{t+T-j}$

$\phi$: benefit replacement rate. $w_{t+T-j}$ wage at time of retirement.
Model details: Government

- Payroll tax: $\tau_t$
- Social Security Benefits: $z^j_t = \phi_t w_{t+T-j}$

Government Debt equation:

$$B_{t+1} = R_t B_t + \sum_{j=T}^{J} N_{t+1-j} \phi_t w_{t+T-j} - \sum_{j=1}^{T-1} N_{t+1-j} \tau_t w_t$$

$\phi$: benefit replacement rate. $w_{t+T-j}$ wage at time of retirement.

$B$: total government bonds. $R_t$: gross interest rate, $N_t$: number of young at time $t$, $T$ retirement age.
Model details: Government

\[ \tau_t = \tau^0_t + \tau^1_t \left( \frac{B_t}{H_t} \right) \]

- \( \tau_t \) payroll tax rate
- \( \tau^0_t \) base tax rate (e.g. 10%)
- \( \tau^1_t \) Leeper tax rate (responds to government debt)
- \( B_t \) government debt, \( H_t \) working population
Rational Expectations Equilibrium

Definition

Given initial conditions $k_0$, $b_0$, $a_{-1}^1$, $a_{-1}^{J-1}$, and an initial population $\sum_{j=1}^J (1 + n)^{1-j} N_0$ (where $N_0$ initial cohort of young), a competitive equilibrium is a sequences of functions for the household savings $\{a_{1t}, a_{2t}, \ldots, a_{Jt}\}_{t=0}^\infty$, production plans for the firm, $\{k_t\}_{t=1}^\infty$, government bonds $\{b_t\}_{t=1}^\infty$, factor prices $\{R_t, w_t\}_{t=0}^\infty$, and government policy variables $\{\tau_t^0, \tau_t^1, \phi_t\}_{t=0}^\infty$, that satisfy the following conditions:

1. Given factor prices and government policy variables, individuals’ decisions solve the household optimization problem

2. Factor prices are derived competitively

3. All markets clear
Rational Expectations Equilibrium

- Households

\[ (R_t a_{t-1}^j + y_t^j - a_t^j)^{-\sigma} = \beta E_t[R_{t+1}(R_{t+1} a_t^j + y_{t+1}^j - a_{t+1}^j)^{-\sigma}] \]

for \( j = 1, \ldots, J - 1 \)

- Asset market

\[ (k_{t+1} + b_{t+1})(1 + n_t) = \frac{\sum_{j=1}^J N_{t+1-j} a_t^j}{H_t} \]

- Government Debt

\[ (1+n_t)b_{t+1} = R_t b_t + \frac{\sum_{j=1}^T N_{t+1-j} \phi_t w_{t+T-j}}{H_t} - (\tau_t^0 + \tau_t^1 (B_t / H_t)) w_t \]
Model details: Saddle-node bifurcation

Zero, one, or two steady states are possible in the model

- Calibrated to have two steady states
- Parameter change that increases the endogenous social security deficit, drives the steady states closer together
- At a critical value of the relevant parameter, only one steady state exists
- Beyond that, no steady states exist

Numerical analysis (Laitner 1990) of linearized system confirms the high-capital steady state is determinate, the low-capital steady state is explosive
Model details: More Stability

Three predetermined variables in the model \((k, b, \text{and } a^{J-1})\) and J-2 free variables \((a^1, \ldots, a^{J-2})\)

Let \(\lambda_i\) indicate an eigenvalue of the linearized system

- **Determinate** \(\lambda_i < 1\) for \(i = 1, 2, 3\); the remaining \(J - 2\) eigs \(\lambda_i > 1\)
- **Indeterminate** \(\lambda_i < 1\) for more than three, the remaining \(\lambda_i > 1\)
- ** Explosive** \(\lambda_i > 1\) for more than \(J - 2\) eigs

Note, complex eigs are possible, consider modulus
Model details: E-stability

- Given constant (potentially incorrect) expectations $p^e = (R^e, w^e, b^e, a_{terminal}^j)^{'}$, the learning dynamics of the FHL model asymptotically converge to $p = (R, w, b, a^j)^{'}$.
Model details: E-stability

- Given constant (potentially incorrect) expectations $p^e = (R^e, w^e, b^e, a^{j, e}_{\text{terminal}})'$, the learning dynamics of the FHL model asymptotically converge to $p = (R, w, b, a^j)'$

$$T : \mathbb{R}^{J-H+3} \rightarrow \mathbb{R}^{J-H+3}$$
Model details: E-stability

- Given constant (potentially incorrect) expectations $p^e = (R^e, w^e, b^e, a^i_{\text{terminal}})'$, the learning dynamics of the FHL model asymptotically converge to $p = (R, w, b, a)'$

$$
T : \mathbb{R}^{J-H+3} \rightarrow \mathbb{R}^{J-H+3}
$$

- A fixed point of the $T$ map is E-stable if it locally stable under the ODE

$$
\frac{dp}{d\tau} = T(p) - p
$$
Model details: E-stability

• Given constant (potentially incorrect) expectations \( p^e = (R^e, w^e, b^e, a^j_{terminal})' \), the learning dynamics of the FHL model asymptotically converge to \( p = (R, w, b, a^j)' \)

\[ T : \mathbb{R}^{J-H+3} \rightarrow \mathbb{R}^{J-H+3} \]

• a fixed point of the \( T \) map is E-stable if it locally stable under the ODE

\[ \frac{dp}{d\tau} = T(p) - p \]

• E-stability requires the real parts the eigenvalues of the derivative matrix \( dT < 1 \)

  • Numerically verified all determinate steady states in the paper are E-stable under FHL learning (at all horizons)
Motivation: short planning horizon

<table>
<thead>
<tr>
<th>Time horizon</th>
<th>Fraction of Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Next few months</td>
<td>0.18</td>
</tr>
<tr>
<td>Next year</td>
<td>0.12</td>
</tr>
<tr>
<td>Next few years</td>
<td>0.27</td>
</tr>
<tr>
<td>Next 5-10 years</td>
<td>0.31</td>
</tr>
<tr>
<td>Longer than 10 years</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Table:** Fraction of HRS survey respondents that selected each time horizon in response to the question “in planning your family’s saving and spending, which time period is most important to you?” Table reports mean across waves 1, 4, 5, 6, 7, 8, 11, and 12.

**Note:** In waves 6, 11, and 12 only respondents younger than 65 were asked this question. In all other waves, the full panel of respondents were asked about their financial planning horizon.
Choice of gain parameter

• Compute consumption of a single rational agent in the learning model
Choice of gain parameter

- Compute consumption of a single rational agent in the learning model
- Choose gain parameter that minimizes the welfare cost to learning agent of not using rational expectations to forecast
Choice of gain parameter

• Compute consumption of a single rational agent in the learning model

• Choose gain parameter that minimizes the welfare cost to learning agent of not using rational expectations to forecast

• Optimal gain parameter near $\gamma = 0.93$
### Choice of gain parameter

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Tax increase</th>
<th>Benefit Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-2.56%</td>
<td>-0.51%</td>
</tr>
<tr>
<td>0.2</td>
<td>-1.56%</td>
<td>-0.43%</td>
</tr>
<tr>
<td>0.3</td>
<td>-1.11%</td>
<td>-0.44%</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.84%</td>
<td>-0.41%</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.64%</td>
<td>-0.34%</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.47%</td>
<td>-0.27%</td>
</tr>
<tr>
<td>0.7</td>
<td>-0.39%</td>
<td>-0.23%</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.30%</td>
<td>-0.18%</td>
</tr>
<tr>
<td><strong>0.9</strong></td>
<td><strong>-0.28%</strong></td>
<td><strong>-0.18%</strong></td>
</tr>
<tr>
<td>1</td>
<td>-0.33%</td>
<td>-0.22 %</td>
</tr>
</tbody>
</table>

- Compares the consumption of a single rational agent (in each cohort) living in a world with life-cycle horizon learners.
- Learning gain parameter $\gamma$ chosen to minimize this cost.
Choice of gain parameter

Compute consumption of a single rational agent in the learning model

this experiment is the announced tax increase
Choice of gain parameter

Finite-Horizon Life-cycle Example capital and bond paths: demographic shock in 1980, tax increase in 2030

Rational
Learn $\gamma=0.2$
Learn $\gamma=0.4$
Learn $\gamma=0.6$
Learn $\gamma=0.9$
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$ number of periods</td>
<td>6</td>
</tr>
<tr>
<td>$T$ retirement date</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$ Capital share of income</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\beta$ Discount factor</td>
<td>$0.995^{10}$</td>
</tr>
<tr>
<td>$\sigma$ Inverse elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$ Depreciation</td>
<td>$1 - (1 - 0.10)^{10}$</td>
</tr>
<tr>
<td>$A$ TFP factor</td>
<td>10</td>
</tr>
</tbody>
</table>
Population growth rate $n$ is calibrated to match the projected ratio of social security beneficiaries to retirees.
References: learning

• Branch, Evans, McGough in *Macroeconomics at the Service of Public Policy* (2013)
References: Anticipated Fiscal Policy

- Evans et al., *Journal of Monetary Economics* (2009)